Instructional Unit Title: Logarithmic Log Jams

The teacher may introduce a scenario to students where a person put $\$ 500$ into an investment account that pays 5\% interest each year for 50 years so tha the students can determine how much they could save over the course of their lives by merely investing a small percentage of their income each year.

The teacher may have students brainstorm why people use credit cards so that the students can understand that there are times when using a credit card can be a benefit. The teacher may then provide students with examples of credit card interest rates and have the student calculate how much their credit card debt would grow over time if they did not pay it off so that students have an idea of the cost of using borrowed money. As a conclusion, the teacher may have the students write a letter to a fictional 16-year old son or daughter explaining how credit cards work and explaining why they think their fictional son or daughter should or should not get a credit card when they turn 18 so that the students can evaluate the costs and benefits of credit.

The teacher may give the students situations of credit card debt where the students have to solve for time (e.g., If I have $\$ 500$ in credit card debt that is compounded annually at $15 \%$ interest, how long until I owe the credit card company $\$ 2000$ ?) so
that students are motivated to find a new mathematical tool for solving an equation where the unknown variable is in the exponent. The teacher may then introduce the idea of a logarithm as a method for rewriting exponential equations so that students can solve the equations using the properties of logarithms or technology.

The teacher provide students with examples of annual percentage rates (APR) that are compounded daily, monthly, or yearly from savings accounts, credit cards, or other financial situations so that students can explore how to generate an exponential equation for daily or monthly compound interest. Students can then use the given APR to calculate the annual
percentage yield (APY), which can be slightly different
depending on how often the interest rate is compounded.

The teacher may provide the students with a variety of annual interest rates for a savings account so that students can determine how long until an investment of $\$ 1000$ doubles in value and discover the doubling formula for investments. Students can finish this experience by explaining how different interest rates in their lives such as credit cards, savings accounts, CDs, car loans, or a mortgage will affect their personal financial decisions. Students can create a comparison of typical interest rates for all of the above so that they have an understanding of how interest rates typically vary among different types of debt instruments or investments.

The teacher may provide students with scenarios where interest on a savings account is compounded more and more often (yearly, monthly, daily, hourly, every minute, every second, etc.) so that students can explore the limit of compounding at a particular interest rate.

The teacher may provide a number of scenarios (from personal finance or other areas of science and math) so that the students can solve equations using logarithms, including the natural logarithms, in context and express their answers in the context of the scenario.

PERFORMANCE ASSESSMENT: You would like to start a small business in your town selling handmade fleece clothing. In order to start the business, you will need to take out a loan of $\$ 200,000$ (at $4 \%$ interest) to purchase equipment and supplies. However, the bank would like see that you will be successful before giving you the loan. Specifically, the bank would like to see a projection of your expected profit. Your current estimate is that you will make $\$ 5000$ in revenue each month with $\$ 3000$ in expenses (not including your loan payment). You expect that your revenue could grow between 3\% and $10 \%$ each year over the next decade while your expenses could grow between $1 \%$ and $5 \%$. You need to model your profit for various scenarios of growth in revenue and expenses. In addition, you need to propose to the bank how long you would like the term of the loan to be (anywhere from 5 to 30 years) and factor how much this will cost into your profit projection. Remember, what you have left for profit is what you will live on. If you are only making \$100 in profit each month, you must live on $\$ 100$ a month. Your business will need a checking account thorough which all your money will flow. You will also need a credit card in order to cover expenses that come up suddenly. Please include these in your business plan as well. Your job is to put together a business plan with all of the above information so that the bank can approve your loan and you can get your business up and running.

 Colorado Teacher-Authored Instructional Unit Sample Storyboard

