Mathematics

Draft Proposed Mathematics Standards Revisions December 2017

*Please note: These are screen shots of the October-December, 2017 online feedback system window for reference only. These are not the final proposed revisions.
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Review the Prepared Graduate Statements

Mathematics

Instructions

In this section, you have the opportunity to provide feedback on the content of the Prepared Graduate Statements (PGS).

On the next page, as you review the grade level expectations, you will be able to provide feedback on the alignment of the PGS with the given grade level expectation.

To leave feedback, click on the comment icon ( ) next to any item. You can then offer feedback and comments.

Once you save your feedback, the icon will change color and show as a checkbox ( ) so you can keep track of your progress.

About Prepared Graduate Statements (PGS)

All of Colorado’s Academic Standards were designed “backwards” from Prepared Graduate Statements. These statements were formerly known as Prepared Graduate Competencies but have been changed to reduce confusion with competency-based learning systems of instruction and assessment practices. The PGS identify the concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting for each content area.

Each grade level expectation of the Colorado Academic Standards aligns to one or more of the PGS.

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.
Mathematics

Grade Level: Preschool
Standard: 1. Number and Quantity

Instructions

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Common Core Information

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Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
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Preschool Learning and Development Expectation:

PCCA. Counting and Cardinality: Know number names and the count sequence.

Indicators of Progress:

By the end of the preschool experience (approximately 60 months/5 years old) students may

a. Count verbally or sign to at least 20 by ones. (similar to ELDG 3-5 1.2)

Examples of High Quality Teaching and Learning Experiences:

Supportive Teaching Practices/Adults May:

1. Count and use numbers as you play together.
2. Take advantage of every opportunity to count with children in a practical and authentic setting.

Examples of Learning/Children May:

1. Read stories, sing songs, and act out poems and finger plays that involve counting, numerals, and shapes.
2. Count their toys, books, pieces of clothing, or simply practice saying a sequence of number words.

Coherence Connections:

1. Between 36-60 months, children say or sign some number words in sequence, starting with one, and understand that counting words are separate words, such as “one,” “two,” “three,” versus “one/wo/three.”
2. In Kindergarten, students count to 100 by ones and tens and count forward from a given number.
3. This GLE is major work of Preschool and is part of several progressions of learning: (a) from saying the counting words to counting out objects and (b) from spoken number words to written base-ten numerals to base-ten understanding.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Preschool Learning and Development Expectation:

P.C.C.B. Counting and Cardinality. Recognize the number of objects in a small set.

Indicators of Progress:

By the end of the preschool experience (approximately 60 months/5 years old) students may

a. Instantly recognize, without counting, small quantities of up to five objects and say or sign the number. (similar to ELDG 3-5 1.1, 1.4)

Examples of High Quality Teaching and Learning Experiences:

Supportive Teaching Practices/Adults May:

1. Hold five or fewer objects in a closed hand, then open it briefly for the child, close it again, and ask, “How many did you see?”

2. Quickly show children a card with five or fewer dots, then hide it and ask who can say how many dots they saw.

Examples of Learning/Children May:

1. Hold three counters in hand and state, “I have three” without counting.

2. Find fewer objects or objects in patterns (like two rows of 2 to make four) easier to subitize.

Coherence Connections:

1. Between 36-60 months, children develop an understanding of what whole numbers mean and become increasingly able to quickly recognize the number of objects in a small set (known as subitizing).

2. In Kindergarten, students count to determine the number of objects.

3. This GLE supports the major work of Preschool by developing an understanding of counting and cardinality.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Preschool Learning and Development Expectation:

P.C.C.C. Counting and Cardinality: Understand the relationship between numbers and quantities.

Indicators of Progress:

By the end of the preschool experience (approximately 60 months/ 5 years old) students may

a. Say or sign the number names in order when counting, pairing one number word that corresponds with one object, up to at least 10. (similar to ELDG 3-5 1.3, 1.4)

b. Count and answer "How many?" questions for approximately 10 objects. (similar to ELDG 3-5 1.4)

c. Accurately count as many as five objects in a scattered configuration. (similar to ELDG 3-5 1.4)

d. Understand that each successive number name refers to a quantity that is one larger. (similar to ELDG 3-5 1.2)

e. Use the number name of the last object counted to represent the number of objects in a set. (similar to ELDG 3-5 1.5)

Examples of High Quality Teaching and Learning Experiences:

Supportive Teaching Practices/Adults May:

1. Play age-appropriate games that involve counting spaces or objects.

2. Count to five from thumb to pinky on an open hand, then close the hand except for the pinky and ask, "How many fingers are still showing?" to see if a child answers one or five.

Examples of Learning/Children May:

1. Match a group of 1 to 10 objects with written and spoken numbers.

2. Play simple games like Candy Land™ that match numbers to a movement of tokens.

Coherence Connections:

1. Between 36-60 months, children coordinate verbal counting with objects by pointing at each object for each number word (known as one-to-one correspondence) and develop an understanding that the last number in the sequence represents how many in the group (known as cardinality).

2. In kindergarten, students count to determine the number of objects using one-to-one correspondence and cardinality for up to 20 objects in a line or 10 scattered objects.

3. This GLE is major work of Preschool and key to several progressions of learning: (a) from saying the counting words to counting out objects and (b) from counting to counting on.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Preschool Learning and Development Expectation:

P.CC.D. Counting and Cardinality: Compare numbers.

Indicators of Progress:

By the end of the preschool experience (approximately 60 months/ 5 years old) students may

a. Compare quantity in two sets of up to five objects and describe the comparison with terms such as more, less, greater than, fewer, or equal to (similar to ELDG 3-5.2.1)

b. Identify and use numbers related to order or position from first to tenth. (similar to ELDG 3-5.1.2, 5.4, 5.5)

Examples of High Quality Teaching and Learning Experiences:

Supportive Teaching Practices/Adults May:

1. Have children group and order materials when cleaning up.

2. Use vocabulary including more than, less than, and equal to to describe quantities.

3. Provide opportunities for children to count, group, and order objects and materials.

Examples of Learning/Children May:

1. Count, group, and sort objects and materials.

2. Be able to express a preference for greater numbers of things (such as candy or toys) when comparing groups of different sizes.

Coherence Connections:

1. Between 36-60 months, children begin to count and compare same-size objects (with adult assistance) and begin to understand that the number of objects is independent of the size of the objects.

2. In Kindergarten, students identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group for up to 10 objects. Students also compare two numbers between 1 and 10 presented as written numerals.

3. This GLE is major work of Preschool and key to several progressions of learning (a) from counting to counting on and (b) from comparison by matching to comparison by numbers to comparison involving adding and subtracting.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Preschool Learning and Development Expectation:

PCC.E. Counting and Cardinality: Associate a quantity with written numerals up to 5 and begin to write numbers.

Indicators of Progress:

By the end of the preschool experience (approximately 60 months/5 years old) students may

a. Associate a number of objects with a written numeral 0-5. (similar to ELDG 3-5 1.3)

b. Recognize and, with support, write some numerals up to 10. (similar to ELDG 3-5 1.3)

Examples of High Quality Teaching and Learning Experiences:

1. Supportive Teaching Practices/Adults May:

   1. Play games with children where spinning a wheel with numbers or the number of dots on a die is associated with the need to count that number of objects or spaces.

   2. Help a child write or trace using any writing tool the numeral corresponding to his or her age.

Examples of Learning/Children May:

1. Match a group of 1 to 5 objects with written and spoken numbers.

2. Copy a printed numeral using their own handwriting.

3. Coherence Connections:

   1. Between 36-60 months, children develop an understanding that a written numeral represents a quantity and uses symbols, like tally marks, to represent numerals.

   2. In Kindergarten, students write numbers from 0 to 20 and associate a number of objects with the written numerals 0-20.

   3. This GLE supports the work of Preschool and is part of a progression of learning from spoken number words to written base-ten numerals to base-ten understanding.
Preschool: Algebra and Functions

**Common Core Information**

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**Prepared Graduates**

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
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**Preschool Learning and Development Expectation:**

POA.A. Operations and Algebraic Thinking: Understand addition as adding to and understand subtraction as taking away from.

**Indicators of Progress:**

By the end of the preschool experience (approximately 60 months/5 years old) students may

a. Represent addition and subtraction in different ways, such as with fingers, objects, and drawings (similar to ELDG 3-5.2.2)

b. Solve addition and subtraction word problems. Add and subtract up to five to or from a given number. (similar to ELDG 3-5.2.3)

c. With adult assistance, begin to use counting on from the larger number for addition. (similar to ELDG 3-5.2.2.2.2.3)

**Examples of High Quality Teaching and Learning Experiences:**

1. **Supportive Teaching Practices/Adults May:**
   1. Use fingers on both hands to represent addition.
   2. Ask a child with five crackers, “If you eat three of your crackers, how many will you have left?”

2. **Examples of Learning/Children May:**
   1. Add a group of three and a group of two, counting “One, two, three…” and then counting on “Four, five!” while keeping track using their fingers.
   2. Take three away from five, counting “Five, four, three, two, one!” while keeping track using their fingers.
   3. Say after receiving more crackers at snack time, “I had two and now I have four.”

3. **Coherence Connections:**
   1. Between 36-60 months, children develop beginning understandings of adding and subtracting with the help of objects and adult support.
   2. In Kindergarten, students add and subtract within 10 using objects or drawings to represent problems and fluently add and subtract within 5.
   3. This GLE is major work of Preschool and part of a progression involving addition and subtraction of increasingly large numbers and increasingly complex problem subtypes (see Table 1 CCSS, p. 85).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Preschool Learning and Development Expectation:

ROA.B. Operations and Algebraic Thinking: Understand simple patterns.

Indicators of Progress:

By the end of the preschool experience (approximately 60 months/5 years old) students may

a. Fill in missing elements of simple patterns (similar to ELDG 3-5 4.1)

b. Duplicate simple patterns in a different location than demonstrated, such as making the same alternating color pattern with blocks at a table that was demonstrated on the rug. Extend patterns, such as making an eight block tower of the same pattern that was demonstrated with four blocks. (similar to ELDG 3-5 4.2)

c. Identify the core unit of sequentially repeating patterns, such as color in a sequence of alternating red and blue blocks. (similar to ELDG 3-5 4.3)

Examples of High Quality Teaching and Learning Experiences:

Supportive Teaching Practices/Adults May:

1. Provide everyday opportunities to explore numbers and patterns, such as helping set the table.

2. Provide opportunities to observe naturally occurring patterns within the indoor and outdoor environments.

3. Introduce songs and movement patterns where children can extend and grow the pattern.

Examples of Learning/Children May:

1. Use art materials and other objects to create patterns (e.g., weaving, stringing beads, stacking blocks).

2. Recognize patterns in a story or song.

3. Name the colors in a sequence of alternating red and blue blocks.

4. Sequence story cards to show beginning, middle, and end.

Coherence Connections:

1. Between 36-60 months, children recognize and work with simple patterns in different forms, such as patterns of objects, numbers, sounds, and movements.

2. In Kindergarten, pattern recognition is embedded in and focused on early numeracy, such as counting by tens, number composition/decomposition, making tens, describing attributes of objects, and classifying objects into categories.

3. This GLE supports the work of Preschool and should generally focus on numeracy skills to benefit mathematics learning.
Colorado Academic Standards Review
MATHEMATICS
Preschool: Data, Statistics, & Probability

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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
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Preschool Learning and Development Expectation:
PMD.A. Measurement and Data: Measure objects by their various attributes using standard and non-standard measurement. Use differences in attributes to make comparisons.

Indicators of Progress:
By the end of the preschool experience (approximately 60 months/5 years old) students may:
a. Measure using the same unit, such as putting together snap cubes to see how tall a book is. (similar to ELDG 3-5 5.3)
b. Compare or order up to five objects based on their measurable attributes, such as height or weight. (similar to ELDG 3-5 5.1 5.2)
c. Use comparative language, such as shortest, heavier, biggest, or longer. (similar to ELDG 3-5 5.1 5.4)

Examples of High Quality Teaching and Learning Experiences:
1. Supportive Teaching Practices/Adults May:
   1. Follow a recipe and let children measure, pour, and stir the ingredients.
   2. Provide opportunities for children to sort, classify, and group household objects and materials.
   3. Ask questions of measurement (e.g., “How many steps does it take to walk from the front door to your cubby?” or “How many blocks long is your arm?”).

Examples of Learning/Children May:
1. Sort objects by physical characteristics such as a color or size.
2. Group objects according to their size, using standard and non-standard forms of measurement (e.g., height, weight, length, color brightness).
3. Explore various processes and units for measurement and begin to notice different results of one method or another.

Coherence Connections:
1. Between 36-60 months, children develop an understanding that attributes can be described and compared in simple ways, such as one child being taller than another.
2. In kindergarten, students describe multiple measurable attributes of an object and make direct comparisons of two objects with a measurable attribute in common.
3. This GLE is in addition to the major work of preschool. These concepts are addressed in kindergarten in a similar way as in preschool.
Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

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Preschool Learning and Development Expectation:

P.G.A. Geometry: Identify, describe, compare, and compose shapes.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/ 5 years old) students may:

- Name and describe shapes in terms of length of sides, number of sides, and number of angles. (similar to ELDG 3-5 3.1)

- Correctly name basic shapes regardless of size and orientation. (similar to ELDG 3-5 3.1)

- Analyze, compare, and sort two- and three-dimensional shapes and objects in different sizes. Describe their similarities, differences, and other attributes, such as size and shape. (similar to ELDG 3-5 3.3)

- Create and build shapes from components. (similar to ELDG 3-5 3.2)

Examples of High Quality Teaching and Learning Experiences.

Supportive Teaching Practices/Adults May:

1. Use a sensory table with various bowls, cups, or other containers to encourage activities.

2. Provide children with puzzles made of simple geometric shapes and encourage saying the names of shapes as they play.

Examples of Learning/Children May:

1. Match, sort, group, and name basic shapes found outside or in the classroom.

2. Use pattern tiles to turn a rectangle into a square.

Coherence Connections:

1. Between 36-60 months, children start by recognizing circles and squares and then add triangles and other shapes. As understanding of shape develops, children identify sides and angles as distinct parts of shapes.

2. In Kindergarten, students identify and describe squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.

3. This GLE is an addition to the major work of Preschool. The geometry learning progression in early elementary has students work with shapes and their attributes in increasingly sophisticated ways over time.
Preschool Learning and Development Expectation:

P.G.B. Geometry: Explore the positions of objects in space.

Indicators of Progress:

By the end of the preschool experience (approximately 50 months / 5 years old) students may:

a. Understand and use language related to directionality, order, and the position of objects, including up/down and in front/behind. (similar to ELDG 3-5 3.4)

b. Correctly follow directions involving their own position in space, such as “Stand up” and “Move forward.” (similar to ELDG 3-5 3.4)

Examples of High Quality Teaching and Learning Experiences:

Supportive Teaching Practices/Adults May:

1. Provide opportunities for conversation using everyday words to indicate space location, shape, and size of objects.

2. Help children organize toys, pointing out concepts such as “in,” “on,” and “beside”.

Examples of Learning/Children May:

1. Use the vocabulary of geometry and position to describe shapes within the room and surrounding environment.

2. Understand relational directions (e.g., “Please put a mat under each plate.”)

Coherence Connections:

1. Between 36-60 months, students develop spatial vocabulary and become able to follow directions involving their own position in space.

2. In Kindergarten, students describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, in front of, behind, and next to.

3. This GLE is an addition to the major work of Preschool. The geometry learning progression in early elementary has students work with shapes and their attributes in increasingly sophisticated ways over time.
Mathematics

Grade Level: Kindergarten
Standard: 1. Number and Quantity

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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
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5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten
K.C.C.A: Counting and Cardinality: Use number names and the count sequence.

Evidence Outcomes
Students Can:
- Count to 100 by ones and by tens (CCSS: K.C.C.A 1)
- Count forward beginning from a given number within the known sequence (instead of having to begin at 1) (CCSS: K.C.C.A 2)
- Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects) (CCSS: K.C.C.A 3)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. MP7: Look for and make use of structure. Students recognize the importance of order in the counting sequence.
2. MP8: Look for and express regularity in repeated reasoning. Students recognize that counting by tens can have advantages compared to repeatedly counting by ones, but can work to achieve the same goal.

Inquiry Questions:
1. When might someone want to count by tens instead of ones?
2. When might someone want to start counting from a number other than one?
3. What is a number we can use to show we have nothing to count?

Coherence Connections:
1. In preschool, students understand that number words have a sequence and that the words are separate (not “onethreethree”).
2. In Grade 1, students will extend the counting sequence to 120.
3. This GLE is major work of kindergarten and key to several progressions of learning: (a) from saying the counting words to counting out objects, (b) from counting to counting on, and (c) from spoken number words to written base-ten numerals to base-ten system understanding.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten

K.CC.B. Counting and Cardinality: Count to determine the number of objects.

Evidence Outcomes

Students Can:

a. Apply the relationship between numbers and quantities and connect counting to cardinality. (CCSS: K.CC.B.4)
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (CCSS: K.CC.B.4.a)
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (CCSS: K.CC.B.4.b)
   c. Understand that each successive number name refers to a quantity that is one larger. (CCSS: K.CC.B.4.c)

b. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. (CCSS: K.CC.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP1: Make sense of problems and persevere in solving them. Students have opportunities to manipulate and organize objects into groups and make sense of how quantity is preserved regardless of the arrangement.

2. MP2: Reason abstractly and quantitatively. Students progress from viewing counting as a process to abstracting quantities into mental objects of their own—especially the quantity 10.

3. MP7: Look for and make use of structure. Students make use of structure by counting on, as well as by recognizing that counting can be made easier when objects are placed in a line, ordered, or when arrangements of objects are repeated.

Inquiry Questions:

1. How is counting to five different than the number five?

2. What is one larger than four? What is one larger than seven?

3. What makes counting ten fingers easier than counting ten pencils or ten shoes?

Coherence Connections:

1. In Preschool, students build conceptions of what whole numbers mean, of subitizing, of one-to-one correspondence between verbal counting and objects, and of cardinality.

2. In Grade 1, students use their understanding of counting and cardinality to add and subtract within 20.

3. This GLE is major work of Kindergarten and key to several progressions of learning: (a) from saying the counting words to counting out objects, (b) from counting to counting on, and (c) from spoken number words to written base-ten numerals to base-ten understanding.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten


Evidence Outcomes

Students Can:

a. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to 10 objects.) (CCSS: K.CC.6)

b. Compare two numbers between 1 and 10 presented as written numerals. (CCSS: K.CC.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP3: Construct viable arguments and critique the reasoning of others. Students make reasoned arguments about the relative sizes of groups.

2. MP6: Attend to precision. Students use precise language to describe why one quantity is less than, greater than, or equal to another, and avoid mixing and misusing different ways of quantifying such as dimension, weight, or magnitude.

Inquiry Questions:

1. How do you know when you have more or less?

2. Is it always correct to say there is more of something in one group than in another based on the number of objects counted?

Coherence Connections:

1. In Preschool, students build an understanding of same versus different numbers of items, numbers of objects versus their size, and ordering from first to tenth.

2. In Grade 1, students build an understanding of ten and place value with two-digit numbers. Students also organize, represent, and interpret data.

3. This GLE is major work of Kindergarten and key to several progressions of learning, (a) from counting to counting on and (b) from comparison by matching to comparison by numbers to comparison involving adding and subtracting.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten

K.NBT.A. Number & Operations in Base Ten: Work with numbers 11-19 to gain foundations for place value.

Evidence Outcomes

**Students Can:**

a. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: K.NBT.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES. Critical Thinking/Problem Solving: Students identify problems.

2. MP1: Make sense of problems and persevere in solving them. Students make sense of quantities and their relationships in problem situations.

3. MP4: Model with mathematics. Students model quantities with drawings or equations.

4. MP7: Look for and make use of structure. Students see the structure of a number as its base-ten units.

Inquiry Questions:

1. How are “add to” and “put together” situations represented in the composition of quantities? (See Table 1 CCSS, p. 88)

2. How are “take from” and “take apart” situations represented in the decomposition of quantities? (See Table 1 CCSS, p. 88)

3. In the number 11, what makes the “1” on the left different from the “1” on the right? What do these quantities look like as objects?

4. What would a number called “ten four” look like? What English language words do we use to represent teen numbers?

Coherence Connections:

1. In Preschool, students develop conceptions of addition and subtraction when working with small collections of objects.

2. In Grade 1, students build an understanding of ten and place value with two-digit numerals.

3. This GLE is major work of Kindergarten and part of a progression from comparison by spoken number words to written base-ten numerals to base-ten system understanding.
Colorado Academic Standards Review
MATHEMATICS
Kindergarten: Algebra and Functions

Instructions
To leave feedback, click on the comment icon ( ) next to any item. You can then offer feedback and comments.
Once you save your feedback, the icon will change color and show as a checkbox ( ) so you can keep track of your progress.

Common Core Information
Colorado’s standards for mathematics and reading, writing and communicating include the Common Core State Standards. Each statement that comes from the Common Core State Standards is followed by the appropriate reference such as (CCSS: 5 NF.2) or (CCSS: SL.K.5). View a reference guide to the CCSS reference codes (PDF).

Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten
K.OA.A. Operations & Algebraic Thinking: Model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings.

Evidence Outcomes
Students Can:

a. Represent addition and subtraction with objects, fingers, mental images, drawings (drawings need not show details, but should show the mathematics in the problem), sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (CCSS: K.OA.A.1)
b. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. (CCSS: K.OA.A.2)
c. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). (CCSS: K.OA.A.3)
d. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS: K.OA.A.4)
e. Fluently add and subtract within 5. (CCSS: K.OA.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:
1. CES: Critical Thinking/Problem Solving. Students identify problems.
2. MP1: Make sense of problems and persevere in solving them. Students make sense of quantities and their relationships in problem situations.
3. MP4: Model with mathematics. Students model quantities with drawings or equations.

Inquiry Questions:
1. How are “add to” and “put together” situations represented in the composition of quantities? (See Table 1 CCSS, p. 88)
2. How are “take from” and “take apart” situations represented in the composition of quantities? (See Table 1 CCSS, p. 88)
3. How are addition and subtraction related?

Coherence Connections:
1. In Preschool, students represent addition and subtraction within 5 with fingers, objects, and drawings.
2. In Grade 1, students use addition and subtraction within 20 and understand properties of operations and the relationship between addition and subtraction.
3. This GLE is major work of Kindergarten and part of a progression involving addition and subtraction of increasingly large numbers and increasingly complex problem subtypes (see Table 1 CCSS, p. 88).
Colorado Academic Standards Review
MATHEMATICS
Kindergarten: Data, Statistics, & Probability

Instructions
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Common Core Information
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten

Evidence Outcomes
Students Can:

a. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. (CCSS: K.MD.A.1)

b. Directly compare two objects with a measurable attribute in common, to see which object has "more of" (less of) the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter. (CCSS: K.MD.A.2)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. CES: Inquiry/Analysis. Students compare and order objects by their attributes as part of making sense of their experiences.
2. MP6: Attent to precision. Students are precise about meanings related to size when measuring an object’s height, weight, or other attribute.

Inquiry Questions:
1. What does it mean for one object to be “bigger” than another?
2. If you are standing on a chair, how should your height be measured differently than if you were standing on the floor?
3. If an object is moved, does that change its size?

Coherence Connections:
1. In Preschool, students develop conceptions of measurable attributes of objects and comparisons based on those attributes.
2. In Grade 1, students measure lengths directly and by iterating length units, and express the length of an object as a whole number of length units.
3. This GLE is in addition to the major work of Grade 1 and contributes to students’ understandings of measurable attributes, comparison, and conservation of length. These concepts have connections to progressions in geometry, the number system, and to ratio and proportion.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten

K.MD.B: Measurement & Data. Classify objects and count the number of objects in each category.

Evidence Outcomes

Students Can:

a. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.) (CCSS: K.MD.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP1: Make sense of problems and persevere in solving them. Students group objects into categories to help make sense of problems.
2. MP2: Reason abstractly and quantitatively. By grouping similar objects together, students abstract individual objects into a new conceptual group.
3. MP5: Use appropriate tools strategically. Students choose appropriate representations of objects and categories.
4. MP6: Attend to precision. Students use appropriate labels and units of measure.

Inquiry Questions:

1. How can numbers of objects be represented to make comparisons?
2. How can objects be categorized in different ways?
3. How can an object’s attributes determine if it does not belong with other objects in a group?

Coherence Connections:

1. In Preschool, students use differences in attributes to make comparisons.
2. In Grade 1, students organize, represent, and interpret data with up to three categories.
3. This GLE supports the work of counting and comparing numbers in Kindergarten and is part of a progression of learning how to analyze categorical data.
Colorado Academic Standards Review
MATHEMATICS
Kindergarten: Geometry

Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

Standard:
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

Select Grade Level & Standard

Instructions
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Common Core Information
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten
K.G.A. Geometry: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Evidence Outcomes
Students Can:
a. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, behind, in front of, behind, and next to. (CCSS. K.G.A.1)
b. Correctly name shapes regardless of their orientations or overall size. (CCSS: K.G.A.2)
c. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”). (CCSS: K.G.A.3)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. MP2: Reason abstractly and quantitatively. Students use their experience with multiple examples of a type of shape to develop a concept image of that shape from which they can abstract common features. (see Tall & Vinner, 1980)
2. MP4: Model with mathematics. Students increasingly describe their physical world from geometric perspectives, e.g., shape, orientation, and spatial relationships.
3. MP6: Attend to precision. Students attend to details about a shape’s attributes when describing the shape.

Inquiry Questions:
1. For a given shape, what attributes make an example of that shape different from a non-example?
2. What are the ways of describing where an object is?

Coherence Connections:
1. In Preschool, students learn about circles, squares, triangles, and their parts.
2. In Grade 1, students will work with these shapes and their attributes in increasingly sophisticated ways.
3. This GLE is an addition to the major work of Kindergarten.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Kindergarten

K.G.B. Geometry: Analyze, compare, create, and compose shapes

Evidence Outcomes

Students Can:

a. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/corners) and other attributes (e.g., having sides of equal length). (CCSS: K.G.B.4)

b. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. (CCSS: K.G.B.5)

c. Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?” (CCSS: K.G.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP4: Model with mathematics. Students model shapes in the world by building them with components or drawing representations of them.

2. MP7: Look for and make use of structure. Students sort a collection of shapes according to how many sides the shapes have.

Inquiry Questions:

1. Can you change a shape into a different kind of shape by rotating it?

2. What kinds of pictures can you make by combining shapes?

Coherence Connections:

1. In Preschool, students understand and use language related to directionality, order, and the position of objects, such as up/down and in front/behind.

2. In Grade 1, students reason with shapes and their attributes in increasingly sophisticated ways.

3. This GLE is an addition to the major work of Kindergarten.
Mathematics

Grade Level: First Grade
Standard: 1. Number and Quantity

Instructions
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Common Core Information
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade
1.NBT.A. Number & Operations in Base Ten. Extend the counting sequence.

Evidence Outcomes
Students Can:

a. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. (CCSS: 1.NBT.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP7: Look for and make use of structure. Students make use of the base ten counting structure when using special words at the decades, like “sixty” and “seventy.”

Inquiry Questions:

1. When might someone want to count by tens instead of ones?
2. Which numbers can be written with two numerals and which numbers are written with three?

Coherence Connections:

1. In Kindergarten, students count to 100 by ones and tens, count forward from a given number, and connect counting to cardinality.
2. In Grade 2, students extend their place value understanding to hundreds and three-digit numbers.
3. This GLE is major work of Grade 1 and is key to learning number words and their irregularities (e.g., how decade words like “fifty” mean “four ten” and are different from “fourteen”) and developing other skills in Number and Operations in Base Ten GLEs.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade

1. NBT.B. Number & Operations in Base Ten: Understand place value.

Evidence Outcomes

Students Can:

a. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases. (CCSS: 1.NBT.B.2)
   a. 10 can be thought of as a bundle of ten ones — called a “ten.” (CCSS: 1.NBT.B.2.a)
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: 1.NBT.B.2.b)
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (CCSS: 1.NBT.B.2.c)

b. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. (CCSS: 1.NBT.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP1: Make sense of problems and persevere in solving them. Students make sense of quantities and their relationships in problem situations.

2. MP2: Reason abstractly and quantitatively. Students abstract 10 ones into a single conceptual object called a ten.

3. MP4: Model with mathematics. Students model ones and tens with objects and mathematical representations.

4. MP7: Look for and make use of structure. Students see the structure of a number as its base-ten units.

Inquiry Questions:

1. What does the position of a digit tell you about its value?

2. What are two ways to describe the number 307?

3. Why was a place value system developed? What might numbers look like without it?

Coherence Connections:

1. In Kindergarten, students decompose numbers from 11 to 19 into ten ones and further ones.

2. In Grade 2, students understand hundreds and place value of three-digit numbers.

3. This GLE is major work of Grade 1 and part of a progression from number words to written base-ten numerals to base-ten system understanding. Saying 67 as “six tens, seven ones” as well as “sixty-seven” can help students progress in their understanding of the tens and ones structure.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade

1.NBT.C. Number & Operations in Base Ten: Use place value understanding and properties of operations to add and subtract.

Evidence Outcomes

Students Can:

a. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. (CCSS: 1.NBT.C.4)

b. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. (CCSS: 1.NBT.C.5)

c. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS: 1.NBT.C.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical Thinking/Problem Solving. Students identify problems.

2. MP1: Make sense of problems and persevere in solving them. Students model quantities with drawings or equations to help them make sense of place value.

3. MP7: Look for and make use of structure. Students use the base-ten structure to add and subtract, including adding and subtracting multiples of ten.

Inquiry Questions:

1. Can you add or subtract ten without having to count by ones?

2. How does modeling addition look different if you add tens and ones separately compared to counting on by tens then by ones?

Coherence Connections:

1. In Kindergarten, students model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings. Students also work with numbers 11-19 to gain foundations for place value.

2. In Grade 2, students understand place value for three-digit numbers and use that understanding and properties of operations to add and subtract within 1000 using models and fluently within 100.

3. This GLE is major work of Grade 1 and part of progression of increasing fluency with addition and subtraction of increasingly large numbers. At this grade, students should focus on subtracting multiples of ten and defer subtraction of other two-digit numbers until Grade 2.
Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precison and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade

1. OA A: Operations & Algebraic Thinking: Represent and solve problems involving addition and subtraction.

Evidence Outcomes

Students Can:

a. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.A.1)

b. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:
1. MP1. Make sense of problems and persevere in solving them. Students make sense of problems by relating objects, drawings, and equations.
2. MP4. Model with mathematics. Students represent situations in word problems using objects, drawings, and equations, including a symbol for an unknown number.

Inquiry Questions:
1. How is comparing two quantities and being asked “how many more” or “how many less” different from situations involving adding to and taking from?
2. How can you represent a problem with phrasing like “Julie has three more apples than Lucy” with objects or equations?
3. How can you represent a problem with phrasing like “How many more apples does Lucy need to have as many as Julie?” with objects or equations?

Coherence Connections:
1. In Kindergarten, students add and subtract within 10 by using objects or drawings to represent problems.
2. In Grade 2, students represent and solve problems involving addition and subtraction within 100, with fluency expected within 20.
3. This GLE is major work of Grade 1 and part of a progression involving addition and subtraction of increasingly large numbers and increasingly complex problem subtypes (see Table 1, CCSS, p. 88).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade

1.OA.B. Operations & Algebraic Thinking: Understand and apply properties of operations and the relationship between addition and subtraction.

Evidence Outcomes

Students Can:

a. Apply properties of operations as strategies to add and subtract. (Students need not use formal terms for these properties.) Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 + 12. (Associative property of addition.) (CCSS: 1.OA.B.3)

b. Understand subtraction as an unknown-addend problem. For example, subtract 10 - 6 by finding the number that makes 10 when added to 6. (CCSS: 1.OA.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP1: Make sense of problems and persevere in solving them. Students make sense of addition and subtraction by applying properties of operations and working with different problem types.

2. MP7: Look for and make use of structure. Students use properties of operations to recognize equivalent forms of equations.

Inquiry Questions:

1. How does an “add to” operation look different when the result is unknown compared to when the change or the start is unknown? (See Table 1 CCSS, p. 88)

2. How does a “take from” operation look different when the result is unknown compared to when the change is unknown or the start is unknown? (See Table 1 CCSS, p. 88)

3. How are addition and subtraction related in “put together” and “take apart” operations? (See Table 1 CCSS, p. 88)

Coherence Connections:

1. In Kindergarten, students add and subtract within 10 by using objects or drawings to represent problems.

2. In Grade 2, students explain why addition and subtraction strategies work, using place value and the properties of operations.

3. This GLE is major work of Grade 1 and not only supports students’ work with operations in Grade 2, but also supports the learning of fractions in Grade 4 and algebraic expressions in Grade 6.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade

1. OA.C. Operations & Algebraic Thinking: Add and subtract within 20.

Evidence Outcomes

Students Can:

a. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (CCSS: 1.OA.C.5)

b. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14), decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 6 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (CCSS: 1.OA.C.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical Thinking/Problem Solving. Students use multiple strategies to think about problems.

2. MP7: Look for and make use of structure. Students make use of the structure of numbers when making tens or creating equivalent but easier or known sums.

Inquiry Questions:

1. Which is easier when adding 4 + 9: counting 9 on to 4 or counting 4 on to 9?

2. Why does knowing doubles like 4 + 4 or 5 + 5 help when adding 4 + 5?

3. How does counting on to add and subtract within 20 make it easier to use fingers even though we only have 10 fingers?

Coherence Connections:

1. In Kindergarten, students understand the relationship between numbers and quantities and connect counting to cardinality.

2. In Grade 2, students fluently add and subtract within 20 using mental strategies and know from memory all sums of two one-digit numbers.

3. This GLE is major work of Grade 1 and part of a progression involving addition and subtraction of increasingly large numbers and increasingly sophisticated solution methods, from direct modeling to counting on to converting to easier problems.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade

1.O.A.D. Operations & Algebraic Thinking: Work with addition and subtraction equations.

Evidence Outcomes

Students Can:

a. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$. (CCSS: 1.OA.D.7)

b. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = _ - 3$, $6 + 6 = _$. (CCSS: 1.OA.D.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP2: Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations.

2. MP3: Construct viable arguments and critique the reasoning of others. Students question assumptions about the meaning of the equals sign and construct viable arguments.

Inquiry Questions:

1. Why is equals not an operation like addition and subtraction?

2. If you have 3 on the left and three on the right, is it true that $3 = 3$? If you add 2 more to the right, how many total do you need on the left to make a true statement, and how would you write that as an equation?

Coherence Connections:

1. In Kindergarten, students represent addition and subtraction with equations without needing to understand the meaning of the equal sign.

2. In Grade 2, students write equations to express an even number as a sum of two equal addends and write equations to express the total number of objects in an array as the sum of equal addends.

3. This GLE is major work of Grade 1 and supports future use of the equals sign with multiplication in Grade 3 and fractions in Grade 4.
Colorado Academic Standards Review
MATHEMATICS
First Grade: Data, Statistics, & Probability

Select Grade Level & Standard

Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

Standard:
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

Change content area

Instructions
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Common Core Information
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade
1. MD.A. Measurement & Data: Measure lengths indirectly and by iterating length units.

Evidence Outcomes
Students Can:

a. Order three objects by length; compare the lengths of two objects indirectly by using a third object. (CCSS: 1.MD.A.1)
b. Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. (CCSS: 1.MD.A.2)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:

1. MP2: Reason abstractly and quantitatively: Students abstract comparisons between lengths using statements like A > B.
2. MP3: Connect viable arguments and critique the reasoning of others. Students use the transitive property to explain if A is longer than B, and B is longer than C, then A must be longer than C.
3. MP6: Attend to precision. Students take care to consider the endpoints of objects when measuring and making comparisons.

Inquiry Questions:
1. How is it possible for 5 sticks placed end-to-end to be equal in length to 6 sticks placed end-to-end?

Coherence Connections:
1. In Kindergarten, students directly compare two objects with a measurable attribute in common.
2. In Grade 2, students measure and estimate lengths in standard units.
3. This GLE is major work of Grade 1 and part of a progression of learning that develops conceptions of comparison, conservation, seriation, and iteration. Students should understand the concepts, procedures, and usefulness of measurement using standard measures before successfully using non-standard measures and the relationships between different units of measurement.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade

1.MD.B. Measurement & Data: Tell and write time.

Evidence Outcomes

Students Can:

a. Tell and write time in hours and half-hours using analog and digital clocks. (CCSS: 1.MD.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Personal Responsibility. Students tell and manage time to be both personally responsible and responsible to the needs of others.

2. MP6: Attend to precision. Students recognize that time is a quantity that can be measured with different degrees of precision.

Inquiry Questions:

1. How long is two half-hours?

2. If the time is 2:30, where would the minute hand be pointing on an analog clock?

Coherence Connections:

1. In Kindergarten, students are not expected to learn how to tell and write time.

2. In Grade 2, students tell and write time from analog and digital clocks to the nearest five minutes.

3. This GLE is an addition to the major work of Grade 1.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade

1. MD.C. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

a. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (CCSS: 1.MD.C.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP1: Make sense of problems and persevere in solving them. Students group objects into categories to help make sense of problems.

2. MP2: Reason abstractly and quantitatively. By grouping similar objects together, students abstract individual objects into a new conceptual group.

3. MP5: Use appropriate tools strategically. Students choose appropriate representations of objects and categories.

4. MP6: Attend to precision. Students use appropriate labels and units of measure.

Inquiry Questions:

1. How do different representations of data indicate there are more objects in one category than in another category?

2. How can objects be categorized in different ways?

3. How can an object’s attributes determine if it does not belong with other objects in a group?

Coherence Connections:

1. In Kindergarten, students classify objects into given categories, count the numbers of objects in each category, and sort the categories by count.

2. In Grade 2, students draw a picture graph and a bar graph to represent a data set with up to four categories, and solve put-together, take-apart, and compare problems using information in a bar graph.

3. This GLE supports the work of adding and subtracting in Grade 1 and is part of a progression of learning how to analyze categorical data.
Mathematics
First Grade: Geometry

Instructions
To leave feedback, click on the comment icon next to any item. You can then offer feedback and comments.
Once you save your feedback, the icon will change color and show as a checkbox so you can keep track of your progress.

Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

Standard:
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

Change content area

Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: First Grade
1. G.A. Geometry: Reason with shapes and their attributes

Evidence Outcomes
Students Can:
a. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes (CCSS: 1.G.A.1)
b. Compose two-dimensional shapes (rectangles, squares, triangles, half-circles, and quarter circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names, such as “right rectangular prism”). (CCSS: 1.G.A.2)
c. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. (CCSS: 1.G.A.3)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. MP1: Make sense of problems and persevere in solving them. Students use their intuitive understandings of measurement, congruence, and symmetry to help them solve puzzles and create shapes.
2. MP2: Reason abstractly and quantitatively. Students solve puzzles and construct designs with shapes to create composite shapes that are conceptualized as independent entities.
3. MP3: Construct viable arguments and critique the reasoning of others. Students justify why a shape belongs in a given category by differentiating between geometrically defining attributes (e.g., “hexagons have six straight sides”) and non-defining attributes (e.g., “color, size, orientation).”

Inquiry Questions:
1. Which attributes matter when deciding if a shape belongs in a given category?
2. If you partition a shape into halves, do you end up with more or fewer pieces than if you partition the shape into quarters?

Coherence Connections:
1. In Kindergarten, students identify, describe, analyze, compare, create, and compose shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
2. In Grade 2, students reason with shapes and their attributes in increasingly sophisticated ways.
3. This CLE is an addition to the major work of Grade 1.
Mathematics
Grade Level: Second Grade
Standard: 1. Number and Quantity

Select Grade Level & Standard
Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Second Grade
2.NBT.A. Number & Operations in Base Ten (Understand Place Value)

Evidence Outcomes
Students Can:

a. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. e.g., 706 equals 7 hundreds, 6 tens, and 6 ones. Understand the following as special cases (CCSS: 2.NBT.A.1)
   a. 100 can be thought of as a bundle of ten tens — called a “hundred” (CCSS: 2.NBT.A.1a)
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (CCSS: 2.NBT.A.1b)

b. Count within 1000; skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.A.2)

c. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form (CCSS: 2.NBT.A.3)

d. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons (CCSS: 2.NBT.A.4)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. MP2: Reason abstractly and quantitatively. Students abstract 10 ones into a single conceptual object called a ten and abstract 100 ones into a single conceptual object called a hundred.
2. MP7: Look for and make use of structure. Students compose, decompose, and compare single-digit numbers according to their base-ten structure.

Inquiry Questions:
1. How many hundreds are in the number “four hundred five”? How do you know? How many tens are in the number “forty-five”? How do you know?
2. How many times do you need to skip count by 5 to reach 100? How many times do you need to skip count by 100 to reach 1000?
3. How many times do you need to skip count by 10 to reach 100? How many times do you need to skip count by 100 to reach 1000?
4. Why is any two-digit number that starts with 5 always larger than a two-digit number that starts with 3?

Coherence Connections:
1. In Grade 1, students understand tens and place value of two-digit numbers.
2. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic.
3. This GLE is major work of Grade 2 and part of a progression from number words to written base-ten numerals to base-ten system understanding.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Second Grade

2.NBT.B. Number & Operations in Base Ten: Use place value understanding and properties of operations to add and subtract.

Evidence Outcomes

Students Can:

a. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.B.5)

b. Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.B.6)

c. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS: 2.NBT.B.7)

d. Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. (CCSS: 2.NBT.B.8)

e. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.) (CCSS: 2.NBT.B.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical Thinking/Problem Solving. Students identify problems.

2. MP1: Make sense of problems and persevere in solving them. Students model quantities with drawings or equations to help them make sense of place value.

3. MP7: Look for and make use of structure. Students use the base-ten structure to add and subtract, composing and decomposing ones, tens, and hundreds as necessary.

Inquiry Questions:

1. How can subtraction be viewed as an unknown addend problem? (e.g., 278 + ? = 425)

2. What might be a more efficient way of adding 38 + 47 + 93 + 62 than adding the addends in order?

Coherence Connections:

1. In Grade 1, students add within 100 using concrete models or drawings and strategies based on place value.

2. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic, including fluently adding and subtracting within 1000.

3. This GLE is major work of Grade 2 and part of a progression of increasing fluency with addition and subtraction of increasingly large numbers.
Colorado Academic Standards Review
MATHEMATICS
Second Grade: Algebra and Functions

Select Grade Level & Standard

Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

Standard:
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves: “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Second Grade
2.O.A.A. Operations & Algebraic Thinking: Represent and solve problems involving addition and subtraction.

Evidence Outcomes
Students Can:

a. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (see CCSS glossary) (CCSS.2.O.AA.1)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. CES: Critical Thinking/Problem Solving. Students identify problems.
2. MP2: Reason abstractly and quantitatively. Students abstract word problems into quantities, operations, and relations to solve for unknowns.
3. MP4: Model with mathematics. Students represent situations in word problems using drawings and equations with symbols for unknown numbers.

Inquiry Questions:
1. In a word problem, what is the unknown quantity?
2. Why are keywords like “more” and “in all” in word problems unreliable indicators of addition and subtraction?

Coherence Connections:
1. In Grade 1, students use addition and subtraction within 20 and understand properties of operations and the relationship between addition and subtraction.
2. In Grade 3, students solve two-step word problems using the four operations.
3. This GLE is major work of Grade 2 and part of a progression of learning how to solve problems of three subtypes: add, take from, put together/take apart, and compare (see Table 1 CCSS, p. 86).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Second Grade

2.OA.B. Operations & Algebraic Thinking. Add and subtract within 20.

Evidence Outcomes

Students Can:

a. Fluently add and subtract within 20 using mental strategies. (See 1.OA.C.6 for a list of strategies.) By end of Grade 2, know from memory all sums of two one-digit numbers. (CCSS: 2.OA.B.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP5: Use appropriate tools strategically. Students recognize those problems that can be done mentally versus those that require the use of objects, diagrams, or equations.

2. MP6: Attend to precision. Students add and subtract within 20 both quickly and accurately.

Inquiry Questions:

1. Which addition and subtraction facts are most easily memorized, and which are quickly derived using other facts and strategies?

2. How do conceptual understanding and procedural fluency support each other in problem solving?

Coherence Connections:

1. In Grade 1, students use addition and subtraction within 20 and understand properties of operations and the relationship between addition and subtraction. Students are expected to use objects, drawings, to support their thinking.

2. In Grade 3 and beyond, students are expected to be fluent with addition and subtraction within 20.

3. This GLE is major work of Grade 2 and the result of multiple years of learning and practice. Fluency describes students who can calculate quickly and accurately. Answers either come from memory, are understood from patterns, or quickly derived using strategies such as mentally composing/decomposing, using doubles, making tens, and relating addition and subtraction.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Second Grade

2.OA.C. Operations & Algebraic Thinking: Work with equal groups of objects to gain foundations for multiplication.

Evidence Outcomes

Students Can:

a. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. (CCSS: 2.OA.C.3)

b. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (CCSS: 2.OA.C.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Creativity/Innovation. Students are curious about the arrangement of objects and how some arrangements give them more mathematical power to solve problems.

2. MP2: Reason abstractly and quantitatively. Students reason with what it means for numbers to be even and odd.

3. MP3: Construct viable arguments and critique the reasoning of others. Students explain why a group of objects is even or odd and if their strategy for deciding works with any group of objects.

Inquiry Questions:

1. What does it mean for a number to be even?

2. Do two equal addends always result in an even sum?

Coherence Connections:

1. In Grade 1, students work with addition and subtraction equations.

2. In Grade 3, students represent and solve problems involving multiplication and division.

3. This GLE supports other work in Grade 2 and future work in multiplication and division.
Colorado Academic Standards Review
MATHEMATICS
Second Grade: Data, Statistics, & Probability

Select Grade Level & Standard

Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

Standard:
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

Change content area

Instructions
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Second Grade
2.MD.A. Measurement & Data: Measure and estimate lengths in standard units.

Evidence Outcomes

Students Can:

a. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (CCSS: 2.MD.A.1)

b. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (CCSS: 2.MD.A.2)

c. Estimate lengths using units of inches, feet, centimeters, and meters. (CCSS: 2.MD.A.3)

d. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS: 2.MD.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:
1. MPS: Use appropriate tools strategically. Students choose between different measurement tools depending on the objects they need to measure.
2. MPS: Attend to precision. Students determine when it is appropriate to estimate an object’s length or when a more precise measurement is needed.

Inquiry Questions:
1. What do the numbers on a ruler represent?
2. What is the appropriate tool for measuring the length of your school hallway, a 1-foot ruler or a 22-foot measuring tape?
3. When is it appropriate to estimate length? When is it not appropriate?

Coherence Connections:
1. In Grade 1, students iterate length units and understand length as the number of same-size length units that span an object with no gaps or overlaps.
2. In Grade 2, students develop an understanding of fractions as numbers related to the partitioning of a whole into equal parts and then iterating those parts.
3. This GLE is a major work of Grade 2 and brings together concepts from counting and cardinality, comparison, and subtraction, while developing a sense for the inverse relationship between the size of a unit and the number of units in the length of a given object, which is important for future work with fractions.
**Prepared Graduates**

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

**Grade Level Expectation: Second Grade**

2.MD.B. Measurement & Data: Relate addition and subtraction to length.

**Evidence Outcomes**

**Students Can:**

a. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.MD.B.5)

b. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. (CCSS: 2.MD.B.6)

**Academic Context and Connections**

**Colorado Essential Skills and Mathematical Practices:**

1. CES. Critical Thinking/Problem Solving. Students recognize problems involving lengths and identify possible solutions.

2. MP2. Reason abstractly and quantitatively. Students build on their experience with measurement tools to understand number lines as a more abstract tool for working with quantities.

3. MP4. Model with mathematics. Students use mathematical representations like drawings and equations to model scenarios described in word problems.

**Inquiry Questions:**

1. When might it be necessary to measure parts of objects and then combine those parts together?

2. How is a number line like a ruler?

**Coherence Connections:**

1. In Grade 1, students add and subtract within 20 and express the length of an object as a whole number of length units.

2. In Grade 3, students develop an understanding of a fraction as a number on a number line.

3. This GLE is major work of Grade 2 and the relationship between measurement and number lines provides a foundation for fraction understanding in future grades.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Second Grade


Evidence Outcomes

Students Can:

a. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS: 2.MD.C.7)

b. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? (CCSS: 2.MD.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES. Personal Responsibility. Students tell and manage time to be both personally responsible and responsible to the needs of others.

2. MP1: Make sense of problems and persevere in solving them. Students make sense of word problems involving money.

3. MP6: Attend to precision. Students recognize that time is a quantity that can be measured with different degrees of precision.

Inquiry Questions:

1. If the time is 2:25, where would the minute hand be pointing on an analog clock?

2. Does the size of a coin indicate the value of the coin?

3. How is money like our base-ten number system, where it takes ten of one unit to make the next unit (ten ones make a ten, ten tens make a hundred)? In what ways is it different?

Coherence Connections:

1. In Grade 1, students tell and write time in hours and half hours using analog and digital clocks.

2. In Grade 3, students tell and write time to the nearest minute and measure time intervals in minutes.

3. This GLE supports other work of Grade 2.
Prepared Graduates

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2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Second Grade

2.M.D. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

a. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS. 2.MD.D.9)

b. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems (see CCSS glossary) using information presented in a bar graph. (CCSS. 2.MD.D.10)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP1: Make sense of problems and persevere in solving them. Students organize objects according to measures or categories to help make sense of problems.

2. MP2: Reason abstractly and quantitatively. Organizing measurement and categorical data helps students abstract differences between individual objects into a representation of variability for a group of objects.

3. MP5: Use appropriate tools strategically. Students choose appropriate representations of objects and categories.

4. MP6: Attend to precision. Students use appropriate labels and units of measure.

Inquiry Questions:

1. How is organizing objects by length measurements, rounded to the nearest unit, similar and different to organizing objects by categories?

Coherence Connections:

1. In Grade 1, students organize, represent, and interpret data with up to three categories and compare how many more or less are in one category than another.

2. In Grade 3, students draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.

3. This GLE supports the work of adding and subtracting in Grade 2 and is part of a progression of learning how to analyze quantitative and categorical data.
Colorado Academic Standards Review
MATHEMATICS
Second Grade: Geometry

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Grade Level Expectation: Second Grade

Evidence Outcomes
Students Can:

a. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (CCSS: 2.G.A.1)

b. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS: 2.G.A.2)

c. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, fourths, half of, a third of, etc., and describe whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. (CCSS: 2.G.A.3)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. MP1. Make sense of problems and persevere in solving them. Students explore decompositions of shapes, such as partitioning a square into fourths in different ways and understanding that each partition, regardless of shape, represents an equal share of the square.

2. MP7. Look for and make use of structure. Students tile a rectangle with rows and columns of squares to understand rectangles as two-dimensional.

3. Inquiry Questions:
   1. How do the rows and columns of a rectangular array help to organize counting?
   2. As you partition a shape into a greater number of equal parts, what happens to the size of the parts?

4. Coherence Connections:
   1. In Grade 1, students distinguish between defining and nondefining attributes, compose two- and three-dimensional shapes, and partition circles and rectangles into halves and fourths.
   2. In Grade 3, students reason with shapes and their attributes in increasingly sophisticated ways.
   3. This GLE is an addition to the major work of Grade 2.
Mathematics

Grade Level: Third Grade

Standard: 1. Number and Quantity

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Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
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Grade Level Expectation: Third Grade

3.NBT.A. Number & Operations in Base Ten: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Evidence Outcomes

Students Can:

a. Use place value understanding to round whole numbers to the nearest 10 or 100. (CCSS: 3.NBT.A.1)
b. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 3.NBT.A.2)
c. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations. (CCSS: 3.NBT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Creativity/Innovation. Students define the problem using a variety of strategies.
2. MP7: Look for and make use of structure. Students recognize and utilize place value and properties of operations.

Inquiry Questions:

1. How is rounding whole numbers to the nearest 10 or 100 useful?
2. Do different strategies for solving lead to different answers when we add or subtract? Why or why not?

Coherence Connections:

1. Use of place value to add and subtract in third grade builds on the major content of second grade and supports students to make generalizations that will lead to a deeper understanding of the algorithms.
2. Multiplying by 10 in third grade prepares students to leverage place value understandings in multiplying multi-digit multiplication in fourth grade.
Prepared Graduates

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Grade Level Expectation: Third Grade

3.NFA. Number & Operations - Fractions: Develop understanding of fractions as numbers.

Evidence Outcomes

Students Can:

a. Describe a fraction \( \frac{1}{4} \) as the quantity formed by 1 part when a whole is partitioned into 4 equal parts; understand a fraction \( \frac{n}{d} \) as the quantity formed by parts \( \frac{1}{d} \) (CCSS: 3.NFA.1)

b. Describe a fraction as a number on the number line; represent fractions on a number line diagram (CCSS: 3.NFA.2)
   a. Represent a fraction \( \frac{1}{2} \) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into 2 equal parts. Recognize that each part has size \( \frac{1}{2} \) and that the endpoint of the part based at 0 locates the number \( \frac{1}{2} \) on the number line. (CCSS: 3.NFA.2.a)
   b. Represent a fraction \( \frac{1}{3} \) on a number line diagram by marking off a length \( \frac{1}{3} \) from 0. Recognize that the resulting interval has size \( \frac{1}{3} \) and that its endpoint locates the number \( \frac{1}{3} \) on the number line. (CCSS: 3.NFA.2.b)

   c. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size (CCSS: 3.NFA.3)
      a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line (CCSS: 3.NFA.3.a)
      b. Recognize and generate simple equivalent fractions, e.g., \( \frac{1}{2} = \frac{2}{4} = \frac{3}{6} \). Explain why the fractions are equivalent, e.g., by using a visual fraction model (CCSS: 3.NFA.3.b)
      c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Ex: Express \( 3 \) in the form \( \frac{n}{1} \); recognize that \( \frac{6}{1} \) is 6, locates \( \frac{6}{1} \) and \( 6 \) at the same point of a number line diagram (CCSS: 3.NFA.3.c)
      d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols \( >, =, < \), and justify the conclusions, e.g., by using a visual fraction model (CCSS: 3.NFA.3.d)

Academic Context and Connections

1. Colorado Essential Skills and Mathematical Practices:
   a. CES: Creativity/Innovation. Students demonstrate curiosity, imagination and eagerness to learn more.
   b. NFM: Reason Abstractly and Quantitatively. Students reason about the number line in a new way - understanding and utilizing fractional parts between whole numbers.
   c. NMF: Attentive to Precision. Two important aspects of fractions provide students the opportunity to attend to precision: specifying the whole, and explaining what is meant by “equal parts.”

Inquiry Questions:

1. How does the denominator of the unit fraction connect to the number of unit fractions that must be added to make a whole?
2. When the numerator is the same, how can the denominator be used to compare two fractions?

Coherence Connections:

1. A deep understanding of the quantity of fractions is crucial for students to use visual models to add and subtract fractions in fourth grade.
2. Students will leverage their understanding of comparing fractions with like numerators or denominators from third grade to compare fractions with unlike denominators in fourth grade.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

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Grade Level Expectation: Third Grade

3.OA.A. Operations and Algebraic Thinking: Represent and solve problems involving multiplication and division.

Evidence Outcomes

Students Can:

a. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7. (CCSS: 3.OA.A.1)

b. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into 8 equal shares of 5 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8. (CCSS: 3.OA.A.2)

c. Use multiplication and division within 100 to solve word problems involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (see Glossary, Table 2) (CCSS: 3.OA.A.3)

d. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × ? = 7. (CCSS: 3.OA.A.4)
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

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5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Third Grade

3.OA.B. Operations and Algebraic Thinking: Apply properties of multiplication and the relationship between multiplication and division.

Evidence Outcomes

Students Can:

a. Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find $8 \times 7$ as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.) (CCSS: 3.OA.B.5)

b. Interpret division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes $32$ when multiplied by $8$. (CCSS: 3.OA.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Creativity/Innovation. Students build on personal experience to specify a challenging problem to investigate.

2. MP6: Attend to precision.

Inquiry Questions:

1. What are ALL of the equations that can be written to represent the relationship between the area of a rectangle and its side lengths.

2. How can the distributive property be modeled using a closed array?

Coherence Connections:

1. Students decompose single digit multiplication problems as a precursor to decomposing based on place value to solve multi-digit multiplication problems in fourth grade.

2. Interpreting division as an unknown factor problem relates to interpreting subtraction as an unknown addend problem in previous grades.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Third Grade

3.OA.C. Operations and Algebraic Thinking: Multiply and divide within 100.

Evidence Outcomes

Students Can:

a. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. (CCSS: 3.OA.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Personal Responsibility. Students discern differences of effective and ineffective processes, communication and tasks.

2. MP7. Look for and express regularity in repeated reasoning.

Inquiry Questions:

1. How can known facts or combinations of known facts be used to solve a multiplication problem (for example, using $5\times4+2\times4$ to solve $7\times4$)?

2. What evidence can be provided that a student is flexible with their understanding of multiplication as an indicator of fluency?

Coherence Connections:

1. By emphasizing flexibility within fluency, students are encouraged to use reasoning to solve problems.

2. The strategies used to fluently solve single digit multiplication problems will be leveraged to support students in solving multi-digit multiplication problems in fourth grade.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

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5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Third Grade

3.OA.D. Operations and Algebraic Thinking: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Evidence Outcomes

Students Can:

a. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).) (CCSS: 3.OA.D.8)

b. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. (CCSS: 3.OA.D.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Information Literacy. Students articulate the most effective options to access information needed for a specific purpose.

2. MP1: Make sense of problems and persevere in solving them.

Inquiry Questions:

1. How can a visual model support students to make sense of and solve word problems?

2. How can the patterns in addition and/or multiplication tables support students to make predictions about probable solutions to a given problem?

Coherence Connections:

1. Order of operations is explored in third grade and will be revisited in fourth and fifth grades.

2. Solving multi step word problems is a focus of every grade. Anchoring sense making in visual models will provide students with a strategy that transcends the operation involved in the particular problem.
Mathematics

Grade Level: Third Grade

Standard: 3. Data, Statistics, and Probability

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2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

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5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Third Grade

3.MD.A Measurement and Data: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Evidence Outcomes

Students Can:

a. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. (CCSS: 3.MD.A.1)

b. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems (problems involving notions of “times as much”, see Glossary, Table 2).) (CCSS: 3.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Information Literacy. Students find and use the most effective tools to access information needed for a specific purpose.

2. MP1: Make sense of problems and persevere in solving them.

Inquiry Questions:

1. How can elapsed time be modeled on a number line to support the connection to addition and subtraction?

2. What concrete models can be used to support students to deeply understand the quantity represented with various units of measure?

Coherence Connections:

1. Students will convert within units of measure in fourth grade. Solving problems using these units in third grade will provide a context for this extension in the next grade level.

2. Problems involving elapsed time is a natural progression from the understandings of telling time developed in first and second grades.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Third Grade

3.MD.B. Measurement and Data: Represent and interpret data.

Evidence Outcomes

Students Can:

a. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. (CCSS: 3.MD.B.3)

b. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (CCSS: 3.MD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Creativity/Innovations. Students investigate to form hypotheses, make observations and draw conclusions.

2. MP4: Model with Mathematics.

Inquiry Questions:

1. How can the work with picture and bar graphs support students to connect mathematics to the world around them?

2. How can students use their work with measuring lengths with halves and fourths to build beginning understandings of addition of fractions with like and unlike denominators?

Coherence Connections:

1. The standard purposely limits the size of each category so that students can focus on understanding the representation of data and comparing quantities represented in a bar graph.

2. Measuring lengths with halves and quarters supports students to build the understanding of the quantity of these common fractions.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Third Grade

3.MD.C. Measurement and Data: Use concepts of area and relate area to multiplication and to addition.

Evidence Outcomes

Students Can:

a. Recognize area as an attribute of plane figures and understand concepts of area measurement. (CCSS: 3.MD.C.6)
   a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. (CCSS: 3.MD.C.6.a)
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. (CCSS: 3.MD.C.6.b)

b. Measure areas by counting unit squares (square cm, square in, square ft, and improvised units). (CCSS: 3.MD.C.6)

c. Use concepts of area and relate area to the operations of multiplication and addition. (CCSS: 3.MD.C.7.a)
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 3.MD.C.7.a)
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (CCSS: 3.MD.C.7.b)
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. (CCSS: 3.MD.C.7.c)
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. (CCSS: 3.MD.C.7.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Professional Skills: Self-Advocacy. Ask questions to develop further personal understanding.

2. MP7. Look for and make use of structure.

Inquiry Questions:

1. How can students transition from concrete models of area to visual models of a closed array and later to an open array as they build their understanding of the connections between repeated addition, multiplicative reasoning and the area of a rectangle?

2. What is the progression of strategies from least sophisticated to most sophisticated for solving a problem involving multiplication using repeated addition and multiplicative reasoning?

Coherence Connections:

1. Relating repeated addition to multiplication with whole numbers in third grade will support students to solve problems involving multiplication of fractions in fourth and fifth grades.

2. Using a visual model to demonstrate the connections between addition and multiplication can be extended to demonstrate the connections between repeated subtraction and division. These connections relate to the composing and decomposing of numbers that students have been doing since kindergarten.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Third Grade

G.M.D. Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Evidence Outcomes

Students Can:

a. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS 3.MD.D.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Inquiry/Analysis. Build on personal experience to specify a challenging problem to investigate.

2. MP1: Make sense of problems and persevere in solving them.

Inquiry Questions:

1. What are all the pairs of side lengths that can create a rectangle with the same area?

2. How can having students write word problems to represent various perimeters support their understanding of area and perimeter?

Coherence Connections:

1. Relating the side length to the area supports students to develop understanding of factors which will support students as they expand the range of numbers and engage in more sophisticated word problems through sixth grade.

2. Transferring the understandings they have developed around perimeter of rectangles to other polygons will support students to develop understandings of geometric shapes and their connection to number and operations.
### Select Grade Level & Standard

**Grade Level:**
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

**Standard:**
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

### Instructions

To leave feedback, click on the comment icon (开发区) next to any item. You can then offer feedback and comments. Once you save your feedback, the icon will change color and show as a checkbox (开发区) so you can keep track of your progress.

### Common Core Information

Colorado's standards for mathematics and reading, writing, and communicating include the Common Core State Standards. Each statement that comes from the Common Core State Standards is followed by the appropriate reference such as (CCSS: 3.NF.2) or (CCSS: 3.L.K-5). View a reference guide to the CCSS reference codes (PDF).

### Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, "Does this make sense?"
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

### Grade Level Expectation: Third Grade


### Evidence Outcomes

**Students Can:**

- Explain that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (CCSS: 3.G.A.1)
- Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape. (CCSS: 3.G.A.2)

### Academic Context and Connections

1. **Colorado Essential Skills and Mathematical Practices:**
   - 1. CES: Creativity/Innovation. Students build on personal experience to specify a challenging problem to investigate.
   - 2. MP2: Reason abstractly and quantitatively.

2. **Inquiry Questions:**
   - 1. How can a shape be partitioned into equal parts in different ways? (For example, will 1/2 of a given shape represent the same area when it looks different?)

3. **Coherence Connections:**
   - 1. Partitioning shapes into equal sizes will support students to use visual models to add, subtract, multiply and divide fractions in future grades.
Mathematics

Grade Level: Fourth Grade
Standard: 1. Number and Quantity

Instructions
To leave feedback, click on the comment icon next to any item. You can then offer feedback and comments. Once you save your feedback, the icon will change color and show as a checkbox so you can keep track of your progress.

Common Core information
Colorado’s standards for mathematics and reading, writing and communicating include the Common Core State Standards. Each statement that comes from the Common Core State Standards is followed by the appropriate reference such as (CCSS: 5.NF.2) or (CCSS: SL.K.5). View a reference guide to the CCSS reference codes (PDF).

Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade
4.NBT.A. Number & Operations in Base Ten: Generalize place value understanding for multi-digit whole numbers.

Evidence Outcomes
Students Can:

a. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division. (CCSS: 4.NBT.A.1)
b. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (CCSS: 4.NBT.A.2)
c. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical Thinking/Problem Solving: Students make connections between information gathered and personal experiences to apply and/or test solutions.
2. MP8: Look for and express regularity in repeated reasoning. Students see that multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.

Inquiry Questions:
1. How can students explore with base ten area pieces or representations to help them understand the magnitude of multiplying by ten or a multiple of ten?

Coherence Connections:
1. Students will rely on their multiplicative reasoning to explore the patterns when multiplying by a multiple of ten.
2. In fifth grade, students will extend their understanding of place value and multiplying by multiples of ten to decimals and dividing by multiples of ten.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade

4 NBT.B Number & Operations in Base Ten: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Evidence Outcomes

Students Can:

a. Fluently add and subtract multi-digit whole numbers using the standard algorithm. (CCSS: 4.NBT.B.4)

b. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.B.5)

c. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical Thinking/Problem Solving. Students make connections between information gathered and personal experiences to apply and/or test solutions.

2. MP3: Construct viable arguments and critique the reasoning of others. Students are able to explain and represent their thinking when multiplying and dividing multi-digit numbers.

3. MP6: Look for and express regularly in repeated reasoning. Repeated reasoning with examples that gain in complexity leads to a general method involving the Grade 4 skill of finding quotients and remainders.

Inquiry Questions:

1. How can a deep understanding of place value support students to leverage the knowledge they have of single digit facts to answer problems involving multi digits?

2. How can the model being used to solve the problem be used to demonstrate the relationship between multiplication and division?

Coherence Connections:

1. Understanding place value and multiplying whole numbers by multiples of ten will support students to multiply and divide multi-digit numbers using place value strategies.

2. Using the area model and arrays to multiply and divide multi-digit numbers in fourth grade will support students to use those same models to explore multiplication and division of fractions and decimals in fifth grade.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade


Evidence Outcomes

Students Can:

a. Explain why a fraction \( \frac{a}{b} \) is equivalent to a fraction \( \frac{nxk}{nxm} \) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (CCSS: 4.N.F.A.1)

b. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as \( \frac{1}{2} \).

Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (CCSS: 4.N.F.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Initiative/Self-Direction. Students apply knowledge to set goals, make informed decisions and transfer to new contexts.

2. MP2: Reason abstractly and quantitatively. Students can reason abstractly about equivalent fractions and fraction comparison.

Inquiry Questions:

1. How does comparing fractions by finding common numerators exemplify understanding of denominators of fractions?

2. How can students demonstrate the fact that multiplying a fraction by \( \frac{n}{n} \) does not change the value (area model)?

Coherence Connections:

1. Finding equivalent fractions by multiplying by \( \frac{n}{n} \) prepares students to find common denominators to add and subtract fractions in fifth grade.

2. Comparing fractions will support students to make reasonable estimates when adding and subtracting fractions with unlike denominators.
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade

4.NF.B. Number & Operations - Fractions: Build fractions from unit fractions.

Evidence Outcomes

Students Can:

a. Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. (CCSS: 4.NF.B.3)
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS: 4.NF.B.3.a)
   b. Decompose a fraction into a sum of fractions with like denominators in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{8} + \frac{2}{8}$. $2\frac{1}{8} = 1 + \frac{1}{8} = \frac{8}{8} + \frac{1}{8}$. (CCSS: 4.NF.B.3.b)
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS: 4.NF.B.3.c)
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (CCSS: 4.NF.B.3.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical Thinking/Problem Solving. Students define the problem using a variety of strategies.

2. MP7: Look for and make use of structure. Students use understanding of the structure of fractions to complete fraction operations.

Inquiry Questions:

1. How can using unit fractions allow students to leverage their knowledge of counting whole numbers when adding and subtracting fractions with like denominators?

2. How can students use repeated addition to exemplify the multiplication of a fraction by a whole number?

Coherence Connections:

1. In sixth and seventh grades, students will develop proportional reasoning which will be supported by the use of unit fractions and scaling up a fraction by multiplying by a whole number.

2. In third grade, students developed a deep understanding of the quantity of fractions through visual models.
b. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS: 4.NF.B.4)
   a. Understand a fraction \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \). For example, use a visual fraction model to represent \( \frac{5}{4} \) as the product \( 5 \times \frac{1}{4} \), recording the conclusion by the equation \( \frac{5}{4} = 5 \times \frac{1}{4} \). (CCSS: 4.NF.B.4.a)
   b. Understand a multiple of \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \), and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express \( 3 \times \frac{2}{5} \) as \( 6 \times \frac{1}{5} \), recognizing this product as \( \frac{6}{5} \).
      \[(\text{In general, } n \times \frac{a}{b} = \frac{nx}{b}.\)]
      (CCSS: 4.NF.B.4.b)
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat \( \frac{3}{8} \) of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? (CCSS: 4.NF.B.4.c)
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade

4.NF.C. Number & Operations - Fractions: Use decimal notation for fractions, and compare decimal fractions.

Evidence Outcomes

Students Can:

a. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) For example, express \( \frac{3}{10} \) as \( \frac{30}{100} \) and add \( \frac{3}{10} + \frac{4}{100} = \frac{34}{100} \) (CCSS: 4.NF.C.5)

b. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as \( \frac{62}{100} \): describe a length as 0.62 meters, locate 0.62 on a number line diagram (CCSS: 4.NF.C.6)

c. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model (CCSS: 4.NF.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Initiative/Self-Direction. Students apply knowledge to set goals, make informed decisions and transfer to new contexts.

2. MP2: Reason abstractly and quantitatively. Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator.

Inquiry Questions:

1. How can expressing tenths as equivalent hundredths in fraction form support students to understand decimal quantities?

2. How can students use a visual model to compare decimals to hundredths?

3. Locating decimals on a number line diagram relates to placing fractions on a number line and prepares students to use the number line as a thinking tool when performing operations on decimals in fifth grade.

Coherence Connections:

1. By exploring fraction decimals in fourth grade, students are more prepared to perform operations on decimals in fifth grade.
Prepared Graduates:

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade

4.OA.A. Operations and Algebraic Thinking: Use the four operations with whole numbers to solve problems.

Evidence Outcomes:

Students Can:

a. Interpret a multiplication equation as a comparison, e.g., interpret 5 × 7 as a statement that 5 × 7 is 35 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.A.1)

b. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See glossary.) (CCSS: 4.OA.A.2)

c. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Creativity/Innovation. Students build on personal experience to specify a challenging problem to investigate.
2. MP1: Make sense of problems and persevere in solving them.

Inquiry Questions:

1. The standard uses the term “times as many as” to describe a multiplicative comparison. How can students use this phrase and problems in context to understand how to distinguish between a multiplicative comparison and an additive comparison?
2. How can students be supported to make sense of word problems so that they will be able to solve problems involving mathematics as they arise in the world around them?

Coherence Connections:

1. In third grade students use repeated addition to engage in multiplicative reasoning. In fifth grade, students will use multiplication to solve problems involving volume.
2. Students will work to better understand place value as they multiply by multiples of ten. This evidence outcome highlights that students will need to have a broader understanding of multiplication as they engage with multiples of all single digit numbers as well.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade

4.OA.B. Operations and Algebraic Thinking: Gain familiarity with factors and multiples.

Evidence Outcomes

Students Can:

a. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS 4.OA.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP7. Look for and make use of structure. Students understand and utilize the relationship between factors and multiples for whole numbers. Students understand the whole number system as composed of prime and composite numbers exclusively.

Inquiry Questions:

1. How can students use an array to explore and determine all of the factors of a given number?

2. How are multiples and factors related to fractional parts of a whole number?

Coherence Connections:

1. Students can use their experiences with exploring area to learn multiplication from third grade to support learning about multiples and factors in fourth grade.

2. Flexibility with the factors and multiples of numbers will support students as they partition whole distances into fractional parts and determine the corresponding distances in fifth grade.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade

4.OA.C. Operations and Algebraic Thinking: Generate and analyze patterns.

Evidence Outcomes

Students Can:

a. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. (CCSS: 4.OA.C.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES. Creativity/Innovation: Students build on personal experience to specify a challenging problem to investigate.

2. MP1. Make sense of problems and persevere in solving them.

Inquiry Questions:

1. How can students explore conjectures about rules?

2. How will students use the rules they generate to solve problems involving unknowns?

Coherence Connections:

1. Generating rules for patterns will support students to understand linear functions in future years.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade

4.MD.A: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Evidence Outcomes

Students Can:

a. Know relative sizes of measurement units within one system of units including km, m, cm, kg, g, lb, oz., l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... (CCSS: 4.MD.A.1)

b. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS: 4.MD.A.2)

c. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. (CCSS: 4.MD.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES. Entrepreneurial Skills: Critical Thinking/Problem Solving. Define the problem using a variety of strategies.

2. MP1: Make sense of problems and persevere in solving them.

Inquiry Questions:

1. How is a deep understanding of place value used to support conversion of units and vice versa?

2. What visual models can students use to make sense of measurement problems and in particular, intervals of time?

3. What experiences can students engage in that enable them to make sense of the magnitude of the largest and smallest conversion measures?

Coherence Connections:

1. Understanding of time is developed across years and the models used to understand and work with time support an underlying understanding of decomposing shapes and fractions of a whole (the whole being 60 minutes).

2. Measurement is leveraged across grade levels to provide students with numbers on which to operate. By anchoring number in the concrete experience of measurement, students are better able to imagine how operations impact larger numbers.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade

4.MD.B. Represent and interpret data.

Evidence Outcomes

Students Can:

a. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. (CCSS: 4.MD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Inquiry/Analysis. Investigate to form hypotheses, make observations and draw conclusions.

2. MP6: Use appropriate tools strategically.

Inquiry Questions:

1. How can informal experiences with data provide foundational understanding to support future data analysis?

2. Where do fractions come up in real life data? Why would it be important to establish the whole when plotting fractions on a line plot? What labels can be used to give the reader all of the context they need to understand the magnitude of the numbers in the line plot?

Coherence Connections:

1. Data is collected in the elementary grades primarily for the purpose of using the numbers.

2. By working with data informally in the elementary grades, students will better be able to engage in conversations about sophisticated data analysis in the middle grades.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fourth Grade


Evidence Outcomes

Students Can:

a. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. (CCSS: 4.MD.C.5)
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. (CCSS: 4.MD.C.5.a)
   b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. (CCSS: 4.MD.C.5.b)

b. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.C.6)

c. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (CCSS: 4.MD.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:


2. MP7: Look for and make use of structure.

Inquiry Questions:

1. How can students use the measure of one angle to tell them important information about another angle?

2. How can the decomposing and composing of angles be related to partitioning and iterating with fractions?

Coherence Connections:

1. Students engage in decomposing and composing throughout mathematics from numbers within 5 to area under a curve. Work with angles can be approached with this same theme to build deeper understanding of number sense.
Mathematics
Grade Level: Fourth Grade
Standard: 4. Geometry

Select Grade Level & Standard

Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

Standard:
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

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1. Prepared Graduates
   1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
   2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
   3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
   4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
   5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

2. Grade Level Expectation: Fourth Grade
4.G.A. Geometry: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Evidence Outcomes
Students Can:

a. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (CCSS: 4.G.A.1)

b. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS: 4.G.A.2)

c. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. (CCSS: 4.G.A.3)

Academic Context and Connections

1. Colorado Essential Skills and Mathematical Practices:
   a. CES Perseverance/Resilience: Students set goals and develop strategies to remain focused on learning goals.
   b. MP7: Look for and make use of structure.

2. Inquiry Questions:
   a. How can the line of symmetry in a figure be connected to the partitioning of a shape?

3. Coherence Connections:
   a. Students have been partitioning shapes to learn the quantity of fractions and understand the relationship of a fraction to its whole.
   b. In fifth grade, students will shift from mere classification of shapes to understand how attributes of a shape can place it in multiple categories at the same time. For example, they will classify quadrilaterals and identify the overlaps in classifications.
Mathematics
Grade Level: Fifth Grade
Standard: 1. Number and Quantity

Select Grade Level & Standard
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, "Does this make sense?"
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade
5.NBTA. Number & Operations in Base Ten. Understand the place value system.

Evidence Outcomes
Students Can:

a. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (CCSS: 5.NBT.A.1)

b. Explain patterns in the number of zeros in the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10 (CCSS: 5.NBT.A.2)

c. Read, write, and compare decimals to thousandths. (CCSS: 5.NBT.A.3)
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × 1/10 + 9 × 1/100 + 2 × 1/1000. (CCSS: 5.NBT.A.3a)
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (CCSS: 5.NBT.A.3b)

d. Use place value understanding to round decimals to any place. (CCSS: 5.NBT.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Perseverance/Resilience. Students set goals and develop strategies to remain focused on learning goals.
2. MP7: Look for and make use of structure. Students understand why multiplying by a power shifts the digits of a whole number or decimal that many places to the left.

Inquiry Questions:

1. How can students use a visual model to understand the place value of tenths, hundredths, and thousandths?
2. How can fractions be used to support students to understand the quantity of decimals?

Coherence Connections:

1. Work with decimal fractions in fourth grade set the groundwork for students to solve problems involving decimals in fifth grade.
2. Extensive place value understanding from prior grades supports students to understand the quantity of decimals in fifth grade.
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade

5.NBT.B. Number & Operations in Base Ten: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Evidence Outcomes

**Students Can:**

a. Fluently multiply multi-digit whole numbers using the standard algorithm. (CCSS: 5.NBT.B.5)

b. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 5.NBT.B.6)

c. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS: 5.NBT.B.7)

Academic Context and Connections

**Colorado Essential Skills and Mathematical Practices:**

1. CES: Times/Time management. Students develop and utilize basic task and time management strategies effectively.

2. MP4. Model with mathematics. Students illustrate and explain their calculations using equations, rectangular arrays, and/or area models.

**Inquiry Questions:**

1. How can using concrete models to perform operations on decimals support students to understand how decimals relate to whole numbers and fractions?

2. How can connecting one visual model (representing the operation they are performing on decimals) to another support students to more clearly understand the operation?

**Coherence Connections:**

1. Students used concrete models and visual models to understand the operations on whole numbers and fractions in past years. Using those same models to understand operations on decimals will support a deeper understanding of the operation and of decimals.

2. Work with decimal fractions in fourth grade will support students to evaluate the reasonableness of solutions to problems involving decimals.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade

6. NFA. Number & Operations - Fractions: Use equivalent fractions as a strategy.

Evidence Outcomes

Students Can:

a. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example,

\[
\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12} \quad \text{(in general,)}
\]

\[
\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \text{(CCSS: 5.NFA.1)}
\]

b. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result

\[
\frac{2}{5} + \frac{1}{2} = \frac{3}{7}, \quad \text{by observing that} \quad \frac{3}{7} < \frac{1}{2}
\]

(CCSS: 5.NFA.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Task/Time Management. Students develop and utilize basic task and time management strategies effectively.

2. MP2: Reason abstractly and quantitatively. Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense.

Inquiry Questions:

1. How can using benchmark fractions help students estimate a reasonable answer to a problem involving fractions?

2. How is the concept of the number of groups and the quantity in each group from division related to the number of partitions represented by a unit fraction?

Coherence Connections:

1. Using equivalent fractions to compare fractions in fourth grade prepared students to use equivalent fractions to find a common denominator in fifth grade.

2. In general, two fractions can be added by subdividing the unit fractions in one using the denominator of the other. If it is not necessary to find a least common denominator to calculate sums of fractions, and the effort of finding a least common denominator can distract from understanding algorithms for adding fractions.
1. Prepared Graduates
   1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
   2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"
   3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
   4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
   5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

2. Grade Level Expectation: Fifth Grade
   5.NF.B. Number & Operations - Fractions: Apply and extend previous understandings of multiplication and division.

3. Evidence Outcomes
   **Students Can:**
   a. Interpret a fraction as division of the numerator by the denominator ( \( \frac{a}{b} = a \div b \)). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret \( \frac{3}{2} \) as the result of dividing 3 by 2, noting that \( \frac{1}{4} \) multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size \( \frac{3}{4} \). If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? (CCSS: 5.NF.B.3)
   b. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (CCSS: 5.NF.B.4)
      a. Interpret the product \( \frac{a}{b} \times q \) as a parts of a partition of \( q \) into \( b \) equal parts, equivalently, as the result of a sequence of operations \( a \times q \div b \). For example, use a visual fraction model to show \( \frac{2}{3} \times \frac{4}{5} = \frac{8}{15} \), and create a story context for this equation. Do the same with \( \frac{2}{3} \times \frac{4}{5} = \frac{8}{15} \) (In general), \( \frac{\frac{4}{5}}{\frac{2}{3}} = \frac{8}{15} \) (CCSS: 5.NF.B.4.a)
      b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS: 5.NF.B.4.b)

4. Academic Context and Connections
   **Colorado Essential Skills and Mathematical Practices:**
   1. CES inquiry/Analysis. Students investigate to form hypothesis, make observations and draw conclusions.
   2. MP6: Attend to precision. Students attend carefully to the underlying unit quantities when solving problems involving multiplication and division of fractions.
   **Inquiry Questions:**
   1. How can students use visual models to find the product of two fractions?
   2. How can students use precise language to describe the division represented by partitioning various wholes into a given number of equal parts?
   **Coherence Connections:**
   1. Dividing a fraction by a whole number or a whole number by a fraction prepares students for dividing fractions by fractions in sixth grade.
c. Interpret multiplication as scaling (resizing), by: (CCSS: 5.NF.B.5)
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. (CCSS: 5.NF.B.5.a)
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number, and relating the principle of fraction equivalence \( \frac{a}{\frac{b}{c}} = \frac{a}{b} \) to the effect of multiplying \( \frac{a}{b} \) by 1. (CCSS: 5.NF.B.5.b)

d. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.B.6)

e. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) (CCSS: 5.NF.B.7)
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for \( \frac{1}{3} \div 4 \) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( \frac{1}{3} \div 4 = \frac{1}{12} \) because \( \frac{1}{3} \times 4 = 1 \frac{1}{3} \). (CCSS: 5.NF.B.7.a)
   b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for \( 4 \div \frac{1}{5} \) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( 4 \div \frac{1}{5} = 20 \) because \( 20 \times \frac{1}{5} = 4 \). (CCSS: 5.NF.B.7.b)
   c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share \( \frac{1}{2} \) lb of chocolate equally? How many \( \frac{3}{2} \)-cup servings are in 2 cups of raisins? (CCSS: 5.NF.B.7.c)
Grade Level: Fifth Grade
Standard: 2. Algebra and Functions

Instructions
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade
5.OA.A. Operations and Algebraic Thinking
Write and interpret numerical expressions.

Evidence Outcomes
Students Can:
a. Use grouping symbols (parentheses, brackets, or braces) in numerical expressions, and evaluate expressions with these symbols. (CCSS: 5.OA.A.1)
b. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18032 + 921) is three times as large as 18032 + 921, without having to calculate the indicated sum or product. (CCSS: 5.OA.A.2)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. CES: Self-Advocacy. Students ask questions to develop further personal understanding.
2. MP2: Reason abstractly and quantitatively.

Inquiry Questions:
1. How can students use grouping symbols and order of operations to maximize or minimize the value of an expression?

Coherence Connections:
1. Exploring with grouping symbols and properties will support students to learn how to solve equations.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade

5.OA.B. Operations and Algebraic Thinking: Analyze patterns and relationships.

Evidence Outcomes

Students Can:

a. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. (CCSS. 5.OA.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CE3. Critical Thinking/Problem Solving. Students make connections between information gathered and personal experiences to apply and/or test solutions.

2. MP7. Look for and make use of structure.

Inquiry Questions:

1. What can students predict about a numerical pattern based on the rule they will use to generate the pattern?

Coherence Connections:

1. In Grade 4, students generated rules from patterns. This year they generate patterns from rules.

2. Generating numerical patterns from rules will prepare students for generating a table for a function in future grades.
Mathematics
Grade Level: Fifth Grade
Standard: 3. Data, Statistics, and Probability

Select Grade Level & Standard

Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

Standard:
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and checking course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, "Does this make sense?"
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade
5.MD.A. Measurement and Data: Convert like measurement units within a given measurement system.

Evidence Outcomes
Students Can:
- Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. (CCSS: 5.MD.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:
1. CES: Information Literacy. Students articulate the most effective options to access information needed for a specific purpose.
2. MP7: Look for and make use of structure.

Inquiry Questions:
1. How is a deep understanding of place value used to support conversion of units and vice versa?
2. What experiences can students engage in that enable them to make sense of the magnitude of the largest and smallest conversion measures?

Coherence Connections:
1. Students have engaged in converting within a given measurement system in prior grades. This year the conversion is used to support the understanding of decimals within the grade level standards.
2. Working with units of measure that are in base ten will support students to use and understand scientific notation in future grades.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade

5.MD.B. Measurement and Data: Represent and interpret data.

Evidence Outcomes

Students Can:

a. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. (CCSS: 5.MD.B.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CE3: Inquiry/Analysis. Students investigate to form hypotheses, make observations and draw conclusions.

2. MP5: Use appropriate tools strategically.

Inquiry Questions:

1. How can line plots be used as a backdrop for conversations around the relationship between various quantities?

2. Using students’ natural curiosities, what inferences can be made about the data they are displaying?

Coherence Connections:

1. Students will be formally analyzing data in sixth and seventh grades. These informal explorations with data using rational numbers will support the data analysis they will encounter in later grades.

2. Using fractions in the line plots will provide additional opportunities for students to make sense of operations on rational numbers which is the major work of the grade.
Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade
5.MD.C. Measurement and Data: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Evidence Outcomes
Students Can:
a. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. (CCSS: 5.MD.C.3)
   a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. (CCSS: 5.MD.C.3a)
   b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. (CCSS: 5.MD.C.3b)

b. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (CCSS: 5.MD.C.4)

c. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. (CCSS: 5.MD.C.5)
   a. Model the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (CCSS: 5.MD.C.5a)
   b. Apply the formulas V = l × w × h and V = b × h for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. (CCSS: 5.MD.C.5b)
   c. Use the additive nature of volume to find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. (CCSS: 5.MD.C.5c)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. CES: Critical Thinking/Problem Solving. Students define the problem using a variety of strategies.
2. MP1: Make sense of problems and persevere in solving them.

Inquiry Questions:
1. How can students use the models for volume to explore and better understand the products of rational numbers?
2. What generalizations can students make about multiplication and division by using the models for volume?

Coherence Connections:
1. Students have worked with area and perimeter in previous grades. An anchoring volume understanding in the concept of the area of the base will support deeper understanding of both concepts.
2. The work that students have been doing with multiplication and division can be reinforced while exploring and learning volume.
Mathematics

Grade Level: **Fifth Grade**

Standard: **4. Geometry**

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### Instructions

To leave feedback, click on the comment icon next to any item. You can then offer feedback and comments.

Once you save your feedback, the icon will change color and show as a checkbox so you can keep track of your progress.

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### Common Core Information

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### Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

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### Grade Level Expectation: Fifth Grade

5.G.A. Geometry: Graph points on the coordinate plane to solve real-world and mathematical problems.

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### Evidence Outcomes

**Students Can:**

- Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). (CCSS: 5.G.A.1)
- Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (CCSS: 5.G.A.2)

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### Academic Context and Connections

**Colorado Essential Skills and Mathematical Practices:**

1. MP2: Reason abstractly and quantitatively.
2. MP4: Model with mathematics.

**Inquiry Questions:**

1. How are the axes of the coordinate plane similar to number lines students have been using as a thinking tool to help them solve problems?
2. How are the streets in Denver Colorado set up in a coordinate plane? Where is the origin?

**Coherence Connections:**

1. Students explore the first quadrant of the coordinate plane in fifth grade in order to provide experiences that help them learn about the four quadrants of the coordinate plane and integers in sixth grade.
2. Work with number line throughout elementary has prepared students to explore the axes of the coordinate plane in fifth and sixth grades.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Fifth Grade

5.G.B. Geometry: Classify two-dimensional figures into categories based on their properties.

Evidence Outcomes

**Students Can:**

a. Explain that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. (CCSS: 5.G.B.3)

b. Classify two-dimensional figures in a hierarchy based on properties. (CCSS: 5.G.B.4)

Academic Context and Connections

**Colorado Essential Skills and Mathematical Practices:**

1. CES: Information Literacy. Students articulate the most effective options to access information needed for a specific purpose.

2. MP2: Reason abstractly and quantitatively.

**Inquiry Questions:**

1. How can the words always, sometimes and never support students to explore classifications of figures?

**Coherence Connections:**

1. Classifying shapes by attributes in previous grades has prepared students to classify figures in a hierarchy based on properties in fifth grade.

2. Using properties to defend classification is an informal experience with proof that will support students in justifying their reasoning when solving mathematics problems.
Prepared Graduates:
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Sixth Grade
6.RPA. Ratios & Proportional Relationships: Understand ratio concepts and use ratio reasoning to solve problems.

Evidence Outcomes
Students Can:

a. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” For every vote candidate A received, candidate B received nearly three votes. (CCSS: 6.RPA.1)

b. Apply the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of $3$ cups of flour to $4$ cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar.” “We paid $75$ for $15$ hamburgers, which is a rate of $5$ per hamburger.” (Expectations for unit rates in this grade are limited to non-complex fractions.) (CCSS: 6.RPA.2)

c. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (CCSS: 6.RPA.3)

(1) a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (CCSS: 6.RPA.3.a)
(2) b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? (CCSS: 6.RPA.3.b)
(3) c. Find a percent of a quantity as a rate per $100$ (e.g., $30\%$ of a quantity means $30\times$ the quantity); solve problems involving finding the whole, given a part and the percent. (CCSS: 6.RPA.3.c)
(4) d. Use ratio reasoning to convert measurement units, manipulate and transform units appropriately when multiplying or dividing quantities. (CCSS: 6.RPA.3.d)
Colorado Academic Standards Review
MATHMATICS
Sixth Grade: Number and Quantity

Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Sixth Grade

6.NS.A. The Number System: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Evidence Outcomes

Students Can:

a. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, if 2/3 of a day is budgeted for a road trip, how long does each person spend on the road trip?

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Entrepreneurial Skills: Critical Thinking/Problem Solving and Creativity/Innovation. Students can create and solve word problems using division of fractions, understanding the relationship of the arithmetic to the problem being solved.
2. MP2: Reason abstractly and quantitatively. In solving word problems involving division of fractions, students will have the opportunity to reason about the meaning of the numbers in context, as well as separate from context as they perform the operations.
3. MP4: Model with mathematics. Students can use division of fractions problems to model real-world situations for which this applies.
4. MP7: Look for and make use of structure. Students often view operations with fractions as an entirely new set of rules and understandings than operations with whole numbers. Within this GLE, there is opportunity for students to understand that the underlying structure of division is consistent across all numbers, including fractions.

Inquiry Questions:

1. When using division, is the quotient always going to be a smaller number than the dividend? Why or why not?
2. What kinds of real-world situations require the division of fractions?
3. How can the division of fractions be modeled visually?

Coherence Connections:

1. In Grade 5, students modeled and computed situations involving whole numbers divided by unit fractions and unit fractions divided by whole numbers. This GLE extends work with adding, subtracting, multiplying, and dividing fractions by focusing on situations that require dividing fractions by fractions and modeling the operation.
2. Students will apply operations with fractions in Grade 7 with negative rational numbers.
3. This GLE requires greater emphasis based on the depth of the topics, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Sixth Grade

6.NS.B. The Number System: Compute fluently with multi-digit numbers and find common factors and multiples.

Evidence Outcomes

Students Can:

a. Fluently divide multi-digit numbers using the standard algorithm. (CCSS: 6.NS.B.2)

b. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (CCSS: 6.NS.B.3)

c. Find the greatest common factor of two whole numbers or equal to 100 and the least common multiple of two whole numbers or equal to 100. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 9 as 4(9 + 2). (CCSS: 6.NS.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Personal Skills: Adaptability/Flexibility. Students can adapt their thinking to be flexible in their use of different algorithms.

2. MP6: Attend to precision. Accuracy and precision are important when students add, subtract, multiply, and divide with decimals.

3. MP7: Look for and make use of structure. This GLE allows students to recognize the structures of factors and multiples of whole numbers in identifying a greatest common factor and least common multiple of two whole numbers. Additional structure can be observed while using the GCF to rewrite the expression using the distributive property.

Inquiry Questions:

1. How do operations with decimals compare and contrast to operations with whole numbers?

2. What does it mean for an algorithm to be efficient, flexible, and accurate?

3. How does rewriting the sum of two whole numbers using the distributive property yield new understanding and insights on the sum?

Coherence Connections:

1. In Grade 5, students developed understanding of dividing whole numbers with two-digit divisors using models and connecting to multiplication. Grade 6 focuses on developing fluency with division, concluding operations with whole numbers developed in Grades K-5. Students have developed understanding and explored various strategies to add, subtract, multiply, and divide decimals in Grades 4 and 5. The focus of Grade 6 is to find efficient, flexible, and accurate algorithms to perform these operations.

2. Students will apply the concept of greatest common factor and rewriting numeric expressions using the distributive property in Grade 6 with simple algebraic expressions containing a variable. This work will continue in Grade 7, laying the foundation for algebra and by connecting properties of whole numbers to expressions containing a variable.
# Colorado Academic Standards Review

## MATHEMATICS

### Sixth Grade: Number and Quantity

#### Grade Level Expectation: Sixth Grade

6.NS.C: The number system. Apply and extend previous understandings of numbers to the system of rational numbers.

#### Evidence Outcomes

**Students Can:**

1. Compare any positive and negative numbers using the coordinate plane or number line.

2. Describe any rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. (CCSS: 6.NS.C.6)
   - a. Use opposite signs of numbers as indicating locations on opposite sides of 0 on the number line. Identify the opposites of a number, e.g., 3 and -3.
   - b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. Explain that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (CCSS: 6.NS.C.6)
   - c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram. Find and position pairs of integers and other rational numbers on a coordinate plane. (CCSS: 6.NS.C.6)
   - d. Order and find absolute value of rational numbers. (CCSS: 6.NS.C.7)
     - a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right. (CCSS: 6.NS.C.7)
     - b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3°C > -7°C to express the fact that -3°C is warmer than -7°C. (CCSS: 6.NS.C.7)
     - c. Define the absolute value of a rational number as its distance from 0 on the number line. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write | -30 | = 30 to describe the size of the debt in dollars. (CCSS: 6.NS.C.7)
     - d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an amount balance less than -30 dollars represents a debt greater than 30 dollars. (CCSS: 6.NS.C.7)
     - e. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Use coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS: 6.NS.C.8)

#### Academic Context and Connections

**Colorado Essential Skills and Mathematical Practices:**

1. **CEP:** Entrepreneurial Skills. Inquiry/Analysis. Students can investigate to form hypotheses, make observations, and draw conclusions.
2. **MF4:** Model with mathematics. Students model real-world situations using the mathematics of rational numbers.
3. **MF4:** Model with precision. Students accurately place negative numbers on a number line and can compare the relative positions of two quantities using precise mathematical language and symbols.

**Inquiry Questions:**

1. Why do we have negative numbers?
2. What relationships exist among positive and negative numbers on the number line?
3. How does the opposite of a number differ from the absolute value of that same number?
4. How does an ordered pair correspond to a point on a coordinate plane?

**Coherence Connections:**

1. This GLE extends the number line to include negative numbers. Students have used number lines in earlier grades to order, compare, and perform operations with positive whole numbers. Number lines have also been used in measurement contexts to describe lengths of objects. Students will be introduced to negative numbers in describing situations where the magnitude and direction of a quantity are natural (e.g., changes in temperature or elevation).
2. Students will extend these initial understandings of negative numbers and relationships on the number line in Grade 7 with operations involving integers and rational numbers.
3. This GLE requires greater emphasis based on the depth of the ideas, the time it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Edward Cardwell, one of the most important figures in the American school system, believed in a system of education that was based on the theory of the four Rs: Reading, Writing, Arithmetic, and Religion. He advocated for the importance of these subjects in the curriculum and their role in the overall development of students. Cardwell believed that a strong foundation in these subjects would prepare students for success in life. He also emphasized the importance of discipline and respect for teachers and authority figures.

The Cardwell system was widely adopted in the United States in the late 19th century, and it remained influential for many years. Cardwell's beliefs and teachings continue to influence the way education is structured and delivered in many schools today. His emphasis on a strong foundation in the four Rs and the importance of discipline and respect for authority figures remains relevant in modern education.
**Prepared Graduates**

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

### Grade Level Expectation: Sixth Grade

6.EE.B. Expressions & Equations: Reason about and solve one-variable equations and inequalities.

### Evidence Outcomes

**Students Can:**

a. Describe solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS: 6.EE.B.5)

b. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem, recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (CCSS: 6.EE.B.6)

c. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p, q$ and $x$ are all nonnegative rational numbers. (CCSS: 6.EE.B.7)

d. Write an inequality of the form $x \geq c$ or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. Show that inequalities of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS: 6.EE.B.8)

### Academic Context and Connections

**Colorado Essential Skills and Mathematical Practices:**

1. CES Entrepreneurial Skills: Inquiry/Analysis. Students can investigate to form hypotheses, make observations, and draw conclusions.

2. MP2: Reason abstractly and quantitatively. When solving algebraic equations that arise from real-world contexts, students have the opportunity to reason about the numbers and operations both within the context and separate from it.

3. MP6: Attend to precision. As with all their work with variables, it is important for students to state precisely the meaning of variables they use when setting up equations.

### Inquiry Questions:

1. What are the different ways a variable can be used in an algebraic equation or inequality? In other words, how are these uses of the variable $x$ different from each other? (a) $x + 5 = 8$, (b) $x = \frac{1}{2}$, (c) $x > 5$.

2. How is the solution to an inequality different than a solution to an equation?

### Coherence Connections:

1. Solving is a process of reasoning to find the numbers which make an equation true, which can include checking if a given number is a solution. Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off, such as in $4x + 3x = 3x + 20$, where they can see that $4x$ must be $20$ to make the two sides equal. This will become a skill of greater importance as the algebra becomes more complex in future grades.

2. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

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5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Sixth Grade

6.EE.C. Expressions & Equations: Represent and analyze quantitative relationships between dependent and independent variables.

Evidence Outcomes

Students Can:

a. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation \( d = 65t \) to represent the relationship between distance and time. (CCSS: 6.EE. C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Inquiry/Analysis. Students can investigate to form hypotheses, make observations and draw conclusions.

2. MP2: Reason abstractly and quantitatively. When analyzing a relationship between two variables, students have an opportunity to reason about the operations that relate numerical and variable quantities.

3. MP4: Model with mathematics. By applying the mathematics of equations and inequalities with variables to real-world situations, students are modeling with mathematics.

Inquiry Questions:

1. How can you determine if a variable is independent or dependent?

2. What are the advantages of showing the relationship between an independent and dependent variable in multiple representations (table, graph, equation)?

Coherence Connections:

1. Through the work of this GLE, students begin to develop a dynamic understanding of variables, the quantities they represent, and how they relate to each other.

2. The work with independent and dependent variables in Grade 6 prepares students for work with graphing relationships and functions in later grades.

3. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Colorado Academic Standards Review
Mathematics
Sixth Grade: Data, Statistics, & Probability

Instructions
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, "Does this make sense?"
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Sixth Grade

Evidence Outcomes
Students Can:

a. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages (CCSS. 6.SPA.1).

b. Demonstrate that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (CCSS. 6.SPA.2)

c. Explain that a measure of center for a numerical data set summarizes all of its values with a single number; while a measure of variation describes how its values vary with a single number. (CCSS. 6.SPA.3)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:

1. CES/Civic/Interpersonal: Global/Cultural Awareness. Students can identify questions that allow multiple perspectives to be collected.
2. MP1. Make sense of problems and persevere in solving them. Students can make sense of practical problems by turning them into statistical investigations.
3. MP2. Reason abstractly and quantitatively. Students can move from context to abstraction and back to context within statistical investigations.

Inquiry Questions:
1. What distinguishes a statistical question from a question that is not a statistical question?
2. Why do we have numerical measures for both the center of a data set and the variation of a data set?

Coherence Connections:
1. In early grades, students displayed data with dot plots (sometimes called line plots) when working with counts or measurements. In Grade 6, students will work to develop a deeper understanding of variability and more precise descriptions of data distributions, using numerical measures of center and spread, and terms such as cluster, peak, gap, symmetry, skew, and outlier.
2. In Grade 7, students will move to the production of data by understanding that good answers to statistical questions depend on quality data collection relevant to the questions of interest.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Sixth Grade


Evidence Outcomes

Students Can:

a. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (CCSS: 6.SP.B.4)

b. Summarize numerical data sets in relation to their context, such as by:
   (CCSS: 6.SP.B.5)
   a. Reporting the number of observations. (CCSS: 6.SP.B.5.a)
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. (CCSS: 6.SP.B.5.b)
   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (CCSS: 6.SP.B.5.c)
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS: 6.SP.B.5.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Civic/Interpersonal Skills: Communication. Students can consider purpose and formality of context and audience when planning content, mode, delivery, and expression.

2. MP2: Reason abstractly and quantitatively. Students can move from context to abstraction and back to context while finding and using measures of center and variability and describing what they mean in the context of the data.

3. MP7: Look for and make use of structure. Students can identify clusters, peaks, and gaps, recognizing common shapes and patterns in these displays of data distribution.

Inquiry Questions:

1. When is one data display better than another?

2. How can different data displays communicate different meanings?

3. When is it better to use the mean as a measure of center? Why?

4. When is it better to use the median as a measure of center? Why?

Coherence Connections:

1. In early grades, students displayed data with dot plots (sometimes called line plots) when working with counts or measurements. Students will extend this to working with larger data sets with the use of histograms and box plots.

2. The work with summarizing numerical data sets in relation to their context prepares students for work with making inferences and justifying conclusions from sample surveys, experiments, and observational studies, as well as interpreting categorical and quantitative data in high school.

3. Grade 7 is where students first work formally with quantitative measures of center and variability.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Sixth Grade


Evidence Outcomes

Students Can:

a. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.1)

b. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = lwh and V = Bh to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.2)

c. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.3)

d. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Critical Thinking/Problem Solving. Students recognize that problems can be identified and possible solutions can be created with respect to using area, surface area, and volume.

2. MP1: Make sense of problems and persevere in solving them. Students must make sense of a given problem by first understanding the correct context of the problem and then use the correct formula.

3. MP2: Reason abstractly and quantitatively. Being able to reason abstractly is a necessary skill when students work with nets and when they draw polygons in order to solve real-world and mathematical problems. The use of formulas provides students with the opportunity to reason quantitatively.

4. MP4: Model with mathematics. Students are able to understand and apply formulas and geometric concepts in a variety of real-world contexts.

5. MP7: Look for and make use of structure. Students understand the relationships between area and volume.

Inquiry Questions:

1. What is the difference between what area measures and what volume measures?

2. How does using decomposition aid in finding the area of composite figures?

3. How are nets of three-dimensional figures used to find surface area?
Coherence Connections:

1. In Grade 5, students plot points in the first quadrant of the coordinate plane as ordered pairs, understanding that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis. In earlier grades, students explore area of rectangles and volume of rectangular prisms, including irregular shapes that can be decomposed into rectangles and rectangular prisms to calculate areas and volumes.

2. Students use their understanding of one-dimensional measures, such as length and width, to determine area. The addition of height to length and width provides students with the opportunity to determine volume and surface area of three-dimensional figures.

3. In Grades 7 and 8, students extend concepts of area and volume to solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms, and use the formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems.
Grade Level: Seventh Grade

Standard: Number and Quantity

Evidence Outcomes

Students Can:

a. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in every 1/4 hour, compute the unit rate as the complex fraction 2/3 miles per hour, equivalently 2 miles per hour. (CCSS: 7.RPA.1)

b. Identify and represent proportional relationships between quantities. (CCSS: 7.RPA.2)
   a. Determine whether two quantities are in a proportional relationship, e.g. by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS: 7.RPA.2.a)
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS: 7.RPA.2.b)
   c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = np. (CCSS: 7.RPA.2.c)
   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. (CCSS: 7.RPA.2.d)

c. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. (CCSS: 7.RPA.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Entrepreneurial Skills. Critical Thinking/Problem Solving and Inquiry/Analysis. Students recognize that problems can be identified and possible solutions can be created; students are able to make predictions.

2. MP1. Make sense of problems and persevere in solving them. This GLE allows students to recognize, identify, and solve problems that involve proportional relationships to make predictions and describe associations among variables.

3. MP2: Reason abstractly and quantitatively. Students analyze and use appropriate quantities and pay attention to units in problems that require reasoning with proportional relationships.

Inquiry Questions:

1. How are proportional relationships related to ratios?

2. How can proportional relationships be expressed using tables, equations, and graphs?

3. What are properties of all proportional relationships when graphed on the coordinate plane? Why are these properties true?

Coherence Connections:

1. Students use their knowledge of ratios, equivalent ratios, and unit rates in Grade 6 to study proportional relationships and representing those relationships in tables, equations, and graphs. Students also apply unit rates to situations with fractions and unlike units and compute multi-step problems with percentages, particularly in personal finance situations. The concept of proportionality is important in presenting graphs, equations, and proportions; they are a conceptual introduction to slope and rate of change in linear situations.

2. In Grade 8, students will extend the concept of proportional relationships in graphs to formally explore slope and the general form of a linear equation with y-intercept at any value b.

3. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and the importance to future mathematics or the demands of college and career readiness.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Seventh Grade

7.NS.A. The Number System. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Evidence Outcomes

Students Can:

a. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers, represent addition and subtraction on a horizontal or vertical number line diagram. (CCSS: 7.NS.A.1)

   a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constitutents are oppositely charged. (CCSS: 7.NS.A.1.a)

   b. Demonstrate \( p + q \) as the number located a distance \(|q|\) from \( p \), in the positive or negative direction depending on whether \( q \) is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses).

   c. Demonstrate subtraction of rational numbers as adding the additive inverse, \( p - q = p + (-q) \). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (CCSS: 7.NS.A.1.b)

   d. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.A.1.d)

b. Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers. (CCSS: 7.NS.A.2)

   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as \((-1)(-1) = 1\) and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.2.a)

   b. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.A.2.b)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Inquiry/Analysis. Students can recognize and describe patterns.

2. CES Personal Skills: Adaptability/Flexibility. Students can adapt their reasoning with integers to flexibly use both models and algorithms.

3. MP3: Construct viable arguments and critique the reasoning of others. As students move from initial to complete understanding of the operations with rational numbers, they would benefit from verbal and written explanations of their understanding.

4. MP8: Look for and express regularity in repeated reasoning. This GLE allows students to recognize the regularity and repeated reasoning as they use previous understanding of the properties of operations to add, subtract, multiply, and divide integers and rational numbers.

Inquiry Questions:

1. How do operations with integers compare and contrast to operations with whole numbers?

2. How can operations with negative integers be modeled visually?

3. How can you determine if a fraction is represented as a terminating or repeating decimal?

Coherence Connections:

1. Students have been learning about the rational number system in earlier grades, using number lines to locate and compare fractions, decimals, and integers. Students also studied the concept of absolute value in Grade 6 and using integers to describe situations. Work in Grade 7 focuses on understanding operations with rational numbers, specifically the integers and negative rational numbers.

2. Students will take decimal expansions and convert them to rational numbers in Grade 8, and will expand their understanding of the number system to include irrational numbers.

3. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If \( p \) and \( q \) are integers, then \( \left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q} \).

   Interpret quotients of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.2.b)

c. Apply properties of operations as strategies to multiply and divide rational numbers. (CCSS: 7.NS.A.2.c)

d. Convert a rational number to a decimal using long division, knowing that the decimal form of a rational number terminates in 0s or eventually repeats. (CCSS: 7.NS.A.2.d)

c. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) (CCSS: 7.NS.A.3)
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Seventh Grade

7 EE.A. Expressions & Equations: Use properties of operations to generate equivalent expressions.

Evidence Outcomes

Students Can:

a. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (CCSS 7.EE.A.1)

b. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.” (CCSS 7.EE.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Interpersonal Skills: Character. Students can state a position and reflect on possible objections to, assumptions and implications of the position.

2. MP7: Look for and make use of structure. Students recognize that the structures of equivalent algebraic expressions provide them with different ways of seeing the same problem.

Inquiry Questions:

1. How is it determined that two algebraic expressions are equivalent?

2. What is the value of having an algebraic expression in equivalent forms?

3. This GLE requires greater emphasis based on the depth of the ideas, the time it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
# Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

# Grade Level Expectation: Seventh Grade

7.EE.B: Expressions & Equations: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

# Evidence Outcomes

<table>
<thead>
<tr>
<th>Students Can:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. For example: if a woman making $25 an hour gets a 10% raise, she will make an additional 2/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 3/4 inches wide, you will need to place the bar about 9 inches from each edge, this estimate can be used as a check on the exact computation. (CCSS: 7.EE.B.3)</td>
</tr>
<tr>
<td>b. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (CCSS: 7.EE.B.4)</td>
</tr>
<tr>
<td>a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? (CCSS: 7.EE.B.4.a)</td>
</tr>
<tr>
<td>b. Solve word problems leading to inequalities of the form px + q &gt; r, px + q ≥ r, px + q &lt; r, or px + q ≤ r where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.</td>
</tr>
</tbody>
</table>

# Colorado Essential Skills and Mathematical Practices:

1. CES Personal Skills: Adaptability/Flexibility. Students can demonstrate ways to adapt and reach workable solutions.
2. UPS: Make sense of problems and persevere in solving them. Students use mental computation and estimation to check the reasonableness of their solutions. They make connections the sequence of operations used in an algebraic approach versus an arithmetic approach, understanding how simply reasoning about the numbers connects to writing and solving a corresponding algebraic equation.
3. UPS: Use appropriate tools strategically. Students calculate using positive and negative rational numbers, using mental math strategies and estimation when appropriate, and calculate when appropriate.
4. UPS: Attend to precision. Students use mathematical symbols accurately to create equations and inequalities that model real-world situations.

# Inquiry Questions:

1. Do algebraic properties work with numbers or just symbols? Why?
2. Why are there different ways to solve equations?
3. In what scenarios might estimation be better than an exact answer?
4. How can the reasonableness of a solution be determined?

# Coherence Connections:

1. Students in Grade 7 are beginning to connect their work with whole numbers, integers, and positive and negative fractions to the system of rational numbers. Creating and solving two-step equations and inequalities builds from work in Grade 6 with one-step equations and inequalities.
2. In Grade 8, students will solve multi-step equations with rational number coefficients, using the distributive property, with variables on both sides.
3. This GLE requires greater emphasis based on the depth of the idea, the time that it takes to master, and/ or the importance to future mathematics or the demands of college and career readiness.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Seventh Grade

7.3PA. Statistics & Probability: Use random sampling to draw inferences about a population.

Evidence Outcomes

Students Can:

a. Understand that statistics can be used to gain information about a population by examining a sample of the population. Explain that generalizations about a population from a sample are valid only if the sample is representative of that population. Explain that random sampling tends to produce representative samples and support valid inferences. (CCSS: 7.SP.A.1)

b. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book, predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. (CCSS: 7.SP.A.2)

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Critical Thinking, Problem Solving, and Inquiry. Analyze. Students can interpret information gathered and draw conclusions based on the best analysis.
2. MPS: Construct viable arguments and critique the reasoning of others. Students can make comparisons that may involve making conjectures about populations parameters and constructing arguments based on data to support the conjectures.

Inquiry Questions:

1. Why would a researcher use sampling for a study or survey?
2. Why does random sampling give more reliable results in a study or survey?
3. How might other methods of obtaining a sample for a study or survey affect the results of the survey?
4. How can a winner be concluded in an election, from a sample, before counting all the ballots?

Coherence Connections:

1. The work within this GLE connects to one of the most important concepts of the grade (Proportional Relationships) in that students will understand that the sample proportion is the best estimate of the population proportion, while realizing that the two are not the same and a different sample will give a slightly different estimate.
2. In earlier grades, students have been using data, both categorical and measurement, to answer simple statistical questions, but have paid little attention to how the data were selected. A primary focus for Grade 7 is the process of selecting a random sample, and the value of doing so.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Seventh Grade


Evidence Outcomes

Students Can:

a. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. (CCSS: 7.SP.B.3)

b. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (CCSS: 7.SP.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Critical Thinking/Problem Solving and inquiry/Analysis. Students can interpret information and gathered and draw conclusions based on the best analysis.

2. MP3: Construct viable arguments and critique the reasoning of others. Students can make comparisons that may involve making conjectures about populations parameters and constructing arguments based on data to support the conjectures.

3. MP5: Use appropriate tools strategically. Students use tools to compute measures of center and variability for purposes of comparing populations.

Inquiry Questions:

1. How do a measure of center (such as mean) and of variability (such as mean absolute deviation) work together to describe a data set?

2. How can we use measures of center and variability to compare two data sets?

Coherence Connections:

1. Students will use their previous understanding of whole numbers and fractions as they reason about and compare measures of center and variability in data sets.

2. The work in Grade 7 prepares students for making inferences and justifying conclusions from random samples in high school.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Seventh Grade

7.SP.C: Statistics & Probability. Investigate chance processes and develop, use, and evaluate probability models.

Evidence Outcomes

Students Can:

a. Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS: 7.SP.C.5)

b. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. (CCSS: 7.SP.C.6)

c. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (CCSS: 7.SP.C.7)
   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected (CCSS: 7.SP.C.7.b).
   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? (CCSS: 7.SP.C.7.b)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Creativity/Innovation. Students can build on personal experience to specify a challenging problem to solve.

2. MP4. Model with mathematics. Students use probability models and simulations to predict outcomes of real-world chance events both theoretically and experimentally.

3. MP5. Use appropriate tools strategically. Students can use technology, manipulatives, and simulations to determine probabilities and understand chance events.

Inquiry Questions:

1. Since the probability of getting heads on the toss of a fair coin is \(\frac{1}{2}\), does that mean for every two tosses of a coin, exactly one of them will be heads? Why or why not?

2. What might a discrepancy in the predicted outcomes and the actual outcomes of a chance event tell us?

Coherence Connections:

1. This is the first grade level where probability concepts are introduced to students.

2. The product rule for counting outcomes for chance events should be used in finite situations like tossing two or three coins or rolling two number cubes. There is no need to go to more formal rules for permutations and combinations at this level. Students should gain experience in the use of diagrams, especially trees and tables, as the basis for organizing counting of possible outcomes from chance processes.

3. The work in Grade 7 with probability is extended in high school to conditional probability, and using probability distributions to solve problems involving expected value. In addition for making inferences and justifying conclusions from random samples within the modeling standards in high school.
d. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (CCSS: 7.SP.C.8)

   a. Explain that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (CCSS: 7.SP.C.8.a)

   b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. (CCSS: 7.SP.C.8.b)

   c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question. If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? (CCSS: 7.SP.C.8.c)
Colorado Academic Standards Review
MATHEMATICS
Seventh Grade: Geometry

Instructions
To leave feedback, click on the comment icon next to any item. You can then offer feedback and comments.
Once you save your feedback, the icon will change color and show as a checkbox, so you can keep track of your progress.

Common Core Information
Colorado's standards for mathematics and reading, writing, and communicating include the Common Core State Standards. Each statement that comes from the Common Core State Standards is followed by the appropriate reference such as (CCSS: 5.NF.2) or (CCSS: SL.K.5). View a reference guide to the CCSS reference codes (PDF).

Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Seventh Grade
7.G.A. Geometry: Draw, construct, and describe geometrical figures and describe the relationships between them.

Evidence Outcomes
Students Can:

a. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS: 7.G.A.1)
b. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS: 7.G.A.2)
c. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in cross sections of right rectangular prisms and right rectangular pyramids. (CCSS: 7.G.A.3)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:

1. CSP Entrepreneurial Skills: Inquiry/Analysis. Students can investigate to form hypotheses, make observations and draw conclusions.
2. MP2: Reason abstractly and quantitatively. Students reason abstractly by deconstructing three-dimensional shapes slice horizontally or vertically and explain the correct cross-section. Students reason quantitatively by solving proportions for scale drawings.
3. MP3: Construct viable arguments and critique the reasoning of others. Students are able to describe, analyze, and generalize about the resulting cross-section of a sliced three-dimensional figure and justify their reasoning.
4. MP5: Use appropriate tools strategically. Students are able to use the correct tools, whether it is with paper, pencil, ruler, compass, and/or protractor, or with the use of technology, to draw geometric shapes.
Inquiry Questions:
1. How are proportions used to solve problems involving scale drawings?
2. How is it possible to draw more than one triangle with the same given conditions? Is this always possible?
3. How are all cross-sections similar?
4. What are some examples of cross-sections whose shapes may be identical but are from different three-dimensional figures?

Coherence Connections:
1. In Grade 5, students classify two-dimensional figures in a hierarchy based on properties, and in Grade 6, students explain correspondences and write an equation to describe a situation using a variable. Students combine prior knowledge of two-dimensional shapes in order to identify the cross-section resulting from slicing three-dimensional figures.

2. Students apply prior knowledge of ratios and proportions to solve scale drawing problems; this skill is also applicable to problems involving similarity between two figures.

3. In Grade 8, students describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. High school students apply the Triangle Inequality Theorem to determine if or when a triangle may exist under given conditions. This also requires an understanding of the minimum information necessary for the existence of a triangle.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Seventh Grade


Evidence Outcomes

Students Can:

a. State the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (CCSS: 7.G.B.4)

b. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (CCSS: 7.G.B.5)

c. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS: 7.G.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Inquiry/Analysis. Students recognize and describe patterns.
2. MP1: Make sense of problems and persevere in solving them. Problem solving for area, volume, and surface area requires students to both make sense of the problem and be able to persevere through the problem solving process.
3. MP4: Model with mathematics. Students model real-world situations involving area, surface area, and volume with mathematics.
4. MP6. Attend to precision. Students are able to calculate a variety of measures (angles, circumference, area, surface area, and volume), and use and reason accurately with corresponding units.

Inquiry Questions:

1. How is the formula for the area of a circle related to the circumference of a circle?
2. What are the angle measure relationships among supplementary, complementary, vertical, and adjacent angles?
3. What are some examples of real-world situations where one would need to find (a) area, (b) volume, and (c) surface area?
Coherence Connections:

1. In Grade 6, students find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes, and find the volume of a right rectangular prism by multiplying the edge lengths of the prism.

2. In Grade 7, students move from simple recall and basic understanding of area and circumference of a circle formulas to application of the formulas in problem solving contexts. Students combine their understanding of supplementary, complementary, vertical, and adjacent angle relationships with solving equation concepts to construct and solve algebraic equations and determine angle measures.

3. In Grade 8, students will know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Select Grade Level & Standard

Grade Level:
- Preschool
- Kindergarten
- First Grade
- Second Grade
- Third Grade
- Fourth Grade
- Fifth Grade
- Sixth Grade
- Seventh Grade
- Eighth Grade
- High School

Standard:
1. Number and Quantity
2. Algebra and Functions
3. Data, Statistics, and Probability
4. Geometry

Change content area

Instructions
To leave feedback, click on the comment icon next to any item. You can then offer feedback and comments.
Once you save your feedback, the icon will change color and show as a checkbox so you can keep track of your progress.

Common Core Information
Colorado's standards for mathematics and reading, writing, and communicating include the Common Core State Standards. Each statement that comes from the Common Core State Standards is followed by the appropriate reference such as (CCSS: 5 MP.2) or (CCSS: SL.K.6). View a reference guide to the CCSS reference codes (PDF).

1. Prepared Graduates
   1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
   2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, "Does this make sense?"
   3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
   4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
   5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

2. Grade Level Expectation: Eighth Grade
   8.NS.A: The Number System. Know that there are numbers that are not rational, and approximate them by rational numbers.

3. Evidence Outcomes
   Students Can:
   a. Demonstrate informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Define irrational numbers as numbers that are not rational. (CCSS: 8.NS.A.1)
   b. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., \(\sqrt{2}\)). For example, by truncating the decimal expansion of \(\sqrt{2}\), show that \(\sqrt{2}\) is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS: 8.NS.A.2)

4. Academic Context and Connections
   Colorado Essential Skills and Mathematical Practices:
   1. CES: Entrepreneurial Skills: Inquiry/Analysis. Students can investigate to form hypotheses, make observations and draw conclusions.
   2. MP7: Look for and make use of structure. The understanding of rational and irrational numbers completes students' study of the real number system, allowing them to describe and work within the structure of the real number system effectively and efficiently.
   3. MP8: Look for and express regularity in repeated reasoning. This GLE allows students to recognize the regularity in converting decimal expansions to rational numbers and recognize when a decimal expansion cannot be represented by a rational number.

5. Inquiry Questions:
   1. How many irrational numbers exist?
   2. Why is there no real number closest to zero?
   3. Why are the locations of irrational numbers on a number line approximations?

6. Coherence Connections:
   1. Students have been learning about the rational number system in prior grades, using number lines to locate and compare fractions, decimals, and integers. In Grade 7, students converted rational numbers to decimals and found patterns in the expansion (SNMP). Using decimal expansions to convert to rational numbers concludes work within the rational numbers and equivalent forms.
   2. Students now have understanding of rational and irrational numbers, which culminates study of the real number system.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Eighth Grade


Evidence Outcomes

Students Can:

a. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 	imes 3^{-5} = 3^{-3} = \frac{1}{3^3} = 9$ (CCSS: 8.EE.A.1)

b. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares (up to 100) and cube roots of small perfect cubes (up to 12). Know that $\sqrt{2}$ is irrational. (CCSS: 8.EE.A.2)

c. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger (CCSS: 8.EE.A.3)

d. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (CCSS: 8.EE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CBS: Entrepreneurial Skills - Critical Thinking/Problem Solving. Students can identify problems and create solutions.
2. MPS: Use appropriate tools strategically. This GLE allows students the opportunity to use calculators or computers to compute and estimate with radicals and roots, and understand how to interpret scientific notation representations within their technology tools.
3. MPS: Look for and express regularity in repeated reasoning. Students explore and generalize the properties of integer exponents to rewrite expressions.

Inquiry Questions:

1. How is performing operations on numbers in scientific notation similar to or different from performing operations on whole numbers?
2. What is the relationship between positive and negative exponents?

Coherence Connections:

1. Students begin denoting whole number powers of ten with exponential notation in Grade 6 and work with expressions containing exponents in Grade 6. This is the first time students are introduced to square roots.
2. The work in Grade 8 with integer exponents expands to understanding the properties of rational exponents in high school.
3. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
1. Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

2. Grade Level Expectation: Eighth Grade

8.EE.B: Expressions & Equations: Understand the connections between proportional relationships, lines, and linear equations.

3. Evidence Outcomes

Students Can:

a. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (CCSS: 8.EE.B.5)

b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intersecting the vertical axis at b. (CCSS: 8.EE.B.6)

4. Academic Context and Connections

5. Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Critical Thinking/Problem Solving and Inquiry/Analysis: Students can make connections between information gathered. Students can also recognize and describe patterns.

2. MP1: Make sense of problems and persevere in solving them. Students make sense of proportional relationships to make comparison statements.

3. MP3: Construct viable arguments and critique the reasoning of others. Students compare and contrast proportional relationships based on properties of equations, tables, and/or graphs.

6. Inquiry Questions:

1. How is it possible to compare two proportional relationships using different representations?

2. How is the unit rate of a proportional relationship related to the slope of its graphical representation?

3. Why are similar triangles effective for describing slope geometrically?

7. Coherence Connections:

1. In Grade 7, students graph proportional relationships, compute the constant of proportionality, and note that proportional relationships go through the origin (0,0).

2. As students in Grade 8 move towards an understanding of the idea of a function, they begin to tie together a number of notions that have been developing over the last few grades: (a) an expression in one variable defines a general calculation in which the variable can represent a range of numbers—an input/output machine with the variable representing the input and the expression calculating the output; (b) choosing a variable to represent the output leads to an equation in two variables describing the relation between two quantities; (c) tabulating values of the expression is the same as tabulating solution pairs of the corresponding equation; and (d) plotting points on the coordinate plane, in which each axis is marked with a scale representing one quantity, affords a visual representation of the relationship between two quantities. Proportional relationships provide a fruitful first ground in which these notions can grow together.

3. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Eighth Grade

8.EE.C. Expressions & Equations. Analyze and solve linear equations and pairs of simultaneous linear equations.

Evidence Outcomes

Students Can:

a. Solve linear equations in one variable. (CCSS: 8.EE.C.7)
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). (CCSS: 8.EE.C.7.a)
   b. Solve linear equations with rational number coefficients, including equations with variables on both sides and whose solutions require expanding expressions using the distributive property and collecting like terms. (CCSS: 8.EE.C.7.b)

b. Analyze and solve pairs of simultaneous linear equations. (CCSS: 8.EE.C.8)
   a. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (CCSS: 8.EE.C.8.a)
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. (CCSS: 8.EE.C.8.b)

Practice:

1. CES Entrepreneurial Skills: Risk Taking. Students can demonstrate flexibility, imagination, and inventiveness in taking on tasks and activities.

2. UP1: Make sense of problems and persevere in solving them. Students solve problems that require solving a system of linear equations in two variables.

3. UP4: Model with mathematics. Students define variables and create and use linear systems of equations to solve real-world problems.

4. UP7: Look for and make use of structure. Students recognize the structures of equations that produce one, infinite, or no solution.

Inquiry Questions:

1. What is meant by a “solution” to a linear equation? What is meant by a “solution” to a system of two linear equations? How are these concepts related?

2. How is it possible for an equation to have more than one solution? How is it possible for an equation to have no solution?

3. Why can’t a system of linear equations have a solution set other than one, zero, or infinitely many solutions?

4. What connections do you see between the graphical solution of a system of linear equations and the algebraic solution?

Coherence Connections:

1. In Grades 6 and 7, students learned to solve one- and two-step equations with rational number coefficients. Work with solving equations in grade 8 extends to variables on both sides, and to using algebraic and graphical methods to solve systems of linear equations. Students are afforded the opportunity to generalize the meaning of “solving” from a single linear equation to a system of two linear equations.

2. In high school, students will continue to solve multi-step linear equations (including inequalities) and will use properties of operations and algebra to solve equations with nonlinear expressions.

3. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Eighth Grade


Evidence Outcomes

Students Can:

a. Define a function as a rule that assigns to each input exactly one output. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required for Grade 8.) (CCSS: 8.F.A.1)

b. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (CCSS: 8.F.A.2)

c. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line. Give examples of functions that are not linear. For example, the function A = πr^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line. (CCSS: 8.F.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Critical Thinking/Problem Solving and Inquiry/Analysis. Students can make connections between information gathered. Students can also recognize and describe patterns.

2. MP1: Make sense of problems and persevere in solving them. This GLE allows students to make connections among real-world descriptions, tables, graphs, and equations of linear functions.

3. MP8: Look for and express regularity in repeated reasoning. This GLE allows students to recognize and state the properties of linear functions through a variety of representations, understanding how the underlying mathematics is connected.

Inquiry Questions:

1. What determines if a mathematical relationship is a function?

2. Why is it important to know if a mathematical relationship is a function or not?

3. What properties do linear functions have that non-linear functions do not have? How can you tell if a function is linear or non-linear?

Coherence Connections:

1. In Grade 7, students graph proportional relationships on the coordinate plane in the form y = ax, where a is the constant of proportionality. In Grade 8, students use that knowledge to study linear functions with nonzero y-intercepts.

2. Students are formally introduced to the concept of a function in Grade 8 with a focus on linear functions, their representations, and their properties. This foundational work prepares students to study function notation, exponential functions, quadratic functions, and writing functions from sequences in high school.

3. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Eighth Grade

8.F.B. Functions: Use functions to model relationships between quantities.

Evidence Outcomes

Students Can:

a. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS: 8.F.B.4)

b. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS: 8.F.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Creativity/Innovation. Students can synthesize ideas in original and surprising ways as they analyze graphs to qualitatively describe relationships.

2. MP4: Model with mathematics. This GLE allows students to generate linear functions to model real-world situations.

3. MP8: Look for and express regularity in repeated reasoning. Students use strategies to calculate the rate of change in a linear function (slope) and use properties of linear functions to create equations.

Inquiry Questions:

1. What is the minimum information needed to write a linear function for a relationship between two quantities?

2. What are some quantitative and qualitative features of graphs of functions?

Coherence Connections:

1. Students in Grade 8 learn about the slope-intercept form of linear functions, composed of the rate of change and initial value. As students calculate these values using tables, graphs, or verbal descriptions, they begin to see the structure of all linear functions (MP7). This builds off work with identifying the constant of proportionality and graphing proportional relationships in Grade 7.

2. Exploring features and properties of linear and non-linear functions are important for high school mathematics where several distinct families of functions are studied.

3. This GLE requires greater emphasis based on the depth of the ideas, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Colorado Academic Standards Review
MATHEMATICS
Eighth Grade: Data, Statistics, & Probability

Instructions
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Common Core Information
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Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Eighth Grade
8 SPA. Statistics & Probability: Investigate patterns of association in bivariate data.

Evidence Outcomes
Students Can:

a. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association (CCSS: 8.SP.A.1).

b. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line (CCSS: 8.SP.A.2).

c. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height (CCSS: 8.SP.A.3).

d. Explain that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores? (CCSS: 8.SP.A.4)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. CES Entrepreneurial Skills: Inquiry/Analysis. Students can recognize and describe patterns in bivariate data.
2. MP2: Reason abstractly and quantitatively. Students can interpret the conditional meaning of slope (and y-intercept, where applicable) in the linear model of bivariate data.
3. MP4: Model with mathematics. Students build statistical models to explore, describe, and generalize the relationship between two variables.
4. MP7: Look for and make sense of structure. Students use scatterplots and/or numerical data to describe possible associations in bivariate data.

Inquiry Questions:
1. In what ways is a scatter plot useful in describing and interpreting the relationship between two quantities?
2. Why would we create a linear model for a set of bivariate data?
3. How do you know when a credible prediction can be made?
4. How can we determine if two non-numerical ideas have an association?

Coherence Connections:
1. Building on the work with decimals and percent, dependent and independent variables in earlier grades, and the ideas of association between measurement variables, students now take a more careful look at the possible association between categorical variables.
2. The work within this GLE connects to the most important concepts of the grade (linear equations and functions), in that students will be able to plot bivariate data as points on a coordinate plane and to make use of the equation of a line in analyzing the relationship between two variables.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Eighth Grade

6.G.A. Geometry: Understand congruence and similarity using physical models, transparencies, or geometry software.

Evidence Outcomes

Students Can:

a. Verify experimentally the properties of rotations, reflections, and translations (CCSS: 8.G.A.1):
   - Lines are taken to lines, and line segments to line segments of the same length. (CCSS: 8.G.A.1.a)
   - Angles are taken to angles of the same measure. (CCSS: 8.G.A.1.b)
   - Parallel lines are taken to parallel lines. (CCSS: 8.G.A.1.c)

b. Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence that exhibits the congruence between them (CCSS: 8.G.A.2).

c. Describe the effect of slanting, translations, rotations, and reflections on two-dimensional figures using coordinates (CCSS: 8.G.A.3).

d. Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations, and dilations, given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (CCSS: 8.G.A.4)

e. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (CCSS: 8.G.A.5)

Colorado Context and Connections

1. CCSS Mathematical Practice: Critical Thinking/Problem Solving and Inquiry/Analysis
   - Students recognize that problems can be identified and possible solutions can be created. Students can also investigate to form hypotheses, make observations and draw conclusions.

2. MP5: Construct viable arguments and critique the reasoning of others. Students are able to explain and justify a proper sequence of transformations for congruent and similar triangles.

3. MP5: Use appropriate tools strategically. Students use physical models, transparencies, geometric software, and/or other appropriate tools to explore the relationships between transformations and congruence and similarity.

4. MP7: Look for and make use of structure. Students are able to specify and apply the properties of rotations, reflections, and translations.

Inquiry Questions:

1. How are rotations, reflections, translations, and dilations connected to the properties of congruence?

2. How are rotations, reflections, translations, and dilations connected to the properties of similarity?

3. Why are angle measures significant regarding the similarity of two figures?

Coherence Connections:

1. In Grade 7, students solve problems involving scale drawings of geometric figures and to draw geometric shapes with given conditions.

2. In Grade 8, students understand congruence and similarity and their relationships to multiple types of transformations. Students also apply their understanding of proportions and similarity to figures that are dilated. Students apply understanding of angle pair relationships formed by a transversal passing through two parallel lines. This supports the following angle pair relationships: alternate interior, alternate exterior, same side interior, and same side exterior.

3. In high school, students verify experimentally the properties of dilations given by a center and a scale factor and also develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

4. This GLZ requires greater emphasis based on the depth of the idea, the time that it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Eighth Grade


Evidence Outcomes

Students Can:

a. Explain a proof of the Pythagorean Theorem and its converse. (CCSS: 8.G.B.6)

b. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS: 8.G.B.7)

c. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS: 8.G.B.8)

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Inquiry/Analysis. Students can investigate to form hypotheses, make observations and draw conclusions.
2. MP4: Model with mathematics. Students apply the Pythagorean Theorem to real-world problems.
3. MP8: Look for and express regularity in repeated reasoning. Students recognize the pattern of Pythagorean Triples.

Inquiry Questions:

1. What is the relationship between the Pythagorean Theorem and its converse? In what ways is each useful?
2. How is the distance formula related to the Pythagorean Theorem?

Coherence Connections:

1. In Grade 6, students draw polygons in the coordinate plane given coordinates for the vertices and use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.
2. The Pythagorean Theorem is one of the most foundational theorems with triangles. The use of the converse of the Pythagorean theorem is essential when trying to determine whether or not a triangle is a right triangle. This skill is vital to trades that work with right angle relationships. Students are able to articulate the derivation of the distance formula using the Pythagorean Theorem, connecting to square roots and solving simple equations in the form $a^2 = p$.
3. In high school, students prove theorems about triangles. Students understand the differences between finding the length of an unknown leg of a right triangle and finding the length of an unknown hypotenuse of a right triangle, and derive the equation of a circle of given center and radius using the Pythagorean Theorem. Understanding the Pythagorean Theorem is a fundamental skill essential to working with the unit circle and trigonometric ratios.
4. This GLE requires greater emphasis based on the depth of the ideas, the time it takes to master, and/or the importance to future mathematics or the demands of college and career readiness.
Colorado Academic Standards Review
MATHEMATICS
Eighth Grade: Geometry

Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: Eighth Grade


Evidence Outcomes

Students Can:

a. State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (CCSS: 8.G.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES Entrepreneurial Skills: Inquiry/Analysis. Students can investigate to form hypotheses, make observations, draw conclusions, and also recognize and describe patterns.

2. MP4: Model with mathematics. Students apply the volume formulas of cones, cylinders, and spheres in real-world problem-solving contexts.

3. MP6: Attend to precision. Preciseness is a requisite for solving problems involving measurement.

Inquiry Questions:

1. How are the formulas of cones, cylinders, and spheres similar? How are they dissimilar?

Coherence Connections:

1. In Grade 7, students solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

2. In high school, students give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone and apply these formulas in modeling situations.
Colorado Academic Standards Review
MATHEMATICS
High School: Number and Quantity

1. Prepared Graduates
   1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
   2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
   3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
   4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
   5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

2. Grade Level Expectation: High School
   HS.N-RN.A. The Real Number System: Extend the properties of exponents to rational exponents.

3. Evidence Outcomes
   Students Can:
   a. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $\sqrt[3]{5}$ to be the cube root of 5 because we want $(\sqrt[3]{5})^3 = \sqrt[3]{5^3}$ to hold, so $(\sqrt[3]{5})^3$ must equal 5. (CCSS: HS.N-RN.A.1)
   b. Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS: HS.N-RN.A.2)

4. Academic Context and Connections

5. Colorado Essential Skills and Mathematical Practices:
   1. CES: Inquiry/Analysis. Students make hypotheses and draw conclusions.
   2. MP7: Look for and make use of structure. This GLE requires students to generalize the properties of integer exponents, familiar from middle school, to rational exponents.
   3. MP8: Look for and express regularity in repeated reasoning. This GLE affords the opportunity to manipulate algebraic expressions utilizing the properties of operations and exponents.

6. Inquiry Questions:
   1. How do we know that the two equations below both represent the same relationship, between the radius of a sphere and its volume? $V = \frac{1}{3} \pi r^3$ and $r = \left( \frac{3 V}{\pi} \right)^{\frac{1}{3}}$

7. Coherence Connections:
   1. Integer exponents (both positive and negative) and radicals were studied in Grade 8. Here, students expand the concept of exponents to include fractional exponents and make the connection to radicals.
   2. In more advanced courses, rational exponents will be extended to irrational exponents by means of exponential and logarithmic functions.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.N-RN.B. The Real Number System: Use properties of rational and irrational numbers.

Evidence Outcomes

Students Can:

a. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. (CCSS: HS.N-RN.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Communication. Students verbally express their understanding in ways appropriate to context and audience.

2. MP3: Construct viable arguments and critique the reasoning of others. This GLE requires students to justify conclusions and communicate them to others.

3. MP7: Look for and make use of structure. This GLE requires students to generalize the properties of integer exponents, familiar from middle school, to rational exponents.

Inquiry Questions:

1. How is it possible that multiplying two irrational numbers gives a product that is not irrational? Why doesn’t this phenomenon apply to rational numbers?

Coherence Connections:

1. Having already extended arithmetic from whole numbers to fractions (Grades 4-6) and from fractions to rational numbers (Grade 7), students in Grade 8 encountered particular irrational numbers such as $\sqrt{2}$. In Algebra I, students will begin to understand the real number system.

2. An important difference between rational and irrational numbers is that rational numbers form a number system. If you add, subtract, multiply or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. Although in applications of mathematics the distinction between rational and irrational numbers is irrelevant, since we always deal with finite decimal approximations (and therefore with rational numbers), thinking about the properties of rational and irrational numbers is good practice for mathematical reasoning habits such as constructing viable arguments (MP3) and attending to precision (MP6).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS N-Q.A. Quantities: Reason quantitatively and use units to solve problems.

Evidence Outcomes

Students Can:

a. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (CCSS: H.S-N.Q.A.1)

b. Define appropriate quantities for the purpose of descriptive modeling. (CCSS: H.S-N.Q.A.2)

c. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (CCSS: H.S-N.Q.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Information Literacy. Students use information accurately for the problem at hand.

2. MP2: Reason abstractly and quantitatively. This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of algebra involves creating abstract algebraic models of problems and then transforming the models via algebraic calculations to reveal properties of the problems.

3. MP4: Model with mathematics. Students will use real-world applications with units as they reason quantitatively, define appropriate quantities, and select and interpret labels for graphs and data displays.

Inquiry Questions:

1. In what ways can the units of a complicated problem help guide us to the solution?

2. Why is “Let z = number of gallons of gas” more accurate than “Let z = gas?”

Coherence Connections:

1. Students in middle grades worked with measurement units, including units obtained by multiplying and dividing quantities. In Algebra 1, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.N-CN.A. The Complex Number System: Perform arithmetic operations with complex numbers

Evidence Outcomes

Students Can:

a. Define complex number \( i \) such that \( i^2 = -1 \), and show that every complex number has the form \( a + bi \) where \( a \) and \( b \) are real numbers. (CCSS. HS.N-CN.A.1)

b. Use the relation \( i^2 = -1 \) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (CCSS. HS.N-CN.A.2)

c. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. (CCSS. HS.N-CN.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Information Literacy. Students use information accurately for the problem at hand.

2. MPT: Look for and make use of structure. As students extend their understanding of the number system to include imaginary and complex numbers, they recognize the underlying structures that connect them to the real number system.

Inquiry Questions:

1. Is the sum of two complex numbers always, sometimes, or never another complex number? Why or why not?

2. Is the product of two complex numbers always, sometimes, or never another complex number? Why or why not?
Coherence Connections:

1. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—in integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

2. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.N-CN.B. The Complex Number System: Represent complex numbers and their operations on the complex plane.

Evidence Outcomes

Students Can:

a. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. (CCSS: HS.N-CN.B.4)

b. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane, use properties of this representation for computation. For example, \((-1 + \sqrt{3}i)^3 = 8\) because \((-1 + \sqrt{3}i)\) has modulus 2 and argument 120°. (CCSS: HS.N-CN.B.5)

c. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. (CCSS: HS.N-CN.B.6)

Academic Context and Connections

Coherence Connections:

1. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
1. Prepared Graduates
   
   1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
   
   2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
   
   3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
   
   4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
   
   5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

2. Grade Level Expectation: High School

   HS.N-CN.C. The Complex Number System: Use complex numbers in polynomial identities and equations.

3. Evidence Outcomes

   Students Can:
   
   a. Solve quadratic equations with real coefficients that have complex solutions. (CCSS: HS N-CN C.7)
   
   b. (+) Extend polynomial identities to the complex numbers. For example rewrite as \( x^2 + 4 = (x + 2i)(x - 2i) \). (CCSS: HS N-CN C.8)
   
   c. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (CCSS: HS N-CN C.9)

4. Academic Context and Connections

   1. Colorado Essential Skills and Mathematical Practices:
      
      1. MP5: Use appropriate tools strategically. Spreadsheets, graphing tools, and many other technologies can be used strategically to gain understanding of quadratic equations with real coefficients and complex solutions.

   2. Inquiry Questions:
      
      1. What differences are evident in the graph of a quadratic equation with real solutions versus the graph of a quadratic equation with complex solutions?

   3. Coherence Connections:
      
      1. When students first apply the quadratic formula to quadratic equations with real coefficients, the square root is a problem if the quantity \( b^2 - 4ac \) is negative. Complex numbers solve that problem by introducing a new number, \( i \), with the property that \( i^2 = -1 \), which enables students to express the solutions of any quadratic equation.

      2. In high school, students extend the real numbers to complex numbers, and one effect is that they now have a complete theory of quadratic equations: Every quadratic equation with complex coefficients has (counting multiplicities) two roots in the complex numbers.

      3. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.N-VM.A Vector and Matrix Quantities: Represent and model with vector quantities.

Evidence Outcomes

Students Can:

a. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \( \mathbf{v} \), \( ||v|| \), \( \mathbf{v} \)). (CCSS: HS.N-VM.A.1)

b. (+) find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. (CCSS: HS.N-VM.A.2)

c. (+) Solve problems involving velocity and other quantities that can be represented by vectors. (CCSS: HS.N-VM.A.3)

Academic Context and Connections

Coherence Connections:

1. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.N-VM.B. Vector and Matrix Quantities: Perform operations on vectors.

Evidence Outcomes

Students Can:

a. (+) Add and subtract vectors. (CCSS: HS.N-VM.B.4)
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. (CCSS: HS.N-VM.B.4.a)
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. (CCSS: HS.N-VM.B.4.b)
   c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. (CCSS: HS.N-VM.B.4.c)

b. (+) Multiply a vector by a scalar. (CCSS: HS.N-VM.B.5)
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g. as $c(v_x, v_y) = (cv_x, cv_y)$. (CCSS: HS.N-VM.B.5.a)
   b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $||c\mathbf{v}|| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$). (CCSS: HS.N-VM.B.5.b)

Academic Context and Connections

Coherence Connections:

1. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.N.VM.C. Vector and Matrix Quantities: Perform operations on matrices and use matrices in applications.

Evidence Outcomes

Students Can:

a. (+) Use matrices to represent and manipulate data, e.g., as when all of the payoffs or incidence relationships in a network. (CCSS: HS.N.VM.C.6)

b. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. (CCSS: HS.N.VM.C.7)

c. (+) Add, subtract, and multiply matrices of appropriate dimensions. (CCSS: HS.N.VM.C.8)

d. (+) Understand that, unlike the multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. (CCSS: HS.N.VM.C.9)

e. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. (CCSS: HS.N.VM.C.10)

f. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimension to produce another vector. Work with matrices as transformations of vectors. (CCSS: HS.N.VM.C.11)

g. (+) Work with $2 \times 2$ matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. (CCSS: HS.N.VM.C.12)

Academic Context and Connections

Coherence Connections:

1. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
1. Prepare Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

2. Grade Level Expectation: High School

HSA-SSE.A. Seeing Structure in Expressions: Interpret the structure of expressions.

3. Evidence Outcomes

Students Can:

a. Interpret expressions that represent a quantity in terms of its context *(CCSS HSA-SSE.A.1)*
   a. Interpret parts of an expression, such as terms, factors, and coefficients *(CCSS HSA-SSE.A.1.a)*
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1 + r)^n as the product of P and a factor not depending on P *(CCSS HSA-SSE.A.1.b)*

b. Use the structure of an expression to identify ways to rewrite it. For example, see \(x^4 - y^4\) as \((x^2)^2 - (y^2)^2\), thus recognizing it as a difference of squares that can be factored as \((x^2 - y^2)(x^2 + y^2)\) *(CCSS HSA-SSE.A.2)*

4. Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Communication. Students interpret expressions representing real world situations and demonstrate their understanding of the parts of the expressions.
2. MP1: Make sense of problems and persevere in solving them. This GLE has students explain and analyze given constraints, and relationships and interpret them in the context of a problem.
3. MP2: Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations. They can create a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them.
4. MP7: Look for and make use of structure. In this GLE, students look closely to discern a pattern or structure. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 \(-3(x - y)^2\) as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers \(x\) and \(y\)

5. Inquiry Questions:

1. How could you show algebraically that the two expressions \((x + 2)^2 - 4\) and \(x^2 + 4n\) are equivalent? How could you show it visually, with a diagram or picture?
1. The separation of algebra and functions in the standards is intended to specify the difference between the two, as mathematical concepts between expressions and equations on the one hand and functions on the other. Students often enter college-level mathematics courses apparently conflating all three of these. For example, when asked to factor a quadratic expression a student might instead find the solutions of the corresponding quadratic equation. A student might attempt to simplify the expression \( \frac{a + \frac{x}{2}}{2} \) by seeing \( \frac{x}{2} \) as 1.

2. Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context.

3. Themes beginning in middle school algebra continue and deepen during high school. In Grades 6 and 7, students began to use the properties of operations to generate equivalent expressions. By Grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving. In High School, these aspects of algebra carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called “mindful manipulation.”
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.A-SSE.B Seeing Structure in Expressions: Write expressions in equivalent forms to solve problems.

Evidence Outcomes

Students Can:

a. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *(CCSS: HS.A-SSE.B.3)*
   
   a. Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS: HS.A-SSE.B.3.a)

   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS: HS.A-SSE.B.3.b)

   c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression $1.15t$ can be rewritten as $(1.15)^{12} \approx 1.1922$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15% (CCSS: HS.A-SSE.B.3.c)

b. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. *(CCSS: HS.A-SSE.B.4)*

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical thinking/problem solving. Students transform expressions to highlight different properties of a expressions and interpret their understanding.

2. MP2: Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations.

3. MP8: Look for and express regularity in repeated reasoning. Noticing the regularity in the way terms combine to make zero when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Inquiry Questions:

1. What does the vertex form of a quadratic equation, $y = a(x - h)^2 + k$, tell us about its graph that the standard form, $y = ax^2 + bx + c$, doesn’t? What does the factored form, $y = a(x - p)(x - q)$, tell us about the graph that the other two forms don’t?

Coherence Connections:

1. As students progress through high school, they should become increasingly proficient with mathematical actions such as “doing and undoing”, for example, looking at expressions generated through the distributive property and identifying expressions that might have led to a given outcome. They use their facility with distributing and comparing factored and expanded expressions to rewrite expressions like $6x + 9xy = 3x^2 + 4x + 2x + 8$, or $x^2 + 2xy + y^2$ in factored form. They might use graphs, tables of values, and properties to reason that two expressions are equivalent, but they rely on definitions and properties of operations to provide a mathematical justification. They do not use “FOIL” as a justification for the multiplication of two binomials, understanding that such mnemonics are not conceptually defensible and do not generalize.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.A.APR.A. Arithmetic with Polynomials and Rational Expressions: Perform arithmetic operations on polynomials

Evidence Outcomes

Students Can:

a. Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (CCSS: HS.A.APR.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Inquiry/Analysis. Students make hypotheses and draw conclusions.

2. MP7: Look for and make use of structure. This GLE requires students to generalize the properties of integers, familiar from middle school, to polynomials.

3. MP8: Look for and express regularity in repeated reasoning. This GLE affords the opportunity to manipulate algebraic expressions utilizing the properties of operations.

Inquiry Questions:

1. $f(x) = x^2$ is a non-negative polynomial because for all values of $x$, $f(x) \geq 0$. If you add two non-negative polynomials together, do you always, sometimes, or never get another non-negative polynomial? What if you multiply them?

Coherence Connections:

1. As students first study algebraic expressions, they use simply numbers in which one or more letters are used to stand for a number which is either unspecified or unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions, particularly polynomial and rational expressions, as naming some underlying thing. There are at least two ways this can go. If the function concept is developed before or concurrently with the study of polynomials, then a polynomial can be identified with the function it defines. In this way, $x^2 - 2x - 3$, $(x - 3)(x + 1)$, and $(x - 1)^2 - 4$ are all the same polynomial because they all define the same function. Another approach is to think of polynomials as elements of a formal number system, in which you introduce the "number" $x$ and see what numbers you can write down with it. In this approach, $x^2 - 2x - 3$, $(x - 3)(x + 1)$, and $(x - 1)^2 - 4$ are all the same polynomial because the properties of operations allow each to be transformed into the others. Each approach has its advantages and disadvantages; the former approach is more common.

2. Either way, polynomials and rational expressions come to form a system in which they can be added, subtracted, multiplied and divided. Polynomials are analogous to the integers; rational expressions are analogous to the rational numbers.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.A-APR.B. Arithmetic with Polynomials and Rational Expressions: Understand the relationship between zeros and factors of polynomials.

Evidence Outcomes

Students Can:

a. State and apply the Remainder Theorem. For a polynomial \( p(x) \) and a number \( a \), the remainder on division by \( x - a \) is \( p(a) \), so \( p(a) = 0 \) if and only if \( (x - a) \) is a factor of \( p(x) \). (CCSS: HS A-APR.B.2)

b. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (CCSS: HS A-APR.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Information Literacy. Students use information accurately for the problem at hand.

2. MP1. Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They transform algebraic expressions to get the information they need.

3. MP2. Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They manipulate the representing symbols as if they have a life of their own.

Inquiry Questions:

1. What is the relationship between factoring a polynomial and finding its zeros?

Coherence Connections:

1. Viewing polynomials as functions leads to explorations of a different nature. Polynomial functions are, on the one hand, very elementary, in that, unlike trigonometric and exponential functions, they are built up out of the basic operations of arithmetic. On the other hand, they turn out to be amazingly flexible, and can be used to approximate more advanced functions such as trigonometric and exponential functions. Experience with constructing polynomial functions satisfying given conditions is useful preparation not only for calculus (where students learn more about approximating functions), but for understanding the mathematics behind curve-fitting methods used in applications to statistics and computer graphics. A simple step in this direction is to construct polynomial functions with specified zeros. This is the first step in a progression which can lead, as an extension topic, to constructing polynomial functions of the function defined by the polynomial functions whose graphs pass through any specified set of points in the plane.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.A-APR.C. Arithmetic with Polynomials and Rational Expressions: Use polynomial identities to solve problems.

Evidence Outcomes

Students Can:

a. Prove polynomial identities. For example, the polynomial identity

\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\]

can be used to generate Pythagorean triples and use them to describe numerical relationships. (CCSS: HS.A-APR.C.4)

b. (+) Know and apply the Binomial Theorem for the expansion of in powers of \(x\) and \(y\) for a positive integer \(n\), where \(x\) and \(y\) are any numbers, with coefficients determined for example by Pascal’s Triangle. (CCSS: HS.A-APR.C.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CST: Critical Thinking/Problem Solving. Students interpret information and draw conclusions based on the form and patterns given.

2. MP8: Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look for patterns, general methods and for shortcuts.

Inquiry Questions:

1. Can you find a case (a specific value of \(x\) and \(y\)) for which the equation

\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\]

does NOT generate a Pythagorean triple?

Coherence Connections:

1. Polynomials form a rich ground for mathematical explorations that reveal relationships in the system of integers. For example, students can explore the sequence of squares

\[1, 4, 9, 16, 25, 36, \ldots \]

and notice that the differences between them—3, 5, 7, 9, 11—are consecutive odd integers. This mystery is explained by the polynomial identity

\[(n+1)^2 - n^2 = 2n + 1.\]

A more complex identity, \((x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\),

allows students to generate Pythagorean triples. For example, taking \(x = 2\) and \(y = 1\) in this identity yields \(5^2 = 3^2 + 4^2\).

2. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should lean in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School


Evidence Outcomes

Students Can:

a. Rewrite simple rational expressions in different forms. Write \( \frac{p(x)}{q(x)} \) in the form \( \frac{r(x)}{s(x)} \), where \( a(x) \), \( b(x) \), \( q(x) \), and \( r(x) \) are polynomials with the degree of \( r(x) \) less than the degree of \( b(x) \), using inspection, long division, or, for the more complicated examples, a computer algebra system. (CCSS: HS.A-APR.D.6)

b. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expressions; add, subtract, multiply, and divide rational expressions. (CCSS: HS.A-APR.D.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical Thinking/Problem Solving. Students transform and interpret information and draw conclusions based on the pattern of data.
2. MP2: Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations. They are able to manipulate representations flexibly as both processes and objects.
3. MP7: Look for and make use of structure. In this GLE, students look closely to discern a pattern or structure to reveal characteristics of rational expressions as they relate to a graphical model of the expression.

Inquiry Questions:

1. How is dividing polynomials like long division with whole numbers?

Coherence Connections:

1. The analogy between polynomials and integers carries over to the idea of division with a remainder. Just as in Oracle 4 students find quotients and remainders of integers, in high school they find quotients and remainders of polynomials. The method of polynomial long division is analogous to, and simpler than, the method of integer long division. A particularly important application of polynomial division is the case where a polynomial \( p(x) \) is divided by a linear factor of the form \( (x - a) \), for a real number \( a \). In this case the remainder is the value \( p(a) \) of the polynomial at \( x = a \). This topic should not be reduced to “synthetic division,” which reduces the method to a matter of carrying numbers between registers, something easily done by a computer, while obscuring the reasoning that makes the result evident. It is important to regard the Remainder Theorem as a theorem, not a technique.

2. Expressing a rational expression in this form allows a student to see different properties of the graph, such as horizontal asymptotes.

3. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.A-CED.A. Creating Equations: Create equations that describe numbers or relationships ∗

Evidence Outcomes

Students Can:

a. Create equations and inequalities
   Include equations arising from linear and quadratic functions, and simple rational and exponential functions in one variable and use them to solve problems. (CCSS: HS.A-CED.A.1)

b. Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. (CCSS: HS.A-CED.A.2)

c. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (CCSS: HS.A-CED.A.3)

d. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$. (CCSS: HS.A-CED.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP2: Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations. They are able to manipulate representations flexibly as both processes and objects.

2. MP4: Model with mathematics. In this GLE students can apply the mathematics they know to model and solve problems arising in everyday life, society, and the workplace. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

3. MPS: Use appropriate tools strategically. In this GLE students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a calculator, a spreadsheet, a computer algebra system, or dynamic geometry software.

4. MP7: Look for and make use of structure. Students discern a pattern or structure and create a model.

Inquiry Questions:

1. What are some similarities in solving all types of equations?

2. How are order of operations and operational relationships (e.g., multiplication and division are inverse operations) important when solving multi-variable equations?
1. At first glance, it might seem that the progression from middle grades to high school is fairly straightforward: the repertoire of functions that is acquired during high school allows students to create more complex equations, including equations arising from linear and quadratic expressions, and simple rational and exponential expressions. Students are no longer limited largely to linear equations in modeling relationships between quantities with equations in two variables, and students start to work with inequalities and systems. Two developments in high school complicate this picture. First, students in high school start using parameters in their equations, to represent whole classes of equations (see HS-F.IF.5) or to represent situations where the equation is to be adjusted to fit data. Second, modeling becomes a major objective in high school. Two of the standards just cited refer to “solving problems” and “interpreting solutions in a modeling context.” And all the standards in the Creating Equations group carry a modeling star, denoting their connection with the Modeling category in high school. This connotes not only an increase in the complexity of the equations studied, but an upgrade of the student’s ability in every part of the modeling cycle.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.A-REI.A. Reasoning with Equations and Inequalities: Understand solving equations as a process of reasoning and explain the reasoning.

Evidence Outcomes

Students Can:

a. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (CCSS: HS.A-REI.A.1)

b. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (CCSS: HS.A-REI.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Communication. Articulate thoughts and ideas effectively using written communication skills.

2. MP3: Construct viable arguments and critique the reasoning of others. This GLE provides students an opportunity to show understanding of a logical flow, use stated assumptions, definitions, and previously established results in constructing arguments.

Inquiry Questions:

1. What types of equations can have extraneous solutions? What types cannot? Why?

2. How and why are extraneous solutions generated?

Coherence Connections:

1. Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. For example adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions.

2. In the process of learning to solve equations, students learn certain standard “if-then” moves, for example “If a = g then a + 2 = g + 2.” The danger in learning algebra is that students emerge with nothing but the moves, which may make it difficult to detect incorrect or made-up moves later on. Thus the first requirement in the standards in this domain is that students understand that solving equations is a process of reasoning.

3. Understanding solving equations as a process of reasoning demystifies “extraneous” solutions that can arise under certain solution procedures. The reasoning begins from the assumption that x is a number that satisfies the equation and ends with a list of possibilities for x. But not all the steps are necessarily reversible, and so it is not necessarily true that every number in the list satisfies the equation. For example, it is true that if x = 2 then x² = 4. But it is not true that if x² = 4 then x = 2 (it might be that x = -2). Squaring both sides of an equation is a typical example of an irreversible step, another is multiplying both sides of the equation by a quantity that might be zero.
**Prepared Graduates**

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

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**Grade Level Expectation: High School**

HS.A-REI.B. Reasoning with Equations and Inequalities. Solve equations and inequalities in one variable.

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**Evidence Outcomes**

<table>
<thead>
<tr>
<th>Students Can:</th>
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<tbody>
<tr>
<td>a. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters (CCSS: HS.A-REI.B.3)</td>
</tr>
<tr>
<td>b. Solve quadratic equations in one variable (CCSS: HS.A-REI.B.4.a)</td>
</tr>
<tr>
<td>a. Use the method of completing the square to transform any quadratic equation into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form (CCSS: HS.A-REI.B.4.a)</td>
</tr>
<tr>
<td>b. Solve quadratic equations (e.g., for $x^2 - 49$) by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers $a$ and $b$ (CCSS: HS.A-REI.B.4.b)</td>
</tr>
</tbody>
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**Academic Context and Connections**

**Colorado Essential Skills and Mathematical Practices:**

1. CES: Inquiry/Analysis. Students solve equations and draw conclusions from their solutions.
2. CES: Perseverance/Resilience. Students work effectively to solve linear and quadratic equations including those with complex solutions.
3. MPS: Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations. They are able to manipulate representations flexibly as both processes and objects.
4. MPS: Construct viable arguments and critique the reasoning of others. Students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They build a logical progression of statements to solve equations. They justify their conclusions, communicate them to others, and respond to the arguments of others.

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**Inquiry Questions:**

1. How does the initial form of a quadratic equation give us to an appropriate solution strategy?

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**Coherence Connections:**

1. With an understanding of solving equations as a reasoning process, students can organize the various methods for solving different types of equations into a coherent picture. For example, solving linear equations involves only steps that are reversible (adding a constant to both sides, multiplying both sides by a non-zero constant, transforming an expression on one side into an equivalent expression). Therefore solving linear equations does not produce extraneous solutions. The process of completing the square also involves only this same list of steps, and so converts any quadratic equation into an equivalent equation of the form $(x - p)^2 = q$ that has exactly the same solutions.

2. It is traditional for students to spend a lot of time on various techniques of solving quadratic equations, which are often presented as if they are completely unrelated (factoring, completing the square, the quadratic formula). In fact, as we have seen, the key step in completing the square involves all its heart factoring. And the quadratic formula is nothing more than an encapsulation of the method of completing the square, expressing the actions repeated in solving a collection of quadratic equations with numerical coefficients with a single formula. Rather than long drills on techniques of dubious value, students with an understanding of the underlying reasoning behind these methods are opportunistic in their application, choosing the method that best suits the situation at hand.
Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

Evidence Outcomes
Students Can:

a. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (CCSS: HS.A.REI.C.6)

b. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (CCSS: HS.A.REI.C.6)

c. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line \( y = –3x \) and the circle \( x^2 + y^2 = 3 \). (CCSS: HS.A.REI.C.7)

d. (+) Represent a system of linear equations as a single matrix equation in a vector variable. (CCSS: HS.A.REI.C.6)

e. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). (CCSS: HS.A.REI.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Communication. Students represent and solve equations that model the world around them by using graphs and algebraic methods.

2. MP2: Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations. They are able to manipulate representations flexibly at both processes and objects.

3. MP3: Construct viable arguments and critique the reasoning of others. Students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They build a logical progression of statements to solve equations. They justify their conclusions, communicate them to others, and respond to the arguments of others.

4. MPS: Use appropriate tools strategically. Students can solve systems of equations and create graphs that display solutions.

Inquiry Questions:

1. How is the solution to a system of equations related to the graph of the system? What if the system has no solution? What if it has infinitely many solutions?

Coherence Connections:

1. Students can solve systems of equations using methods that can include, but are not limited to graphical, elimination/linear combination, substitution, and modeling.

2. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or apps to model and find approximate solutions for systems of equations.

3. Student work with solving systems of equations starts the same way as work with solving equations in one variable: with an understanding of the reasoning behind the various techniques. An important step is realizing that a solution to a system of equations must be a solution of all the equations in the system simultaneously. Then the process of adding one equation to another is understood as “if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum of the left sides of the two equations is equal to the sum of the right sides.”

4. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+)
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School


Evidence Outcomes

Students Can:

a. Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (CCSS: HS.A-REI.D.10)

b. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x). (Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.) Find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (CCSS: HS.A-REI.D.11)

c. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (CCSS: HS.A-REI.D.12)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Communication. Students represent and solve equations that model the world around them by using graphs and algebraic methods.
2. MP1: Make sense of problems and persevere in solving them: Depending on the context of the problem, students transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. They explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.
3. MP6: Attend to precision. Students are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

Inquiry Questions:

1. How many different ways can you find to solve $x^2 = (2x - 9)^2$?

Coherence Connections:

1. Students need to understand that approximate solutions can be obtained using numerical solution methods (data in a table used to approximate an algebraic function) and graphical solutions.
2. Algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions.
3. The Reasoning with Equations and Inequalities GLEs ask students not only to carry out a procedure for solving a quadratic equation, but to understand such a procedure as a logical process in which an equality or a set of possible equalities at each step follows from the previous steps. Teachers should model this reasoning process when discussing problems with students.
Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School
HS.F-IF.A.1 Interpreting Functions: Understand the concept of a function and use function notation.

Evidence Outcomes
Students Can:

a. Explain that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).
(CCSS: HS.F-IF.A.1)

b. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
(CCSS: HS.F-IF.A.2)

c. Demonstrate that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1 \), \( f(n + 1) = f(n) + f(n - 1) \) for \( n \geq 1 \).
(CCSS: HS.F-IF.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP2: Reason abstractly and quantitatively. Students use symbolic notation and manipulate the representations as both processes and objects, without necessarily attending to their referents.

2. MP7: Look for and make use of structure. Students to see algebraic expressions as single objects or as being composed of several objects.

Inquiry Questions:
1. Besides the notation we use, what makes a function different from an equation?

Coherence Connections:

1. The notion of a function is introduced in Grade 8. Linear functions are a major focus, but note that students are also expected to give examples of functions that are not linear. In high school, students deepen their understanding of the notion of function, expanding their repertoires to include quadratic and exponential functions, and increasing their understanding of correspondences between geometric transformations of graphs of functions and algebraic transformations of the associated equations.

2. A key advantage of function notation is that the correspondence is built into the notation. For example, \( f(5) \) is shorthand for “the output value of \( f \) when the input value is 5.”

3. Functions are a fundamental idea in mathematics that are the basis of most mathematical applications yet are challenging to learn and teach. Understanding the concept of functions requires developing and applying an understanding of the definition of function as a set of ordered pairs that maps from a set of inputs (domain) to a set of outputs (range) where each input is mapped to exactly one output. Understanding functions also requires developing a mental image that enables one to simultaneously think of what is happening to two quantities, which is cognitively complex yet foundational to understanding functions. The notion of functions is broad and flexible allowing it to apply to many situations. In the study of calculus (considered beyond essential mathematics standards), the concept of function together with the rate of change are integral to reason about how variables operate together.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.F-IF.B. Interpreting Functions. Interpret functions that arise in applications in terms of the context.

Evidence Outcomes

Students Can:

a. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features. Key features include: intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (CCSS: HS.F-IF.B.4)

b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $A(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function $A$. (CCSS: HS.F-IF.B.5)

c. Calculate and interpret the average rate of change presented symbolically or as a table, of a function over a specified interval. Estimate the rate of change from a graph. (CCSS: HS.F-IF.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP5. Use appropriate tools strategically. As students graph functions and interpret key features of their graphs, or use key features to construct a graph, they engage with mathematical tools such as graph paper, graphing calculators, or computer algebra systems.

2. MP7. Look for and make use of structure. Students make structural comparisons between linear, exponential, quadratic and higher order polynomial, rational, radical and trigonometric functions and observe communities between functions with unrestricted domains and consistency in end behavior across function types.

Inquiry Questions:

1. In what ways does a real-world context influence the domain of the function that models it?
2. How are slope and rate of change related?

Coherence Connections:

1. Students should be able to interpret and analyze how the dependent and independent variables for a function vary together, with extensive attention to linear, exponential and quadratic functions. At the same time, experience with various functions is critical to fully develop conceptual understanding of functions and rate of change as students compare and contrast the behavior and attributes of different functions.

2. Given a function, students should be able to identify whether the function is linear, exponential, quadratic or none of these. Building upon prior knowledge of tables, students should use tables to explore and analyze how functions behave including how the independent variable changes as the dependent variables changes.

3. Graphs help us reason about rates of change. Students learned in Grade 8 that the rate of change of a linear function is equal to the slope of the line that is its graph. And because the slope of a line is constant, that is, between any two points it is the same, the “rate of change” has an unambiguous meaning for a linear function. For nonlinear functions, however, rates of change are not constant, and so we talk about average rates of change over an interval.

4. As a way for students to make sense of and demonstrate their understanding, students should construct sketches/graphs from a given context where two variables are changing dynamically (e.g., examining the height as compared to the volume of an urn as it fills with water, which is more complex than graphing a relationship between time and height).

5. Technology should be used as a tool to assist students in their ability to visualize and understand how various functions behave as experienced in different representations and in context.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.F-IF.C: Interpreting Functions. Analyze functions using different representations.

Evidence Outcomes

Students Can:

a. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (CCSS: HS.F-IF.C.7)
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS: HS.F-IF.C.7.a)
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (CCSS: HS.F-IF.C.7.b)
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (CCSS: HS.F-IF.C.7.c)
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. (CCSS: HS.F-IF.C.7.d)
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (CCSS: HS.F-IF.C.7.e)

b. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (CCSS: HS.F-IF.C.8)
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (CCSS: HS.F-IF.C.8.a)
   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as \( y = (1.05)^t \), \( y = (0.9)^t \), \( y = (1.01)^t \), \( y = (1.2)^t \), and classify them as representing exponential growth or decay. (CCSS: HS.F-IF.C.8.b)

c. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (CCSS: HS.F-IF.C.9)

Academic Context and Connections

1. Colorado Essential Skills and Mathematical Practices:
   1. MP2: Reason abstractly and quantitatively. Analyzing functions requires the ability to think and reason abstractly and understand the inherent connections between the symbolic representation, a table of values, the graph of a function, and key features of the graph.
   2. MP5: Use appropriate tools strategically. Students use calculators or other graphing software to see the graphs of more complicated functions. With a firmer understanding of these underlying concepts, graphing by hand becomes more straightforward in many cases.
   3. MP6: Attend to precision. Students attend to important terms, definitions, and symbols.

Inquiry Questions:

1. How can we adjust the way we write a function to illustrate its key features? Give an example of a function written two different ways—one where one or more key features is evident, and another where they are not.

Coherence Connections:

1. Building upon understanding of linear functions as one family of functions and experience with connecting representations, students begin to explore other functions in greater depth. Through high school and across their mathematics courses, it is crucial that students be given multiple and varied opportunities to compare and contrast functions as they reason about the structure inherent in functions in general and the structure within specific families of functions. However, each and every student should delve more deeply into characteristics specifically of exponential and quadratic families of functions to model and solve problems. A card sort activity in which students are given different representations of various functions can be used to highlight similarities and differences between the functions. Such an activity can be a launch to a discussion of families of functions and defining characteristics of function families (and some subfamilies).

2. While important that students are able to recognize, construct, and apply attributes of exponential and quadratic functions, perhaps more important is that students see each of these families of functions in a more general sense as a way to model and explain phenomena. The inclusion of other functions (e.g., square root, periodic, polynomial, rational) is a powerful instructional and learning opportunity. When students compare and contrast different characteristics of functions, students develop a more complex, connected understanding of the structure of functions in general and structures that make families of functions unique.

3. In analyzing families of functions, students identify and describe such behaviors as the following: (a) whether a function is increasing/decreasing over a particular interval; (b) when the value of the function is positive or negative, (c) local minima and maxima, (d) intercepts, and (e) end behavior.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS F-B.F.A: Building Functions: Build a function that models a relationship between two quantities.

Evidence Outcomes

Students Can:

a. Write a function that describes a relationship between two quantities.*
   (CCSS: HS F-B.F.A.1)
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   (CCSS: HS F-B.F.A.1.a)
   b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (CCSS: HS F-B.F.A.1.b)
   c. (+) Create functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $H(t)$ is the height of a weather balloon as a function of time, then $T(H(t))$ is the temperature at the location of the weather balloon as a function of time. (CCSS: HS F-B.F.A.1.c)

b. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
   (CCSS: HS F-B.F.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP4: Model with mathematics. This GLE connects directly to mathematical modeling, where students apply their understanding of functions to real-world contexts.

Inquiry Questions:

1. Why does a function require one output for every input?
2. How can the ideas of cause and effect be developed through the building of functions?

Coherence Connections:

1. By the end of middle school, students should have a working understanding of how a function is defined as well as modeling situations between two variables in context. This can be seen as an understanding of how two variables relate to each other in terms of input and output.
2. High school teachers need to build from the understanding of input and output by ensuring that students understand the dependence in mathematical relationships. This is developed by multiple opportunities for students to represent mathematical relationships through tables and graphs with the outcome being a mathematical understanding of cause and effect. In high school, students need to be able to analyze dependence and independence in linear, quadratic, and exponential situations. Rate of change in both linear and non-linear patterns needs to be analyzed in contextualized and de-contextualized situations.
3. Linear functions are a foundation for analyzing and interpreting (with and without technology) non-linear functions.
4. Building new functions in context using mathematical operations to solve problems further develops understanding of function.
5. Some functions have inverses, analogous to “undoing” used to solve equations.
6. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS F-BF.B: Building Functions: Build new functions from existing functions.

Evidence Outcomes

Students Can:

a. Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( kf(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) both positive and negative, find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (CCSS: HS.F-BF.B.3)

b. Find inverse functions. (CCSS: HS.F-BF.B.4)
   a. Solve an equation of the form \( f(x) = c \) for a simple function \( f \) that has an inverse and write an expression for the inverse. For example, \( f(x) = 2^x \) or \( f(x) = x^3 \) for \( x \neq 0 \). (CCSS: HS.F-BF.B.4.a)
   b. (+) Verify by composition that one function is the inverse of another. (CCSS: HS.F-BF.B.4.b)
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (CCSS: HS.F-BF.B.4.c)
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain. (CCSS: HS.F-BF.B.4.d)
   c. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (CCSS: HS.F-BF.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP3: Construct viable arguments and critique the reasoning of others. As students gain an understanding of transformations of functions, they will create verbal and written explanations of the generalities they find across function families.

2. MP6: Attend to precision. Students use accurate terms, definitions and mathematical symbols when building functions.

Inquiry Questions:

1. What is a function family?

2. What are the similarities and differences between function families?

3. Describe cases where the inverse of a function is only a function when the domain is restricted.

Coherence Connections:

1. Whereas an equation is based on the ideas of equality where two values (numeric or symbolic) are set equal to each other, a function is the mapping of set of inputs to a set of outputs.

2. Functions can be presented in many different forms and using algebraic skills and knowledge combined with knowledge of function and function families, functions can be rewritten in forms conducive to analysis and problem solving.

3. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.F-LE.A. Linear, Quadratic and Exponential Models. Construct and compare linear, quadratic, and exponential models and solve problems.

Evidence Outcomes

Students Can:

a. Distinguish between situations that can be modeled with linear functions and with exponential functions. (CCSS: HS.F-LE.A.1)
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (CCSS: HS.F-LE.A.1.a)
   b. Identify situations in which one quantity changes at a constant rate per unit interval relative to another. (CCSS: HS.F-LE.A.1.b)
   c. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (CCSS: HS.F-LE.A.1.c)

b. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (CCSS: HS.F-LE.A.2)

c. Use graphs and tables to describe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (CCSS: HS.F-LE.A.3)

d. For exponential models, express as a logarithm the solution to \( ab^x = d \) where \( a \), \( c \), and \( d \) are numbers and the base \( b \) is 2, 10, or \( e \) evaluate the logarithm using technology. (CCSS: HS.F-LE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP1: Make sense of problems and persevere in solving them. Students reason about and with situations that can be modeled by functions. In high school, focused study on multiple function types adds complexity to the reasoning required.

2. MP7: Look for and make use of structure. Students distinguish between situations that can be modeled with linear functions and with exponential functions using understandings of rates of growth and factors of growth over equal intervals.

Inquiry Questions:

1. How can a table, graph, and function notation be used to represent and solve function problems?

2. Why is an understanding of the connections between table, graph, and function notation important to forming a conceptual understanding of functions?

Coherence Connections:

1. To prove that a linear function grows by equal differences over equal intervals, students draw on the understanding developed in Grade 8 that the ratio of the rise and run for any two distinct points on a line is the same (see the Expressions and Equations Progression) and recall it in terms of function inputs and outputs. An interval can be seen as determining two points on the line whose inputs (x-coordinates) occur at the boundaries of the intervals. The equal ratios can be seen as the runs for two pairs of points. Because these runs have equal length and the ratio of run to rise is the same for any pair of distinct points, the differences of the corresponding outputs (the rises) are the same.

2. At both the middle school and high school level, many students’ difficulty in working with functions is the shift from numeric to symbolic representations. However, when students see numerals as symbols of one quantity they can more readily see letters as symbols of many quantities.

3. While the similarity between functions and algebra are obvious in terms of representations of the patterns they create, the inability to recognize the distinction between them can be an obstacle in a student’s understanding of mathematics.

4. Hand-held and computer-based technology should be accessed by students in both middle school and high school levels to assist in visualizing the patterns created by functions.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.F-LE.B. Linear, Quadratic, and Exponential Models: Interpret expressions for functions in terms of the situation they model.

Evidence Outcomes

Students Can:

a. Interpret the parameters in a linear or exponential function in terms of a context. (CCSS: HS.F-LE.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP2. Reason abstractly and quantitatively. As they work within contextual applications, students will both decontextualize - abstract a given situation and representing it symbolically and manipulate the representing symbols without necessarily attending to their referents - and contextualize - pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Inquiry Questions:

1. What are the similarities and differences in the ways that graphs, tables, and equations model a function?

2. What does the linear component, \( bx + c \), of a quadratic expression determine about the quadratic function?

3. How do the \( a \) and \( b \) values in the exponential expression \( a \left( bx^{2}\right) \) compare to the \( a \) and \( b \) values in the linear expression \( a + bx \)?

Coherence Connections:

1. While middle school students will come to high school with some experiences in modeling linear relationships with functions, high school teachers can take these descriptive skills for solving through graph and table representations.

2. Students should describe rate of change between two quantities as well as initial values both within and apart from context. An understanding of how the interval remains the same in a linear situation as well as how the interval increases or decreases in a nonlinear situation is developed in high school. The use of recursive reasoning should be accessed for students to analyze pattern and structure in tables in order to create functions which model the situation in the context.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.F-TFA. Trigonometric Functions: Extend the domain of trigonometric functions using the unit circle.

Evidence Outcomes

Students Can:

a. Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: HS.F-TFA.1)

b. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: HS.F-TFA.2)

c. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4, and π/6, and use the unit circle to express the values sine, cosine, and tangent for π, π + x, and 2π – x and in terms of their values for x where x is any real number. (CCSS: HS.F-TFA.3)

d. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. (CCSS: HS.F-TFA.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP7: Look for and make use of structure. Students measure distance around a circle in units the length of the radius of the circle, or radians, and see how this measure stays the same for all equivalent angles, regardless of the circle’s size.

Inquiry Questions:

1. How does the relationship of the lengths of both legs in a right triangle support both degree and radian measure?
2. How is the relationship of the lengths of both legs in a right triangle seen in the unit circle?

Coherence Connections:

1. Students begin their study of trigonometry with right triangles. Right triangle trigonometry is concerned with ratios of sides of right triangles, allowing functions of angle measures to be defined in terms of these ratios. This limits the angles considered to those between 0 degrees and 90 degrees. After study of trigonometric ratios in right triangles, students expand the types of angles considered. Students learn, by similarity, that the radian measure of an angle can be defined as the quotient of arc length to radius. As a quotient of two lengths, therefore, radian measure is “dimensionless.” That is why the “unit” is often omitted when measuring angles in radians.

2. Trigonometry is a component of mathematics unique to high school where the functions standard and geometry standard overlap and support each other. The Grade 8 standard developing understanding with the Pythagorean Theorem (Understand and Apply the Pythagorean Theorem: 8.G.B) is a critical support for the right angle trigonometry developed in high school.

3. The unit circle is one model to access for making sense of right angle trigonometry. Using this model requires an understanding of fractions first developed in the elementary grades as well as an understanding of the value π first developed in Grade 8. A main use of the unit circle is to see angles in terms of radian measures so this can be compared to degree measure.
**Prepared Graduates**

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

**Grade Level Expectation: High School**

HS.F-TF.B. Trigonometric Functions: Model periodic phenomena with trigonometric functions.

**Evidence Outcomes**

**Students Can:**

- **a.** Model periodic phenomena with trigonometric functions with specified amplitude, frequency, and midline. *(CCSS: HS.F-TF.B.5)*

- **b.** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. *(CCSS: HS.F-TF.B.6)*

- **c.** (+) Use inverse function to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. *(CCSS: HS.F-TF.B.7)*

**Academic Context and Connections**

**Colorado Essential Skills and Mathematical Practices:**

1. MP2. Reason abstractly and quantitatively. This GLE provides students an opportunity to make sense of periodic quantities and their relationships in problem situations.

2. MP4. Model with mathematics. This GLE provides students an opportunity to apply the mathematics they know to model periodic situations arising in everyday life, society, and the workplace.

**Inquiry Questions:**

1. How does an understanding of the unit circle support an understanding of periodic phenomena?

2. What are examples of phenomena that can be modeled using trigonometric functions?

**Coherence Connections:**

1. Periodic phenomena is a direct result of motion around the unit circle. Students come from middle school with understandings of circles and can find area and circumference of circles. This understanding needs to be access by high school teachers as they develop the ideas of periodic motion as simply being the graph of the movement around the circle.

2. Transformations of this graph are made by changing the $a$ and $b$ values in the parent function $y = a \sin(b)x$. Transformations on the coordinate plane are first seen in Grade 8 (Ventry experimentally the properties of rotations, reflections, and translations 8.G.A.1) and should be accessed by high school teachers in the development of trigonometric transformations.

3. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
**Prepared Graduates**

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

**Grade Level Expectation: High School**

HS.F-TF.C. Trigonometric Functions: Prove and apply trigonometric identities.

**Evidence Outcomes**

**Students Can:**

a. Prove the Pythagorean identity \(\sin^2(\theta) + \cos^2(\theta) = 1\) and use it to find \(\sin(\theta), \cos(\theta),\) or \(\tan(\theta)\) given \(\sin(\theta), \cos(\theta),\) or \(\tan(\theta)\) and the quadrant of the angle. (CCSS: HS.F-TF.C.8)

b. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. (CCSS: HS.F-TF.C.9)

**Academic Context and Connections**

**Colorado Essential Skills and Mathematical Practices:**

1. CES: Communication. Students effectively explain the relationship between algebra and trigonometry.

2. MP2: Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problem situations.

3. MP7: Look for and make use of structure. Students see algebraic expressions as single objects or as being composed of several objects.

**Inquiry Questions:**

1. How is the Pythagorean identity related to the Pythagorean Theorem?

2. How is the identity \(\sin^2(\theta) + \cos^2(\theta) = 1\) related to the equation of a circle centered at the origin?

**Coherence Connections:**

1. An identity is a statement of equality between two expressions that is true for all values of the variables for which the expressions are defined. The use of the distributive property and a factored quadratic expression are examples of identities students bring from middle school to high school. High school teachers can access this middle school level understanding to further develop algebraic as well as trigonometric identities.

2. The Pythagorean Identity is a foundational trigonometric identity that must be understood through its components both in and out of context.

3. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+)
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.S.ID.A: Interpreting Categorical and Quantitative Data: Summarize, represent, and interpret data on a single count or measurement variable.

Evidence Outcomes

a. Model data in context with plots on the real number line (dot plots, histograms, and box plots). (CCSS: HS.S.ID.A.1)

b. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (CCSS: HS.S-ID.A.2)

c. Interprets differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (CCSS: HS.S-ID.A.3)

d. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and identify data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. (CCSS: HS.S-ID.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices

1. CES: Information Literacy: Students understand statistical descriptors of data and can interpret and be critical of the use of statistics in their lives outside of school.

2. MP4: Model with mathematics. The contextual nature of statistics at this level affords the opportunity for students to use mathematics to model, describe, interpret, and generalize about the real-world.

3. MPS: Use appropriate tools strategically. Students should be aware that technology now allows easy computation of any area under a normal curve.

Inquiry Questions:

1. How would you describe the difference between the distributions of two data sets with the same measure of center but different measures of spread?

2. What questions might a statistician ask about extreme data points? How do they/should they affect the interpretation of the data?

Coherence Connections:

1. In connection with the mean as a measure of center, the standard deviation is introduced as a measure of variation. The standard deviation is based on the squared deviations from the mean, but involves much the same principle as the mean absolute deviation (MAD) that students learned about in Grades 6–8. Students should see that the standard deviation is the appropriate measure of spread for data distributions that are approximately normal in shape, as the standard deviation then has a clear interpretation related to relative frequency.

2. At this level, students are not expected to fit normal curves to data. (In fact, it is rather complicated to rescale data plots to be density plots and then find the best fitting curve.) Instead, the aim is to look for broad approximations, with application of the rather rough “empirical rule” (also called the 68%-95%-99.7% Rule) for distributions that are somewhat bell-shaped. The better the bell, the better the approximation. Using such approximations is partial justification for the introduction of the standard deviation.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.S-ID.B. Interpreting Categorical and Quantitative Data: Summarize, represent, and interpret data on two categorical and quantitative variables.

Evidence Outcomes

Students Can:

a. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (CCSS: HS.S-ID.B.5)

b. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (CCSS: HS.S-ID.B.6)
   a. Fit a function to the data: use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (CCSS: HS.S-ID.B.6.a)
   b. Informally assess the fit of a function by plotting and analyzing residuals. (CCSS: HS.S-ID.B.6.b)
   c. Fit a linear function for a scatter plot that suggests a linear association. (CCSS: HS.S-ID.B.6.c)

   Distinguish between correlation and causation. (CCSS: HS.S-ID.B.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Use Information and Communications Technologies. Students create, interpret and demonstrate their results using technology.

2. MP4: Model with mathematics. As with univariate data analysis, students take a deeper look at bivariate data, using their knowledge of functions to fit models to quantitative data.

3. MP5: Use appropriate tools strategically. Students’ work with categorical and quantitative data should rely on the use of technology, freeing them up to analyze, describe, and make determinations about the data.

4. MP7: Look for and make use of structure. As with univariate data analysis, students now take a deeper look at bivariate data, using their knowledge of proportions to describe categorical associations.

Inquiry Questions:

1. Does a high correlation (close to 1) in the data of two quantitative variables mean that one causes a response in the other? Why or why not?

2. In what way(s) does a plot of the residuals help us consider the best model for a data set?

Coherence Connections:

1. As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts. In a modeling context, they might informally fit a quadratic function to a set of data, graphing the data and the model function on the same coordinate axes. They also draw on skills they first learned in middle school to apply basic statistics and simple probability in a modeling context. For example, they might estimate a measure of center or variation and use it as an input for a rough calculation.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.S-ID.C. Interpreting Categorical and Quantitative Data: Interpret linear models.

Evidence Outcomes

Students Can:

- Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (CCSS: HS.S-ID.C.7)

- Using technology, compute and interpret the correlation coefficient of a linear fit. (CCSS: HS.S-ID.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Information Literacy. Students understand can interpret and be critical of the use of statistics in their lives outside of school.

2. CES: Use Information and Communications Technologies. Students create statistical correlations using technology and can interpret their results as they relate to use outside of school.

3. MP2: Reason abstractly and quantitatively. Students reason about the contextual meaning of slope and intercept of linear models of real-world data, and reason non-contextually when finding the linear model.

4. MP5: Use appropriate tools strategically. Students use technology to compute, model, and reason about linear models of bivariate data.

Inquiry Questions:

1. How is it possible for the intercept of a linear model to not have meaning in the context of the data?

2. What does the correlation coefficient of a linear fit tell us?

Coherence Connections:

1. Students have seen scatter plots in Grade 8 and now extend that knowledge to fit mathematical models that capture key elements of the relationship between two variables and to explain what the model tells us about that relationship. Some of the data should come from science, as in the examples about cricket chirps and temperature, and tree growth and age, and some from other aspects of their everyday life, e.g., cost of pizza and calories per slice.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.S-IC.A. Making Inferences and Justifying Conclusions. Understand and evaluate random processes underlying statistical experiments.

Evidence Outcomes

**Students Can:**

a. Describe statistics as a process for making inferences about population parameters based on a random sample from that population. (CCSS: HS.S-IC.A.1)

b. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (CCSS: HS.S-IC.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Information Literacy. Students understand how statistics serve to make inferences about a population.

2. MP3: Construct viable arguments and critique the reasoning of others. Students understand that the process of fitting and interpreting models for discovering possible relationships between variables requires insight, good judgment and a careful look at a variety of options consistent with the questions being asked in the investigation. They use a variety of statistical tools to construct and defend logical arguments based on data.

3. MP6: Attend to precision. Students understand and describe the differences between statistics (derived from samples) and parameters (characteristic of the population).

Inquiry Questions:

1. What is the difference between a statistic and a parameter?

2. How could we ensure that the results of a simulation closely approximate the theoretical model?

Coherence Connections:

1. In Grade 7, students approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

2. Students move beyond the analyzing data of middle school to making sound statistical decisions based on probability models. The reasoning process is as follows: develop a statistical question in the form of a hypothesis (supposition) about a population parameter; choose a probability model for collecting data relevant to that parameter; collect data; compare the results seen in the data with what is expected under the hypothesis. If the observed results are far away from what is expected and have a low probability of occurring under the hypothesis, then that hypothesis is called into question. In other words, the evidence against the hypothesis is weighed by probability.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.S-IC.B. Making Inferences and Justifying Conclusions. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Evidence Outcomes

Students Can:

a. Identify the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (CCSS: HS.S-IC.B.5)

b. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. (CCSS: HS.S-IC.B.4)

c. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (CCSS: HS.S-IC.B.5)

d. Evaluate reports based on data. Define and explain the meaning of significance, both statistical (using p-values) and practical (using effect size). (CCSS: HS.S-IC.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CSS. Critical Thinking/Problem Solving. Students apply statistical methods to interpret information and draw conclusions in real-world contexts.
2. MP3. Construct viable arguments and critique the reasoning of others. Students consider sampling, design, and results of sample surveys, experiments, and observational studies and justify reasonable responses and mistraining or inaccurate results.
4. MP8. Look for and express regularity in repeated reasoning. Students observe regular patterns in distributions of sample statistics and use them to make generalizations about the population parameter.

Inquiry Questions:

1. How can the results of a statistical investigation be used to support an argument?
2. What happens to sample-to-sample variability when you increase the sample size?
3. How does randomization minimize bias?
4. Can the practical significance of a given study matter more than statistical significance? Why is it important to know the difference?
5. Why is the margin of error in a study important?

Coherence Connections:

1. In grades 6-8, students engage with statistics to: (a) draw informal comparative inferences about two populations; (b) informally assess degree of visual overlap of two numerical data distributions; (c) use measures of center and measure of variability for numerical data from random samples to draw comparative inferences; and (d) generate or simulate multiple samples to gauge variation in estimates and predictions. These concepts are extended and formalized in high school.

2. Students' understanding of random sampling is the key that allows the computation of margins of error in estimating a population quantity can now be extended to the random assignment of treatments to available units in an experiment.
**Prepared Graduates**

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

**Grade Level Expectation: High School**

**HS.S.CPA.1** Conditional Probability and the Rules of Probability: Understand independence and conditional probability and use them to interpret data.

**Evidence Outcomes**

**Students Can:**

a. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or “and,” “or,” “not”). (CCSS: HS.S.CPA.1)

b. Explain that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS: HS.S.CPA.2)

c. Using the conditional probability of A given B as \( P(A | B) \), interpret the independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (CCSS: HS.S.CPA.3)

d. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (CCSS: HS.S.CPA.4)

e. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (CCSS: HS.S.CPA.5)

**Academic Context and Connections**

**Colorado Essential Skills and Mathematical Practices:**

1. **CES:** Critical Thinking/Problem Solving. Students understand and apply probability and data collection methods to the real world.

2. **MP3:** Construct viable arguments and critique the reasoning of others. Students can explain their reasoning about independent events, conditional probabilities, and interpret frequency tables. They can explain these mathematical concepts in the context of everyday situations.

3. **MP4:** Model with mathematics. Students understand that in real-world applications, the probabilities of events are often approximated by data about those events.

**Inquiry Questions:**

1. Can probability be used to model all types of uncertain situations? Why or why not?

2. How and why are simulations used to determine probability when the theoretical probability is unknown?

**Coherence Connections:**

1. In Grades 7 and 8, students encountered the development of basic probability, including chance processes, probability models, and sample spaces.

2. In high school, the relative frequency approach to probability is extended to conditional probability and independence, rules of probability and their use in finding probabilities of compound events, and the use of probability distributions to solve problems involving expected value. As seen in the Making Inferences & Justifying Conclusions GLEs, there is a strong connection between statistics and probability. This will be seen again in this section with the use of data in selecting values for probability models.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.S-CP.B: Conditional Probability and the Rules of Probability. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Evidence Outcomes

Students Can:

a. Find the conditional probability of $A$ given $B$ as the fraction of $B$'s outcomes that also belong to $A$, and interpret the answer in terms of the model. (CCSS: HS.S-CP.B.6)

b. Apply the Addition Rule,
   \[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]
   , and interpret the answer in terms of the model. (CCSS: HS.S-CP.B.7)

c. (+) Apply the general Multiplication Rule in a uniform probability model,
   \[ P(A \text{ and } B) = P(A) P(B | A) = P(B) P(A | B) \]
   , and interpret the answer in terms of the model. (CCSS: HS.S-CP.B.8)

d. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. (CCSS: HS.S-CP.B.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. CES: Critical Thinking/Problem Solving. Students understand and apply probability to the real world.

2. MP2: Reason abstractly and quantitatively. Students consider probability concepts in context and mathematically, and can make connections between both types of reasoning.


Inquiry Questions:

1. What is an everyday situation that helps explain the Addition Rule? How does the context help you understand the subtraction of $P(A \text{ and } B)$?

Coherence Connections:

1. Studying and understanding probability, which is always in a context, provides high school students with a mathematical structure for dealing with the many changes they will experience as part of life.

2. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
 Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

 Grade Level Expectation: High School

HS.S-MD.A. Using Probability to Make Decisions: Calculate expected values and use them to solve problems.

 Evidence Outcomes

 Students Can:

a. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. (CCSS: HS.S-MD.A.1)

b. (+) Calculate the expected value of a random variable: interpret it as the mean of the probability distribution. (CCSS: HS.S-MD.A.2)

c. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. (CCSS: HS.S-MD.A.3)

d. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? (CCSS: HS.S-MD.A.4)

 Academic Context and Connections

 Coherence Connections:

1. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School


Evidence Outcomes

Students Can:

a. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. (CCSS: HS.S-MD.B.5)
   a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or game at a fast-food restaurant. (CCSS: HS.S-MD.B.5.a)
   b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or major accident. (CCSS: HS.S-MD.B.5.b)

b. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (CCSS: HS.S-MD.B.6)

c. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). (CCSS: HS.S-MD.B.7)

Academic Context and Connections

Coherence Connections:

1. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G.CO.A. Congruence: Experiment with transformations in the plane.

Evidence Outcomes

Students Can:

a. State precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (CCSS: HS.G.CO.A.1)

b. Represent transformations in the plane using e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not. (e.g., translation versus horizontal stretch). (CCSS: HS.G.CO.A.2)

c. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (CCSS: HS.G.CO.A.5)

d. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (CCSS: HS.G.CO.A.4)

e. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using appropriate tools. e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (CCSS: HS.G.CO.A.5)

Colorado Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MPS: Use appropriate tools strategically. Students explore transformations in the plane using concrete and technological tools. The use of tools allows students to attend to the relationships without the burden of difficulty with precision.

2. MPS: Attend to precision. Because concepts such as rotation, reflection, and translation were treated in the Grade 8 standards mostly in the context of hands-on activities, and with an emphasis on geometric intuition, high school geometry will put equal weight on precise definitions.

Inquiry Questions:

1. What is the relationship between functions and geometric transformations?

2. How is a figure's symmetry connected to congruence transformations?

Coherence Connections:

1. Rotations, reflections and translations are developed experimentally in Grade 8, and this experience should be built upon in high school, giving greater attention to precise definitions and formal reasoning.

2. Transformations can be studied in terms of functions, where the inputs and outputs are points in the plane. Rotations are studied again in the context of circles.

3. Using previous comparisons and descriptions of transformations, develop and understand the meaning of rotations, reflections, and translations based on angles, circles, perpendicular lines, parallel lines, and line segments.

4. Communicating reasoning in geometry involves formal proof/precise language, informal explanation/construction, and strategic experimentation to verify/refute claims, and students must have experiences with all three.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G.CO.B. Congruence: Understand congruence in terms of rigid motions.

Evidence Outcomes

Students Can:

a. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (CCSS: HS.G.CO.B.6)

b. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (CCSS: HS.G.CO.B.7)

c. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (CCSS: HS.G.CO.B.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP3: Construct viable arguments and critique the reasoning of others. Students justify their reasoning regarding congruence in terms of rigid motions.

2. MP6: Attend to precision. Students salvage conjectures that fail in particular cases by more precisely defining the conditions under which the conjecture is true. For example, two angle measures and a side length do not determine a triangle, but a certain configuration of these parts leads to the angle-side-angle theorem.

Inquiry Questions:

1. How can transformations be used to prove that two triangles are congruent?

2. What is the minimum amount of information you need to know about two triangles in order to determine if they are congruent? Why is that the minimum?

Coherence Connections:

1. Graphs of functions and other curves provide opportunities to make congruence and similarity arguments as well as connect algebra and geometry

2. Figures, including graphs of functions, are defined by their attributes, and showing that two figures are congruent involves finding a rigid motion (translation, rotation, reflection, or glide reflection) or sequence of rigid motions that maps one figure to the other.
Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision, and comparison while continually asking themselves, "Does this make sense?"
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School
HS.G-CO.C. Congruence. Prove geometric theorems.

Evidence Outcomes
Students Can:

a. Prove theorems about lines and angles. Theorems include: vertical angles are congruent, when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (CCSS: HS.G-CO.C.9)

b. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (CCSS: HS.G-CO.C.10)

c. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (CCSS: HS.G-CO.C.11)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:
1. MP2 Reason abstractly and quantitatively. Abstraction is used in geometry when, for example, students use a diagram of a specific isosceles triangle as an aid to reason about all isosceles triangles.
2. MP3 Construct viable arguments and critique the reasoning of others. Students explain their thinking and consider alternate approaches when working with formal and informal geometric proofs.

Inquiry Questions:
1. Can some theorems be proven without using other previously proven theorems? If so, what does that imply about a system of theorems?

Coherence Connections:
1. Properties of lines and angles, triangles and parallelograms were investigated in Grades 7 and 8. In high school these properties are revisited in a more formal setting, giving greater attention to precise statements of theorems and establishing these theorems by means of formal reasoning.
2. The theorem about the midline of a triangle can be connected to concepts of similarity. The proof can be based on the similarity property that corresponding sides of similar triangles are proportional.
3. Proof is sometimes formatted with a two-column approach, with one column headed "statements" and the other column headed "reasons." Although this may sometimes be a valuable way for a student to express reasoning systematically, other approaches can and should be used. Not all proofs are best communicated by the two-column approach. Students may also, for example, write sentences (paragraph proof), or use boxes (flow proof), or they may employ other formats, or combine formats, for communicating proof. Essential for both career and postsecondary education is students' ability to communicate reasoning by means of a constructed argument in which one step leads logically to the next, complete with justification.
4. Proofs of theorems can be made from a synthetic point of view by using a deductive approach as well as from an analytical point of view by using an algebraic approach. Both approaches can be useful, and in some cases one approach may provide a more understandable argument than the other.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-CO.D. Congruence: Make geometric constructions.

Evidence Outcomes

Students Can:

a. Make formal geometric constructions with a variety of tools and methods. (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)

   Copying a segment, copying an angle, bisecting a segment, bisecting an angle, constructing perpendicular lines, including the perpendicular bisector of a line segment, and constructing a line parallel to a given line through a point not on the line. (CCSS: HS.G-CO.D.12)

b. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (CCSS: HS.G-CO.D.13)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP5: Use appropriate tools strategically. Students use a variety of tools to make geometric constructions.
2. MP6: Attend to precision. Students precisely use construction tools to communicate their results using mathematical language and logic.

Inquiry Questions:

1. How is a geometric construction like a proof?

Coherence Connections:

1. Drawing geometric shapes with rulers, protractors and technology is developed in Grade 7. In high school, students perform formal geometric constructions using a variety of tools. Students will utilize proofs to justify validity of their constructions.
2. Students can use geometric constructions to precisely locate the line line of reflection between and image and its pre-image and to accurately draw a figure under a translation or rotation and justify its validity.
3. Communicating reasoning in geometry involves formal proof/precise language, informal explanation/construction, and strategic experimentation to verify/refute claims; and students must have experiences with all three.
Prepared Graduates
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School
HS.G-SRT.A. Similarity, Right Triangles, and Trigonometry: Understand similarity in terms of similarity transformations.

Evidence Outcomes
Students Can:

a. Verify experimentally the properties of dilations given by a center and a scale factor. (CCSS: HS.G-SRT.A.1)
   a. Show that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. (CCSS: HS.G-SRT.A.1.a)
   b. Show that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. (CCSS: HS.G-SRT.A.1.b)

b. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar, explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (CCSS: HS.G-SRT.A.2)

c. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (CCSS: HS.G-SRT.A.3)

Academic Context and Connections
Colorado Essential Skills and Mathematical Practices:
1. MP2: Reason abstractly and quantitatively. When reasoning about similarity, students use auxiliary lines not part of the original figure.

2. MP5: Use appropriate tools strategically. Students use geometric tools and technology (including dynamic geometric software) in exploring and verifying the properties of dilations and in understanding the properties of similar figures.

3. MP8: Look for and express regularity in repeated reasoning. Students recognize and use repeated reasoning in exploring and verifying the properties of dilations and similarity, and in establishing the AA criterion for similar triangles.

Inquiry Questions:
1. How can we use the concepts of similarity to measure real-world objects that are difficult or impossible to measure directly?

2. How are similarity and congruence related to one another?

Coherence Connections:
1. Dilations and similarity, including the AA criterion, are investigated in Grade 8, and these experiences should be built upon in high school with greater attention to precise definitions, careful statements and proofs of theorems and formal reasoning.

2. Congruence is a special case of similarity. Similarity with a scale factor equal to 1 becomes congruency.

3. Figures, including graphs of functions, are defined by their attributes, and showing that two figures are similar involves finding a scaling transformation (dilation or composition of dilation with a rigid motion) or sequence of scaling transformations that maps one figure to the other.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-SRT.B. Similarity, Right Triangles, and Trigonometry: Prove theorems involving similarity.

Evidence Outcomes

Students Can:

a. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (CCSS. HS.G-SRT.B.4)

b. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (CCSS: HS.G-SRT.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP3. Construct viable arguments and critique the reasoning of others. Students justify their reasoning using logical, cohesive steps when proving theorems and solving problems in geometry.

2. MP6. Attend to precision. Students use precise geometric and other mathematical terms and symbols as they construct proofs and solve problems in geometry.

Inquiry Questions:

1. How does the Pythagorean Theorem support the case for triangle similarity?

Coherence Connections:

1. The Pythagorean Theorem and its converse are proven in Grade 8. In high school, another proof, based on similar triangles, is presented.

2. The alternate interior angle theorem and its converse, as well as properties of parallelograms, are established informally in Grade 8 and proved formally in high school.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justificiation and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-SRT.C: Similarity, Right Triangles, and Trigonometry: Define trigonometric ratios and solve problems involving right triangles.

Evidence Outcomes

Students Can:

a. Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (CCSS: HS.G-SRT.C.6)

b. Explain and use the relationship between the sine and cosine of complementary angles. (CCSS: HS.G-SRT.C.7)

c. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (CCSS: HS.G-SRT.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP2: Reason abstractly and quantitatively. Students use abstract thinking to reason from the properties of similar triangles to the definitions of the trigonometric ratios for acute angles, as they recognize that the proportionality of side measures creates a single ratio based on the angle measure, regardless of the size of the right triangle.

2. MP4: Model with mathematics. Students apply the trigonometric ratios and the Pythagorean Theorem to model and solve real-world problems.

3. MP7: Look for and make use of structure. Students make use of structure as they connect triangle similarity and the trigonometric ratios for acute angles.

Inquiry Questions:

1. How are the trigonometric ratios for acute angles connected to the properties of similar triangles?

2. What visual representation(s) explains why the sine of an acute angle is equivalent to the cosine of its complement?

Coherence Connections:

1. Trigonometry is not introduced until high school. Right triangle trigonometry (a geometry topic) has implications when studying algebra and functions. Trigonometric ratios are functions of the size of an angle, the Pythagorean Theorem can be used to show that \( \sin^2(\theta) + \cos^2(\theta) = 1 \).

2. By similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
 Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

 Grade Level Expectation: High School

HS.G-SRT.D. Similarity, Right Triangles, and Trigonometry: Apply trigonometry to general triangles.

 Evidence Outcomes

 Students Can:

a. (+) Derive the formula \( A = \frac{1}{2}ab\sin(C) \) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (CCSS: HS.G-SRT.D.9)

b. (+) Prove the Laws of Sines and Cosines and use them to solve problems. (CCSS: HS.G-SRT.D.10)

c. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (CCSS: HS.G-SRT.D.11)

 Academic Context and Connections

 Coherence Connections:

1. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
3 Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

3 Grade Level Expectation: High School

HS.G.C.A. Circles: Understand and apply theorems about circles.

3 Evidence Outcomes

Students Can:

a. Prove that all circles are similar. (CCSS: HS.G.C.A.1)

b. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (CCSS: HS.G.C.A.2)

c. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (CCSS: HS.G.C.A.3)

d. (+) Construct a tangent line from a point outside a given circle to the circle. (CCSS: HS.G.C.A.4)

3 Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP3: Construct viable arguments and critique the reasoning of others. Students justify their reasoning and use logical, cohesive steps when proving theorems and solving problems in geometry.

2. MP5: Use appropriate tools strategically. Students will use geometric tools and technology (including dynamic geometry software) in exploring relationships in circles and in circle-related constructions.

3. MP6: Attend to precision. Students use precise geometric terms, definitions, and symbols when working with circles and their related concepts.

Inquiry Questions:

1. Draw or find examples of several different circles. In what ways are they related geometrically? How can you describe these relationships in terms of transformations?
6 Coherence Connections:

1. Circles can be related to similarity because all circles are similar. Relate circle properties to geometric constructions.

2. Instruction should focus on constructing objects to support an understanding of measurement and should support students in discovering formulas for the area and circumference of a circle. These activities can provide opportunities to discuss how constructing approximating polygons and right triangles are useful for measurement.

3. Students may explain the formulas for the area and circumference of a circle by using regular polygons with increasing numbers of sides inscribed in the circle. The essential idea of the explanation is the use of inscribed regular polygons and the right triangles within to approximate the circles. As the number of sides increases, the polygons approximate the circle, and so the area and perimeter of the polygons should approximate the circle’s area and circumference, respectively. Combining this idea with facts about triangles, the process of finding the formulas for area and circumference of a circle can foster understanding of the formulas. The approximating shapes can be thought of as a union of line segments, triangles, or rectangles, providing a useful strategy for approximations in life as well as a conceptual connection to calculus.

4. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-C.B. Circles: Find arc lengths and areas of sectors of circles.

Evidence Outcomes

Students Can:

a. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality. Derive the formula for the area of a sector. (CCSS: HS.G.C.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP3: Construct viable arguments and critique the reasoning of others. Students use verbal and written arguments using similarity to justify arc lengths and radian measures.

2. MP6: Attend to precision. Students attend to precise mathematical definitions, relationships, and symbols to describe and solve problems involving arc lengths and areas of sectors of circles.

3. MP7: Look for and make use of structure. Students use collective understandings of the area of a circle and the meaning of a central angle to synthesize the formula for the area of a sector.

Inquiry Questions:

1. In what ways is it more accurate and more convenient to use radian measure for a central angle rather than degree measure?

Coherence Connections:

1. Formulas for area and circumference of a circle, as well as proportional reasoning, were developed in Grade 7. Here the formulas are generalized to fractional parts of a circle and will prepare students for the study of trigonometry.

2. A similarity transformation with scale factor $k$ transforms any length by a scale factor of $k$, including the arc length. So, just as the radius gets multiplied by $k$, so does the arc length. This means that the ratio between the arc length and the radius stays equivalent no matter what the radius; in other words, the arc length is proportional to the radius. Since arc length is equal to the constant times the radius, setting the radius equal to 1 tells us that the constant is the arc length for a circle of radius 1, which is the radian measure of the angle.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-GPE.A. Expressing Geometric Properties with Equations: Translate between the geometric description and the equation for a conic section.

Evidence Outcomes

Students Can:

a. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (CCSS: HS.G-GPE.A.1)

b. Derive the equation of a parabola given a focus and directrix. (CCSS: HS.G-GPE.A.2)

c. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. (CCSS: HS.G-GPE.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP2: Reason abstractly and quantitatively. Students use abstract and quantitative reasoning to apply the Pythagorean Theorem to the equations of conic sections, particularly circles and parabolas.

2. MP7: Look for and make use of structure. Students see the underlying structure of the equations for conic sections and their connection to the Pythagorean Theorem.

Inquiry Questions:

1. How does the Pythagorean Theorem connect to the general equation for a circle with center \((a, b)\) and radius \(r\)? How can this be illustrated with a diagram?

2. How does the Pythagorean Theorem connect to the equation for a parabola? How can this be illustrated with a diagram?
1. In Grade 8, the Pythagorean theorem was applied to find the length of an unknown side of a right triangle. This understanding later extends to find the distance between two particular points. In high school, the application is generalized to obtain formulas related to conic sections.

2. Quadratic functions and completing the square are studied in the domain of interpreting functions. The methods are applied here to transform a quadratic equation representing a conic section into standard form.

3. The power of analytic geometry to reduce geometric relationships to algebraic ones creates a danger in teaching them because students can lose sight of the geometric meaning of the formulas. Thus the equation for a circle with center \((a, b)\) and radius \(r\),
\[
(x - a)^2 + (y - b)^2 = r^2
\]
can become disconnected from the Pythagorean Theorem, even though it is a direct statement of that theorem for any right triangle with radius of the circle as its hypotenuse. As another example, students sometimes get the impression that the word “parabola” is the name for the graph of a quadratic function, whereas a parabola is a geometric object with a geometric definition. The interplay between geometry and algebra is evident in the derivation of the equation from this definition.

4. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.
2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”
3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-GPE.B. Expressing Geometric Properties with Equations: Use coordinates to prove simple geometric theorems algebraically.

Evidence Outcomes

Students Can:

a. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 3)$. (CCSS: HS.G-GPE.B.4)

b. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (CCSS: HS.G-GPE.B.5)

c. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (CCSS: HS.G-GPE.B.6)

d. Use coordinates and the distance formula to compute perimeters of polygons and areas of triangles and rectangles. (CCSS: HS.G-GPE.B.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP2: Reason abstractly and quantitatively. Students use coordinate proof to connect geometric theorems to the coordinate plane.
2. MP3: Construct viable arguments and critique the reasoning of others. Students use verbal and written justifications to prove theorems involving distance and ratio.
3. MP7: Look for and make use of structure. Students apply understandings of distance and perpendicularity to polygons.

Inquiry Questions:

1. What mathematical concepts and tools become available when coordinates are applied to geometric figures?

Coherence Connections:

1. From their work in Grade 8, students are familiar with the idea that two points $(x_1, y_1)$ and $(x_2, y_2)$ in the coordinate plane determine a right triangle whose hypotenuse is the line segment between the two points and whose legs are parallel to the axes. Two important geometric facts about these triangles lead to foundational formulas in analytic geometry. First, for all the pairs of distinct points on a given line, the corresponding triangles are similar. This can be shown using the AA criterion for similarity. Because the horizontal (or vertical) grid lines are all parallel to each other, and the line is transversal to those parallel lines, the ratio of the vertical side to the horizontal side does not depend on which two points are chosen, and so it is a characteristic of the line itself, called its slope, $m$. The algebraic manifestation of this is the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. The relationship between the slopes of parallel and perpendicular lines is a nice example of the interplay between geometry and algebra. Second, the Pythagorean Theorem applies: the length of the hypotenuse is the distance between the two points, and the lengths of the legs can be calculated as differences between the coordinates. The algebraic manifestation of the Pythagorean Theorem is the formula for the distance, $d$, between the two points: Students can use the distance formula to prove simple facts about configurations of points in the plane.

2. The algebraic manifestation of the Pythagorean Theorem is the formula for the distance, $d$, between the two points: Students can use the distance formula to prove simple facts about configurations of points in the plane.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-GMD.A. Geometric Measurement and Dimension: Explain volume formulas and use them to solve problems.

Evidence Outcomes

Students Can:

a. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments. (CCSS: HS.G-GMD.A.1)

b. (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. (CCSS: HS.G-GMD.A.2)

c. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (CCSS: HS.G-GMD.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP3: Construct viable arguments and critique the reasoning of others. Students justify their reasoning and understanding of the formulas for circumference of a circle, area of a circle, and volumes of cylinders, pyramids, and cones.

2. MP4: Model with mathematics. Students apply volume formulas for cylinders, pyramids, cones, and spheres to real-world contexts to solve problems.

3. MP6: Use appropriate tools strategically. Students use technologies as appropriate to estimate and compute areas and volumes.

Inquiry Questions:

1. How could you use other geometry relationships to explain why the volume of a cylinder is \( V = \pi r^2 h \)?

2. How could you use algebra to prove that a right cylinder and a corresponding oblique cylinder have the same volume?
Coherence Connections:

1. In Grade 6, students were required to know and use the formulas for volumes of cylinders, cones, and spheres. In Grade 7, students informally derived the formula for the area of a circle from the circumference. In this cluster those formulas are derived by a combination of concrete demonstrations and formal reasoning.

2. Lengths, areas, and volumes of an object can be computed by determining how the object might be constructed by using products, intersections, unions, and complements of other objects and their volumes. For example, the volume of a cylinder is the product of the area of a cross section and its height, and the area of a ring-shaped figure is the complement of the area of the inner circle.

3. When an object is the image of a known object under a scaling transformation, the length, area, or volume of the image can be computed by using proportional relationships.

4. The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).
1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, “Does this make sense?”

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-GMD.B. Geometric Measurement and Dimension: Visualize relationships between two-dimensional and three-dimensional objects.

Evidence Outcomes

Students Can:

a. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (CCSS: HS.G-GMD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP2: Reason abstractly and quantitatively. Students reason abstractly to visualize, describe, and justify their understanding of cross-sections of three-dimensional objects and three-dimensional objects generated by rotations of two-dimensional objects.

Inquiry Questions:

1. Will the shape of a cross-section of a three-dimensional object be the same for all planes that intersect the object? Why or why not?

Coherence Connections:

1. In Grades 6–8, students apply geometric measurement to real-world and mathematical problems, making use of properties of figures as they dissect and rearrange them in order to calculate or estimate lengths, areas, and volumes. Use of geometric measurement continues in high school. Students examine it more closely, giving informal arguments to explain formulas used in earlier grades. These arguments draw on the abilities they have developed in earlier grades: dissecting and rearranging two- and three-dimensional figures, and visualizing cross-sections of three-dimensional figures.
Prepared Graduates

1. Make meaning of a problem and plan a solution pathway, while monitoring and evaluating their progress and changing course if necessary.

2. Communicate precisely to others, using clear definitions to justify their reasoning. They are able to use reasoning skills that rely on estimation, precision and comparison while continually asking themselves, "Does this make sense?"

3. Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

4. Communicate effective logical arguments using justification and proof. Argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

5. Use critical thinking to recognize problematic aspects of situations, create models, and present and defend solutions.

Grade Level Expectation: High School

HS.G-MG.A. Modeling with Geometry: Apply geometric concepts in modeling situations.

Evidence Outcomes

Students Can:

a. Use geometric shapes, their measures, and their properties to describe objects. (e.g., modeling a tree trunk or a human torso as a cylinder).* (CCSS: HS.G-MG.A.1)

b. Apply concepts of density based on area and volume in modeling situations. e.g., persons per square mile, BTUs per cubic foot.* (CCSS: HS.G-MG.A.2)

c. Apply geometric methods to solve design problems. (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).* (CCSS: HS.G-MG.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. MP4: Model with mathematics. Students apply geometric properties and relationships to make sense of and solve real-world problems.

2. MP5: Use appropriate tools strategically. Students model and solve problems using technology and dynamic geometry software.

Inquiry Questions:

1. Why is it useful to describe everyday objects as geometric shapes?

Coherence Connections:

1. Any mathematical object that represents a situation from outside mathematics and can be used to solve a problem about that situation is a mathematical model. Modeling often involves making simplifying assumptions that ignore some features of the situation being modeled. If a population grows by approximately the same percentage each year, sometimes a bit above, sometimes a bit below, students might choose to fit an exponential function to the data and use it to make predictions. In geometry, in order to study how the illuminated percentage of the moon's surface varies during a month, students might represent the moon as a rotating sphere, half black and half white.
2. Geometric modeling can be used in Fermi problems, problems which ask for rough estimates of quantities. Such problems often involved estimates of densities. Of all the subjects students learn in geometry, trigonometry may have the greatest application in college and career. Students in high school should see authentic applications of trigonometry to many different contexts.

3. Applying abstract geometric concepts involving congruence, similarity, measurement, trigonometry, and other related areas to solving problems situated in real-world contexts provides a means of building understanding about concepts and experiencing the usefulness of geometry.

4. The mathematical modeling cycle using geometric concepts and methods introduces geometric techniques, tools, and points of view that are valuable to problem solving.