

# Unit Title: Fraction Frenzy

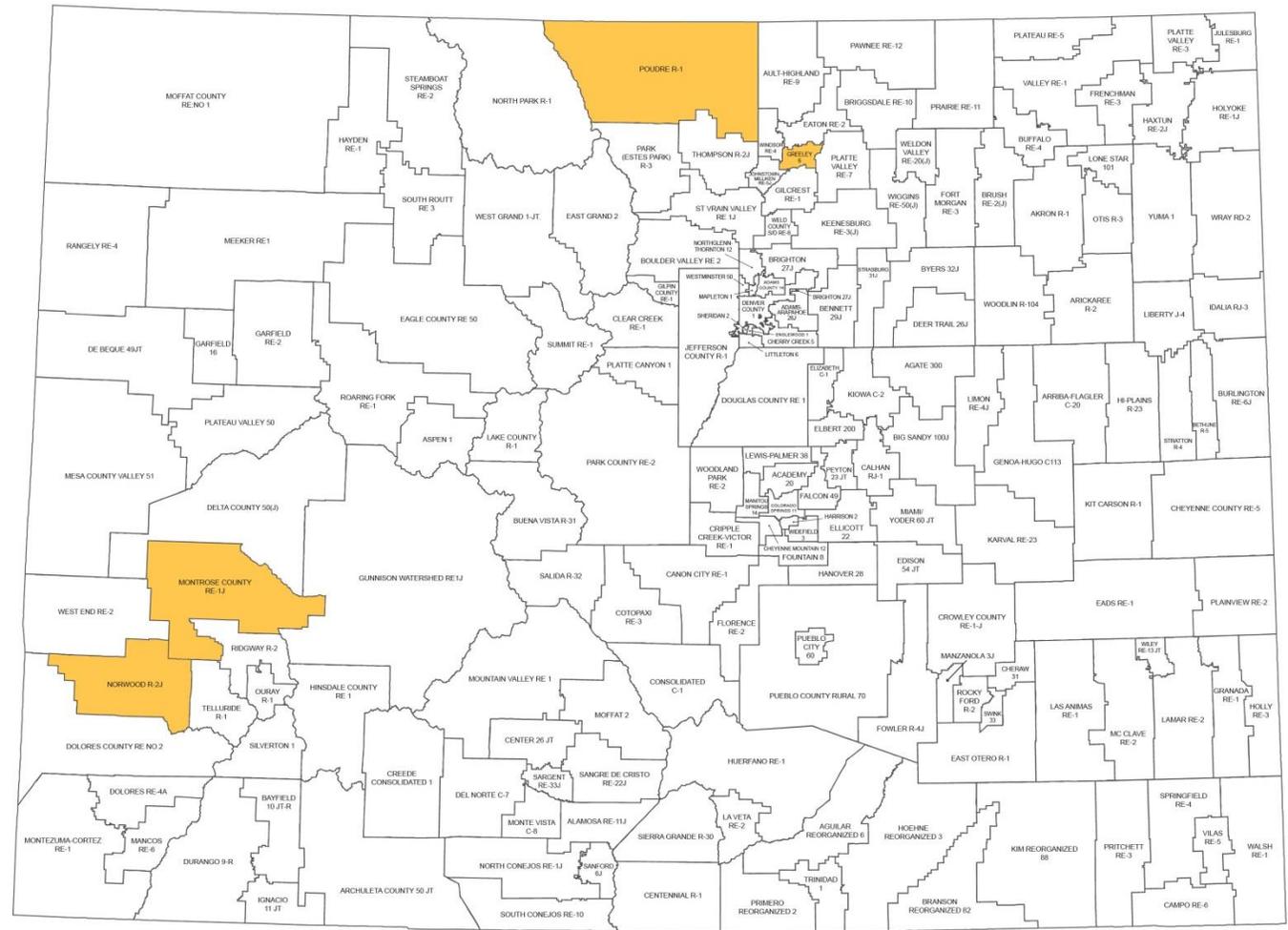
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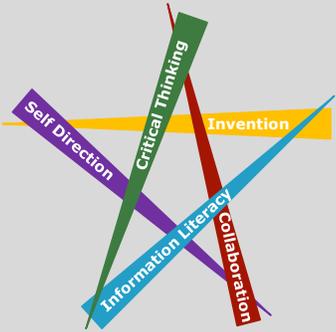
## BASED ON A CURRICULUM OVERVIEW SAMPLE AUTHORED BY

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*This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.*

**Curriculum Development Course at a Glance  
Planning for 4<sup>th</sup> Grade Mathematics**

Content Area	Mathematics	Grade Level	4 <sup>th</sup> Grade
Course Name/Course Code			
Standard	Grade Level Expectations (GLE)	GLE Code	
1. Number Sense, Properties, and Operations	1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms	MA10-GR.4-S.1-GLE.1	
	2. Different models and representations can be used to compare fractional parts	MA10-GR.4-S.1-GLE.2	
	3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency	MA10-GR.4-S.1-GLE.3	
2. Patterns, Functions, and Algebraic Structures	1. Number patterns and relationships can be represented by symbols	MA10-GR.4-S.2-GLE.1	
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to represent data	MA10-GR.4-S.3-GLE.1	
4. Shape, Dimension, and Geometric Relationships	1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time	MA10-GR.4-S.4-GLE.1	
	2. Geometric figures in the plane and in space are described and analyzed by their attributes	MA10-GR.4-S.4-GLE.2	
<p align="center"><b>Colorado 21<sup>st</sup> Century Skills</b></p>  <p><b>Critical Thinking and Reasoning:</b> <i>Thinking Deeply, Thinking Differently</i></p> <p><b>Information Literacy:</b> <i>Untangling the Web</i></p> <p><b>Collaboration:</b> <i>Working Together, Learning Together</i></p> <p><b>Self-Direction:</b> <i>Own Your Learning</i></p> <p><b>Invention:</b> <i>Creating Solutions</i></p>		<p><b>Mathematical Practices:</b></p> <ol style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with mathematics.</li> <li>5. Use appropriate tools strategically.</li> <li>6. Attend to precision.</li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>	
Unit Titles	Length of Unit/Contact Hours	Unit Number/Sequence	
Fraction Frenzy	5 weeks	4	

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Unit Title</b>	Fraction Frenzy		<b>Length of Unit</b>	5 weeks
<b>Focusing Lens(es)</b>	Representation	<b>Standards and Grade Level Expectations Addressed in this Unit</b>	MA10-GR.4-S.1-GLE.2 MA10-GR.4-S.3-GLE.1	
<b>Inquiry Questions (Engaging-Debatable):</b>	<ul style="list-style-type: none"> <li>• What would the world be like without fractions? (MA10-GR.4-S.1-GLE.2-IQ.4)</li> <li>• Why are fractions useful? (MA10-GR.4-S.1-GLE.2-IQ.3)</li> </ul>			
<b>Unit Strands</b>	Number and Operations – Fractions, Measurement and Data			
<b>Concepts</b>	Increasing, decreasing, numerators, denominators, fractions, factor, equivalence, comparison, referent unit, whole, benchmark fractions, estimation, relative size, common denominator/numerator, decompose, sum, addition, subtraction, properties of operations, relationship, mixed number, equivalent fractions, decimals, word problems, joining, separating, unit fractions, multiple,			

<b>Generalizations</b> My students will <b>Understand</b> that...	<b>Guiding Questions</b>	
	<b>Factual</b>	<b>Conceptual</b>
Equivalent fractions describe the same part of a whole by using different fractional parts (MA10-GR.4-S.1-GLE.2-EO.a.i)	How can you show that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions?	Why (or when) are equivalent fractions necessary or helpful?
Increasing or decreasing both the numerators and denominators of a fraction by the same factor creates equivalent fractions (MA10-GR.4-S.1-GLE.2-EO.a.i)	How can you justify two fractions are equivalent by using visual fraction models? What happens when you multiply both the numerator and denominator by the same number?	Why are there multiple names for the same fraction? How can different fractions represent the same quantity?(MA10-GR.4-S.1-GLE.2-IQ.1) Why do you need to know equivalent fractions?
Decisions about the size of a fraction relative to another fraction often involves the comparison of the fractions' denominators (if their numerators are equal), or numerators (if their denominators are equal) or the creation of common denominators or numerators for the fractions (MA10-GR.4-S.1-GLE.2-EO.a.iii)	What are examples of benchmark fractions and how are they useful in comparing the size of fractions? When do you need to find a common denominator or common numerator?	Why is it possible to compare fractions with either a common denominator or common numerator?
As with whole numbers, mathematicians compose fractions by joining/combining fractions (with the same denominator) as sums and decompose/separate fractions (with the same denominator) as differences in multiple ways (MA10-GR.4-S.1-GLE.2-EO.b.i.1)	How can you record decompositions of fractions with an equation? How can you justify decompositions of fractions using visual fraction models?	Why when you decompose fractions do you only break apart the numerator and not the denominator? How is decomposing fractions similar and different from decomposing whole numbers?

**Colorado Teacher-Authored Sample Instructional Unit**

<p>To add and subtract mixed numbers with like denominators requires the use of properties of operations (MA10-GR.4-S.1-GLE.2-EO.b.i.2)</p>	<p>How can you write the number 1 as a fraction? Any whole number?          How can you rewrite a mixed number as an equivalent fraction?          In a mixed number, what operation is happening between the whole number and the fraction?</p>	<p>Why might you need to rewrite a mixed number as an equivalent fraction in order to perform addition or subtraction?</p>
<p>Place value (and its understanding) provides an efficient means to express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100 (MA10-GR.4-S.1-GLE.2-EO.a)</p>	<p>How can you add two fractions with a denominator of 10 and 100 respectively (e.g. <math>3/10 + 5/100</math>)?</p>	<p>Why is it easy to change a fraction with a denominator of 10 to 100?</p>
<p>Word problems and contexts involving joining and separating parts of the same (size) whole require the addition and subtraction of fractions (MA10-GR.4-S.1-GLE.2-EO.b.i.3)</p>	<p>How does solving the problem “How much pizza does Thomas have left if gives away half of his pizza at lunch and then eats half of what he has left?” require the use of fractions?</p>	<p>How word problems involving the addition and subtraction of fractions similar and different from those of whole numbers?</p>
<p>The multiplication of a fraction (<math>1/b</math>) by a whole number (<math>a</math>) creates a fraction that is a multiple of the original fraction (MA10-GR.4-S.1-GLE.2-EO.b.i, b.ii.1, 2)</p>	<p>How can the repeated addition of <math>1/4</math>, 3 times, show that <math>3/4</math> is multiple of <math>1/4</math>?          How can you multiply a whole number by a fraction?          What different types of word problems represent the product of a whole number times a fraction? (MA10-GR.4-S.1-GLE.2-EO.b.ii.3)</p>	<p>When multiplying a whole number by a unit fraction is the result larger or smaller than the original whole number?          Why is every fraction both a multiplication and division problem at the same time?</p>

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Key Knowledge and Skills: My students will...</b>	<i>What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.</i>
<ul style="list-style-type: none"> <li>• Explain why a fraction <math>a/b</math> is equivalent to a fraction <math>(n \times a) / (n \times b)</math> by using visual fractions models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size (MA10-GR.4-S.1-GLE.2-EO.a.i)</li> <li>• Generate equivalent fractions (MA10-GR.4-S.1-GLE.2-EO.a.ii)</li> <li>• Compare two fractions with different numerators and different denominators by creating common denominators or numerators or comparing to a benchmark fraction and record the results of the comparisons with symbols <math>&gt;</math>, <math>=</math>, <math>&lt;</math> and justify the conclusions (MA10-GR.4-S.1-GLE.2-EO.a.iii)</li> <li>• Recognize that comparisons are valid only when the two fractions refer to the same whole (MA10-GR.4-S.1-GLE.2-EO.a.iii)</li> <li>• Understand addition and subtraction of fractions as joining and separating parts referring to the same whole (MA10-GR.4-S.1-GLE.2-EO.b.i)</li> <li>• Decompose a fraction into a sum of fractions with the same denominator in more than one way, record each decomposition by an equation and justify the decomposition (MA10-GR.4-S.1-GLE.2-EO.b.i.1)</li> <li>• Add and subtract mixed numbers with like denominators (MA10-GR.4-S.1-GLE.2-EO.b.i.2)</li> <li>• Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators (MA10-GR.4-S.1-GLE.2-EO.b.i.3)</li> <li>• Understand a fraction <math>a/b</math> as a multiple of <math>1/b</math> and a multiple of <math>a/b</math> as a multiple of <math>1/b</math> (MA10-GR.4-S.1-GLE.2-EO.b.ii.1)</li> <li>• Multiply a fraction by a whole number and solve word problems involving multiplication of a fraction by a whole number (MA10-GR.4-S.1-GLE.2-EO.b.ii)</li> <li>• Express a fraction with denominator of 10 as an equivalent fraction with a denominator 100, and use this technique to add two fractions with respective denominators 10 and 100 (MA10-GR.4-S.1-GLE.1-EO.b.i)</li> <li>• Make a line plot to display a data set of measurements in fractions of a unit (<math>1/2</math>, <math>1/4</math>, <math>1/8</math>) and solve problems involving addition and subtraction of fractions by using information presented in line plots (MA10-GR.4-S.3-GLE.1-EO.a, b)</li> </ul>	

<b>Critical Language:</b> includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline. EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: <i>“Mark Twain exposes the hypocrisy of slavery through the use of satire.”</i>	
<b>A student in _____ can demonstrate the ability to apply and comprehend critical language through the following statement(s):</b>	<i>I can compare two fractions by having first determining if there are smaller or larger than a benchmark fraction like <math>\frac{1}{2}</math> or finding a common numerator or denominator; if two fractions have the same numerator then the fraction with the smaller denominator is larger and vice versa if they have same denominator.</i>
<b>Academic Vocabulary:</b>	Apply, explain, generate, compare, express, understand, increasing, decreasing, estimation
<b>Technical Vocabulary:</b>	Solve, equivalent, mixed numbers, numerator, denominator, unit fraction, benchmark fraction, whole, part, multiple, equivalent fractions, common numerator, common denominator, decompose, sum, addition, subtraction, joining, separating

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Unit Description:</b>	In this unit students explore fractions as quantities. Students build on their prior knowledge with whole numbers to compose and decompose fractions in multiple ways; the early development of addition and subtraction of fractions. Students also explore the concept of finding a fraction amount of a whole number. In addition, students begin to compare fractions by recognizing the need to examine both the numerator and denominator and the importance of maintaining a consistent unit or whole. Throughout the unit students are focusing on the quantitative, verbal and symbolic parts of a number to provide a strong foundation for the formalization of fraction operations in fifth and sixth grade. Finally, students explore the concept of equivalent fractions and the special cases of fractions with denominators of 10 or 100 to prepare them for their work with decimals.
<b>Unit Generalizations</b>	
<b>Key Generalization:</b>	Word problems and contexts involving joining and separating parts of the same (size) whole require the addition and subtraction of fractions
<b>Supporting Generalizations:</b>	Equivalent fractions describe the same part of a whole by using different fractional parts
	Increasing or decreasing both the numerators and denominators of a fraction by the same factor creates equivalent fractions
	Decisions about the size of a fraction relative to another fraction often involves the comparison of the fractions' denominators (if their numerators are equal), or numerators (if their denominators are equal) or the creation of common denominators or numerators for the fractions
	As with whole numbers, mathematicians compose fractions by joining/combining fractions (with the same denominator) as sums and decompose/separate fractions (with the same denominator) as differences in multiple ways
	To add and subtract mixed numbers with like denominators requires the use of properties of operations
	Place value (and its understanding) provides an efficient means to express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100
	The multiplication of a fraction (1/b) by a whole number (a) creates a fraction that is a multiple of the original fraction

<b>Performance Assessment:</b> <i>The capstone/summative assessment for this unit.</i>	
<b>Claims:</b> (Key generalization(s) to be mastered and demonstrated through the capstone assessment.)	Word problems and contexts involving joining and separating parts of the same (size) whole require the addition and subtraction of fractions
<b>Stimulus Material:</b> (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)	As an artist you have been hired to design a beaded safety pin to sell on the internet ( <a href="http://www.makingfriends.com/safety_pin_projects.htm">http://www.makingfriends.com/safety_pin_projects.htm</a> ). Your safety pin design should include 36 beads and six colors and no two colors can have the same quantity. You will create an advertisement for your design to include on the website, that will include a description of the process you used to create the pin (i.e., color and pattern choices) as well as the fractional parts that each of the colors represent in the whole safety pin. Good luck!
<b>Product/Evidence:</b> (Expected product from students)	Students will create a 6 by 6 design of different colored beads by drawing it on graph paper. In your advertisement you must include: <ul style="list-style-type: none"> <li>• An addition equation showing the fractional amount for each color (e.g., <math>\frac{3}{36}</math> yellow + <math>\frac{20}{36}</math> red + <math>\frac{6}{36}</math> green + <math>\frac{2}{36}</math> blue + <math>\frac{1}{36}</math> purple + <math>\frac{4}{36}</math> orange = <math>\frac{36}{36}</math>)</li> <li>• A comparison between at least two colors (e.g., I have more orange than blue in my design, <math>\frac{4}{36} &gt; \frac{1}{36}</math>)</li> </ul>

**Colorado Teacher-Authored Sample Instructional Unit**

	<ul style="list-style-type: none"> <li>• A subtraction equation showing what fraction of your design is not a particular color (e.g., My design is mainly red, <math>36/36 - 20/36 = 13/36</math>)</li> <li>• A color written as a fraction that is a denominator different than 36 (e.g., <math>1/4</math> of my design is green)</li> </ul>
<b>Differentiation:</b> (Multiple modes for student expression)	<p>Students can create an advertisement for design created by the teacher.</p> <p>Students can complete a template for the advertisement created by the teacher.</p> <p>Students can create several different designs and compare them in the advertisement.</p>

Texts for independent reading or for class read aloud to support the content	
Informational/Non-Fiction	Fiction
N/A	<p><i>Apple Fractions</i> by Jerry Pallotta (Lexile level 490)</p> <p><i>The Hershey’s Milk Chocolate Bar Fractions Book</i> by Jerry Pallotta (Lexile level 710)</p> <p><i>If You Were a Fraction</i> by Trisha Speen Shaskan (Lexile level 710)</p> <p><i>Working with Fractions</i> by David Adler (Lexile level 690)</p> <p><i>Whole-y Cow!: Fractions are Fun</i> by Taryn Sounders (Lexile level 600)</p>

Ongoing Discipline-Specific Learning Experiences				
1.	Description:	Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others [Mathematical Practice 3]	Teacher Resources:	<a href="http://schools.nyc.gov/NR/rdonlyres/D0A70F2D-1133-418C-B68F-95E6D714F357/0/NYCDOEG5MathStuffedwithPizza_Final.pdf">http://schools.nyc.gov/NR/rdonlyres/D0A70F2D-1133-418C-B68F-95E6D714F357/0/NYCDOEG5MathStuffedwithPizza_Final.pdf</a> (lesson plans contains exemplars that could be replicated for students to critique the reasoning of others)
			Student Resources:	N/A
	Skills:	Construct and communicate a complete and concise response, justify a conclusion using correct vocabulary, interpret and critique the validity of other’s conclusions and reasoning, and identify errors and present correct solutions	Assessment:	Students analyze and defend their solutions for each major learning experience. Careful attention should be paid to precise use of vocabulary and symbols. Periodically throughout the unit, students could be provided with flawed solutions and asked to identify, describe, and correct the flaw.
2.	Description:	Think/work like a mathematician – Engaging in the practice of modeling the solution to real world problems [Mathematical Practice 4]	Teacher Resources:	<a href="https://www.sites.google.com/a/cmpso.org/caccss-resources/k-8-modeling-task-force/k-8-modeling-resources">https://www.sites.google.com/a/cmpso.org/caccss-resources/k-8-modeling-task-force/k-8-modeling-resources</a> (examples of modeling problems and resources for teachers on teaching and scoring them) <a href="http://www.insidemathematics.org/index.php/standard-4">http://www.insidemathematics.org/index.php/standard-4</a> (video examples of students modeling with mathematics) <a href="http://learnzillion.com/lessons/1722-solve-multistep-word-problems-using-model-drawing">http://learnzillion.com/lessons/1722-solve-multistep-word-problems-using-model-drawing</a> (video about modeling)

**Colorado Teacher-Authored Sample Instructional Unit**

			Student Resources:	N/A
	Skills:	Devise a plan to apply mathematics to solve everyday problems by using stated assumptions. Map relationships between important quantities by selecting appropriate tools to create models. Analyze mathematical relationships between important quantities to draw conclusions. Determine if the results make sense. If necessary, change the model	Assessment:	Modeling Problems Students utilize visual and symbolic fraction models such as number lines, bar models and array/area models to represent and analyze relationships of real world problems to draw conclusions and interpret results in relation to the context of the problem.
3.	Description:	Mathematicians fluently add and subtract multi-digit whole numbers.	Teacher Resources:	<a href="http://melrose.patch.com/groups/margaret-adamss-blog/p/bp--activities-at-home-for-increasing-math-fluency">http://melrose.patch.com/groups/margaret-adamss-blog/p/bp--activities-at-home-for-increasing-math-fluency</a> (recommendations to support fluency) <a href="http://www.mathwire.com/numbersense/bfacts.html">http://www.mathwire.com/numbersense/bfacts.html</a> (math games for the classroom to build fluency with basic facts)
			Student Resources:	<a href="http://www.coolmath-games.com/1-number-games-addition-subtraction.html">http://www.coolmath-games.com/1-number-games-addition-subtraction.html</a> (games to help students build fluency) <a href="http://www.sheppardsoftware.com/mathgames/fruitshoot/fruitshoot_subtraction.htm">http://www.sheppardsoftware.com/mathgames/fruitshoot/fruitshoot_subtraction.htm</a> (subtraction practice) <a href="http://www.sheppardsoftware.com/mathgames/fruitshoot/fruitshoot_addition.htm">http://www.sheppardsoftware.com/mathgames/fruitshoot/fruitshoot_addition.htm</a> (addition practice)
	Skills:	Add and subtract multi-digit whole numbers.	Assessment:	Fluency Problems Students build fluency with the addition and subtraction algorithm by practicing addition and subtraction basic facts and multi-digit problems involving regrouping.

**Prior Knowledge and Experiences**

Student familiarity with addition, subtraction, multiplication and division of whole numbers provides a strong foundation for this unit. These concepts are reinforced for students in relation to fractions providing additional practice for students. It is also helpful for students to have comfort with number line and area models with whole numbers, as these models will be extended to fractions throughout the unit.

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Learning Experience # 1</b>	
<p>The teacher may provide models for decomposing and composing a fraction so that students can relate the decomposition and composition of a fraction to their work with and knowledge of whole numbers.</p> <p><i>Enactive:</i> Students can fill a fraction with multicolored manipulatives to show the decomposition/composition of a fraction and then verbalize to their partner the composition (e.g., This egg carton shows this four-twelfths is composed of one-twelfth yellow and three-twelfths green).</p> <p><i>Iconic:</i> Students can use ten frames to show the decomposition of a fraction on a ten-frame and verbalize their decomposition to partner (e.g., This ten frame shows three-tenths decomposed into one-tenth and two-tenths).</p> <p><i>Symbolic:</i> Students can represent their egg carton and ten-frame fractions with equations (e.g., <math>4/12+8/12=12/12</math>; <math>10/10 = 3/10+ 4/10+ 3/10</math>; <math>12/12 - 4/12 = 8/12</math>)</p>	
<b>Teacher Notes:</b>	As students create their decompositions it may be helpful to ask questions that connect their previous work with whole number decomposition to fraction decomposition (i.e., 12 can be broken into 2+10 or 6+6 and 12/12 can be broken into 2/12+10/12 or 6/12+6/12). It is also helpful to point out that fractions with denominators of ten play an important role similar to the number ten in whole numbers. This learning experience can be revisited throughout the unit using other models such as pattern blocks and tangrams.
<b>Generalization Connection(s):</b>	As with whole numbers mathematicians compose fractions by joining/combining fractions (with the same denominator) as sums and decompose/separate fractions (with same denominator) as differences in multiple ways Place value (and its understanding) provides an efficient means to express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100
<b>Teacher Resources:</b>	<p><a href="http://catalog.mathlearningcenter.org/files/media/MathMindEye_Gr5-10_Samples/MME/MME-Unit-IV-Act1s.pdf">http://catalog.mathlearningcenter.org/files/media/MathMindEye_Gr5-10_Samples/MME/MME-Unit-IV-Act1s.pdf</a> (egg carton fractions)</p> <p><a href="http://www.k-5mathteachingresources.com/ten-frames.html">http://www.k-5mathteachingresources.com/ten-frames.html</a> (ten frames)</p> <p><a href="http://learnzillion.com/lessons/1423-decompose-fractions">http://learnzillion.com/lessons/1423-decompose-fractions</a> (video showing decomposition of fractions)</p> <p><a href="http://www.npd117.net/cms/lib02/IL01001910/Centricity/Domain/11/12-14-12%204th%20-%20Decomposing%20Fractions.pdf">http://www.npd117.net/cms/lib02/IL01001910/Centricity/Domain/11/12-14-12%204th%20-%20Decomposing%20Fractions.pdf</a> (visual representation of decomposing a fraction using a tape diagram)</p> <p><a href="https://www.khanacademy.org/math/cc-fourth-grade-math/cc-4th-fractions-topic/cc-4th-decomposing%20fractions/v/decomposing-a-fraction-visually">https://www.khanacademy.org/math/cc-fourth-grade-math/cc-4th-fractions-topic/cc-4th-decomposing%20fractions/v/decomposing-a-fraction-visually</a> (video representation of decomposing fractions)</p> <p><a href="http://www.docstoc.com/docs/44388324/Pattern-block-patterns">http://www.docstoc.com/docs/44388324/Pattern-block-patterns</a> (free printable template of pattern blocks)</p> <p><a href="http://maccss.ncdpi.wikispaces.net/file/view/4thGradeUnit.pdf">http://maccss.ncdpi.wikispaces.net/file/view/4thGradeUnit.pdf</a> (unit on fraction, lesson 7 decomposes tangrams)</p>
<b>Student Resources:</b>	N/A
<b>Assessment:</b>	<p>Students mastering the concept and skills of this lesson should be able to answer questions such as:</p> <p>How can you decompose a fraction in multiple ways?</p> <p>How is decomposing a fraction (e.g. 8/12) similar or different than decomposing a whole number (8)?</p> <p>Why might fractions with denominators of ten be an important type of fraction?</p> <p>Why do you only break apart the numerator and not the denominator when decomposing fractions?</p> <p>Why is it important when adding and subtracting fractions to compose or decompose fractions referring to the same whole (i.e., Why is 1/2 of a ten-frame added to 1/2 of an egg carton not equal to one egg carton or one ten-frame)?</p>

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.mathlearningcenter.org/sites/default/files/documents/sample_materials/BR4-TG-U3-M2.pdf">http://www.mathlearningcenter.org/sites/default/files/documents/sample_materials/BR4-TG-U3-M2.pdf</a> (provides a visual for using string as a partition on p. 22)	Students can use string to partition the egg carton so they visually represent the denominator
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can decompose a fraction into fractions with unlike denominators (e.g., $8/12 - 1/3 = 3/12$ )
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Understand addition and subtraction of fractions as joining and separating parts referring to the same whole</li> <li>• Decompose a fraction into a sum of fractions with the same denominator in more than one way, record each decomposition by an equation and justify the decomposition</li> <li>• Express a fraction with denominator of 10 as an equivalent fraction with a denominator 100, and use this technique to add two fractions with respective denominators 10 and 100</li> </ul>	
<b>Critical Language:</b>	Decompose, compose, fractions, addition, subtraction, denominator, numerator, equation, tenths, ten-frames, whole	

**Learning Experience # 2**

<p>The teacher may use skip counting games (e.g., Buzz or Zap) so that students can strengthen their composition skills by developing mental and verbal concepts of fractions.</p> <p><i>Enactive:</i> Students can skip count by a fraction forwards and backwards saying Buzz whenever they land on a whole number.</p> <p><i>Iconic:</i> Students can represent on a number line skip counting by a fraction from a number (e.g., students might show skip counting forwards by <math>2/3</math> starting at zero and circle every time they land on a whole a number or they might show skip counting backwards by <math>1/10</math> starting at 4 and circle every time the land on a whole number).</p> <p><i>Symbolic:</i> Students can write composition or decomposition of fractions based on their number lines (e.g., <math>2/3 + 2/3 + 2/3 = 2</math>; <math>4 - 1/2 - 1/2 = 3</math>).</p>	
<b>Teacher Notes:</b>	This learning experience can be repeated throughout the unit by changing the starting number and the amount by which to skip count (e.g., starting at $1/3$ skip count by $2/3$ saying Buzz at every whole number). Students can also practice counting by tenths and hundredths, which will support their work later in the year with decimals. Students do not need to state a fraction greater than one as a mixed number, changing to a mixed number during the initial stages of the game might create too high of a cognitive load and obscure the pattern for some students.
<b>Generalization Connection(s):</b>	As with whole numbers mathematicians compose fractions by joining/combining fractions (with the same denominator) as sums and decompose/separate fractions (with same denominator) as differences in multiple ways Place value (and its understanding) provides an efficient means to express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100
<b>Teacher Resources:</b>	<a href="http://spoonful.com/family-fun/buzz">http://spoonful.com/family-fun/buzz</a> (Buzz using fractions)
<b>Student Resources:</b>	N/A

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Assessment:</b>	Students mastering the concept and skills of this lesson should be able to answer questions such as: What patterns do you notice when skip counting by a unit fraction (i.e., if skip counting by one-third, why does every third person say Buzz)? Why is skip counting forward related to addition? Why is skip counting backwards related to subtraction? How many 1/10ths are in the number one? If you skip counted by 1/100 <sup>th</sup> , how many skips would it take to reach the number one? On a number line, how is the whole represented?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://theworksheetsonline.com/numline.html">http://theworksheetsonline.com/numline.html</a> (fraction number lines)	Students can be skip count by a fraction using a closed fraction number line (i.e., a pre-made number line with fractions)
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can skip count by mixed numbers and/or say Buzz on multiples of quantities other than whole numbers such as 1/2
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Understand addition and subtraction of fractions as joining and separating parts referring to the same whole</li> <li>• Decompose a fraction into a sum of fractions with the same denominator in more than one way, record each decomposition by an equation and justify the decomposition</li> <li>• Express a fraction with denominator of 10 as an equivalent fraction with a denominator 100, and use this technique to add two fractions with respective denominators 10 and 100</li> </ul>	
<b>Critical Language:</b>	Decompose, compose, fractions, addition, subtraction, denominator, numerator, equation, tenths, hundredths, number line, whole	

**Learning Experience # 3**

The teacher may utilize number line activities (e.g., a life-size number line) so that students can begin comparing fractions to benchmark quantities (e.g., 0, 1/2, 1).

*Enactive:* Students can be given a fraction to plot on a life-size number line in relation to the quantities of 0, 1/2, and 1 and justify the placement of their fraction.

*Iconic:* Students can use poster paper to plot sets of fractions on a number line with a partner and explain their strategies for comparing the fractions. Students can then share their number lines and strategies for making comparisons with their fellow students.

*Symbolic:* Students can do a gallery walk examining the number lines of their fellow students and write comparison statements about the fractions from each poster using the less than and greater than symbols (i.e., < >).

**Teacher Notes:** This learning experience reviews and extends the work from third grade on comparing fractions. During the enactive stage of the lesson the teacher can begin by placing the benchmark quantities of 0, 1/2, and 1 on the number line. Each student can then be given a card to plot on the number line. Initially students can sit down after they have plotted their fractions and justified their choice of location. Towards the end of the enactive stage the teacher can give two students cards and ask the students to plot the fractions at the same time in relation to each other and the benchmark quantities. The paired fractions should be chosen

**Colorado Teacher-Authored Sample Instructional Unit**

	carefully to bring out key ideas such as comparing fractions with the same denominator or numerator or fractions just one piece more or less than $\frac{1}{2}$ . During the iconic stage students can be given sets of fractions to develop key strategies for comparing fractions without creating common denominators (e.g., common denominators, common numerators, fractions close to zero/unit fractions, fractions close to one, fractions slightly larger or smaller than $\frac{1}{2}$ ). The goal of the iconic stage is for students develop strategies for comparing a variety of fractions without creating common denominators. The concept of common denominators is explored in the next two learning experiences.	
<b>Generalization Connection(s):</b>	Decisions about the size of a fraction relative to another fraction often involves the comparison of the fractions' denominator (if their numerators are equal), or numerators (if the denominators are equal), or the creation of common denominators or numerators for the fractions	
<b>Teacher Resources:</b>	<a href="http://eric.ed.gov/?id=EJ668697">http://eric.ed.gov/?id=EJ668697</a> <i>Developing number sense on the number line</i> by Jennifer Bays <a href="http://www.teacherspayteachers.com/Browse/Search:fraction%20cards">http://www.teacherspayteachers.com/Browse/Search:fraction%20cards</a> (variety of fractions cards that can be printed to create fractions for the number line activities)	
<b>Student Resources:</b>	N/A	
<b>Assessment:</b>	Students mastering the concept and skills of this lesson should be able to answer questions such as: How can you determine if a fraction will be close to one or zero? If a fraction is close to one-half, how can you determine if it is slightly larger or smaller than one-half? Why is comparing fractions with the same denominator similar to comparing whole numbers? Why is comparing fractions with the same numerator different from comparing whole numbers?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.teacherspayteachers.com/Product/Free-Fraction-Strips-14938">http://www.teacherspayteachers.com/Product/Free-Fraction-Strips-14938</a> (fraction bar template)	Students can be compare fractions using fraction manipulatives or drawings during the enactive and iconic stages Students can be compare fractions close to zero or one during the enactive stage Students can be compare sets of fractions during the iconic stage with common denominators
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can compare pairs of fractions during the enactive stage for fractions around $\frac{1}{2}$ Students can compare sets of fractions during the iconic stage that include equivalent fractions and/or fractions centered around the benchmark $\frac{1}{2}$
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Compare two fractions with different numerators and different denominators by creating common denominators or numerators or comparing to a benchmark fraction and record the results of the comparisons with symbols <math>&gt;</math>, <math>=</math>, <math>&lt;</math> and justify the conclusions</li> </ul>	
<b>Critical Language:</b>	Numerator, denominator, compare, benchmark fractions, less than, greater than, equal to	

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Learning Experience # 4</b>	
<p>The teacher may provide pairs of fractions with different denominators and different numerators (e.g., <math>\frac{2}{3}</math> and <math>\frac{3}{5}</math>) so students can investigate ways to create common denominators to compare equivalent fractions [e.g., <math>\frac{1}{2} = \frac{(2 \times 1)}{(2 \times 2)}</math>].</p> <p><i>Enactive:</i> Students can work with a partner to compare two fractions using counters by using trial and error to find a quantity of counters that can be used to represent both fractions (e.g., students might first try ten or twelve counters to show <math>\frac{2}{3}</math> and <math>\frac{3}{5}</math> but eventually they will find a multiple of fifteen works to show these two fractions).</p> <p><i>Iconic:</i> Students can use grid paper to show visually how to compare the two fractions provided in the enactive stage (e.g., students comparing <math>\frac{2}{3}</math> to <math>\frac{3}{5}</math> on grid paper can draw two 1 by 15 arrays with ten squares shaded in one and nine squares shaded in the other).</p> <p><i>Symbolic:</i> Students can write the equivalent fractions they created during the enactive and iconic stage (i.e., <math>\frac{2}{3} = \frac{10}{15}</math> and <math>\frac{3}{5} = \frac{9}{15}</math>) and explore an efficient method for creating equivalent fractions.</p>	
<b>Teacher Notes:</b>	<p>Students will likely struggle during the enactive stage to find a quantity of counters that will work to show both fractions. It is important for students to preserve while finding a common denominator but the teacher may want to limit the amount of counters students have access to initially to keep their trial and error manageable. Several key ideas are explored during this learning experience. First, students can recognize the need to compare fractions referring to same whole (e.g., the number of counters). Second, students can begin to explore patterns for finding a common size “whole” or denominator. The teacher may want to have each group of students compare two different fractions and then hold a class discussion about the patterns they notice between the new common denominators and denominators of the original fractions. Students may notice that the original denominators are both factors of the new common denominator or vice versa the new common denominator is a multiple for both of the original denominators. The formal process for creating equivalent fractions occurs in fifth grade the focus during fourth grade is on creating visual representations that support explanations about why two fractions are equivalent. After students have focused on fractions with smaller denominators and noticed patterns, students can be provided fractions with denominators of tenths and hundredths and asked to compare them.</p>
<b>Generalization Connection(s):</b>	<p>Equivalent fractions describe the same part of a whole by using different fractional parts</p> <p>Increasing or decreasing both the numerators and denominators of a fraction by the same factor creates equivalent fractions</p> <p>Decisions about the size of a fraction relative to another fraction often involves the comparison of the fractions’ denominators (if their numerators are equal), or numerators (if their denominators are equal) or the creation of common denominators or numerators for the fractions</p> <p>Place value (and its understanding) provides an efficient means to express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100</p>
<b>Teacher Resources:</b>	<p><a href="http://incompetech.com/graphpaper/">http://incompetech.com/graphpaper/</a> (free downloadable grid paper)</p> <p><a href="http://learnzillion.com/lessons/1434-compare-fractions-with-different-denominators-using-area-models">http://learnzillion.com/lessons/1434-compare-fractions-with-different-denominators-using-area-models</a> (video about comparing fractions with different denominators)</p> <p><a href="http://learnzillion.com/lessons/2970-generate-equivalent-fractions-using-a-grid-model">http://learnzillion.com/lessons/2970-generate-equivalent-fractions-using-a-grid-model</a> (video about generating equivalent fractions)</p> <p><a href="http://learnzillion.com/lessons/2970-generate-equivalent-fractions-using-a-grid-model">http://learnzillion.com/lessons/2970-generate-equivalent-fractions-using-a-grid-model</a> (video comparing fractions with tenths and hundredths)</p> <p><a href="http://learnzillion.com/lessons/2841-use-a-number-line-to-show-how-fractions-with-denominators-10-and-100-are-equivalent">http://learnzillion.com/lessons/2841-use-a-number-line-to-show-how-fractions-with-denominators-10-and-100-are-equivalent</a> (video using a number line to show fraction equivalence with denominators of 10 and 100)</p>
<b>Student Resources:</b>	<p><a href="http://www.ixl.com/math/grade-4/compare-fractions">http://www.ixl.com/math/grade-4/compare-fractions</a> (practice questions to compare fractions)</p> <p><a href="http://www.ixl.com/math/grade-4/fractions-with-denominators-of-10-100-and-1000">http://www.ixl.com/math/grade-4/fractions-with-denominators-of-10-100-and-1000</a> (finding equivalent fractions for tenths and hundredths)</p>

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Assessment:</b>	Students mastering the concept and skills of this lesson should be able to answer questions such as: Why is it important to compare fractions referring to the same whole (i.e., is $1/2$ a large pizza equal to $1/2$ a small pizza)? How can you compare fractions with different denominators and numerators? What patterns did you notice when finding a common denominator? When creating an equivalent fraction, why do both the numerator and denominator change? How can equivalent fractions be created? What relationship did you notice between the numerators and denominators of equivalent fractions? How many hundredths are there in one-whole? How many hundredths are there in one tenth? Why is $3/10$ larger than $3/100$ ?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can compare fractions whose common denominator is less than ten to limit the amount of trial error necessary to find a common denominator
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can compare fractions with denominators of tenths and hundredths
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Explain why a fraction <math>a/b</math> is equivalent to a fraction <math>(n \times a) / (n \times b)</math> by using visual fractions models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size</li> <li>• Generate equivalent fractions</li> <li>• Compare two fractions with different numerators and different denominators by creating common denominators or numerators or comparing to a benchmark fraction and record the results of the comparisons with symbols <math>&gt;</math>, <math>=</math>, <math>&lt;</math> and justify the conclusions</li> <li>• Express a fraction with denominator of 10 as an equivalent fraction with a denominator 100, and use this technique to add two fractions with respective denominators 10 and 100</li> <li>• Recognize that comparisons are valid only when the two fractions refer to the same whole</li> </ul>	
<b>Critical Language:</b>	Numerator, denominator, compare, benchmark fractions, less than, greater than, equal to, equivalent fractions	

<b>Learning Experience # 5</b>
<p>The teacher may provide fraction models larger than one so that students can begin decomposing and composing mixed numbers.</p> <p><i>Enactive:</i> Students can create fractional quantities larger than one using a variety of manipulatives (e.g., egg cartons, ten-frames, fraction circles, fraction bars, base ten blocks, money).</p> <p><i>Iconic:</i> Students can show either a decomposition/composition of a fraction greater than one on a number line (e.g., students might first show <math>14/10</math> on two ten-frames and then show a jump from 1 to zero on their number line and then a jump from 1 to <math>14/10</math> to show the composition of <math>1 + 4/10</math>).</p> <p><i>Symbolic:</i> Students can write addition and subtraction equations for fractions greater than one (e.g., <math>14/10 = 10/10 + 4/10</math>; <math>14/10 = 1 + 4/10</math>; <math>14/10 - 5/10 = 9/10</math>).</p>

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Teacher Notes:</b>	Students can practice writing fractions greater than one as a mixed number and in fractional form (e.g., improper fractions) and making connections between the two formats. It may also help for students to write the number one as a fraction (e.g., $\frac{4}{4}$ ) and as the number 1. A common misconception for students when working with mixed numbers is to assume $1\frac{4}{10}$ means $1 \times \frac{4}{10}$ and not $1 + \frac{4}{10}$ . The addition symbol is silent in a mixed number but you may want to require students to write it until they grow comfortable with the meaning of the mixed number. By having students write compositions of fractions greater than one in a variety of ways students will begin to develop a foundation for operating on mixed numbers and “improper” fractions. For example, the fraction $\frac{23}{5}$ could be written as $\frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{3}{5}$ or $1 + 1 + 1 + 1 + \frac{3}{5}$ or as $4 + \frac{3}{5}$ all of these representations are equivalent and provide students with new insight into the concept of a fraction greater than one. Students can use models such as base-ten blocks and money to model fractions of $\frac{1}{10}$ and $\frac{1}{100}$ (e.g., a penny can represent $\frac{1}{100}$ of a dollar and a dime $\frac{1}{10}$ of a dollar).	
<b>Generalization Connection(s):</b>	To add and subtract mixed numbers with like denominators requires the use of properties of operations. As with whole numbers, mathematicians compose fractions by joining/combining fractions (with the same denominator) as sums and decompose/separate fractions (with the same denominator) as differences in multiple ways Place value (and its understanding) provides an efficient means to express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100	
<b>Teacher Resources:</b>	<a href="http://catalog.mathlearningcenter.org/files/media/MathMindEye_Gr5-10_Samples/MME/MME-Unit-IV-Act1s.pdf">http://catalog.mathlearningcenter.org/files/media/MathMindEye_Gr5-10_Samples/MME/MME-Unit-IV-Act1s.pdf</a> (egg carton fractions) <a href="http://www.k-5mathteachingresources.com/ten-frames.html">http://www.k-5mathteachingresources.com/ten-frames.html</a> (ten frames) <a href="http://learnzillion.com/lessonsets/343-add-and-subtract-mixed-numbers-with-like-denominators">http://learnzillion.com/lessonsets/343-add-and-subtract-mixed-numbers-with-like-denominators</a> (video describing how to add and subtraction mixed numbers with like denominators)	
<b>Student Resources:</b>	N/A	
<b>Assessment:</b>	Students mastering the concept and skills of this lesson should be able to answer questions such as: Why are fractions greater than one important? What some examples from life of fractions greater than one? Why are $1\frac{4}{10}$ , $\frac{14}{10}$ and $1 + \frac{4}{10}$ all equivalent quantities? How many pennies are in a dime? What fraction is a penny of a dollar? What fraction is a dime of a dollar? What relationship does $\frac{1}{10}$ have to 1? What relationship does $\frac{1}{10}$ have to $\frac{1}{100}$ ? What fraction of a dollar is 23 dimes? What are at least two different ways to find the total quantity of $1\frac{3}{5} + \frac{9}{5}$ ? What is the relationship between a mixed number and improper fraction?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can decompose and compose fractions greater than one into just their whole number and fractional components using only one fractional model (e.g., ten-frames)
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.mathgoodies.com/lessons/fractions/mixed_to_fr_actions.html">http://www.mathgoodies.com/lessons/fractions/mixed_to_fr_actions.html</a> (algorithm for changing mixed numbers)	Students can create a generalized method (e.g., algorithm) for turning any mixed number into a fraction greater than one and vice versa

### Colorado Teacher-Authored Sample Instructional Unit

<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Add and subtract mixed numbers with like denominators</li> <li>• Express a fraction with denominator of 10 as an equivalent fraction with a denominator 100, and use this technique to add two fractions with respective denominators 10 and 100</li> </ul>
<b>Critical Language:</b>	Addition, subtractions, sum, difference, decompose, compose, mixed number, improper fraction, fraction greater than one, equivalence, penny, dime, dollar

#### Learning Experience # 6

The teacher may provide addition and subtraction fraction word problems so that students can use decomposing and composing fractions in real world contexts.

*Enactive:* Students can use manipulatives to solve addition and subtraction word problems involving fractions with like denominators and fractions greater than one (e.g., mixed numbers and improper fractions) with like denominators.

*Iconic:* Students can use a number line to estimate the size of the sum or difference of fraction word problem and then use the number line to find the sum or difference.

*Symbolic:* Students can write an addition or subtraction equation that represents the word problem.

<b>Teacher Notes:</b>	It is important for students to work with fractional quantities greater than and less than one. The teacher may want to start with story problems for fractions less than one and then moved to quantities greater than one. For word problems involving fractions greater than one, it is helpful for students work with quantities written in the form of mixed numbers and “improper” fractions. Students should also work with word problems involving tenths and hundredths. These word problems can involve both tenths and hundredths (i.e., unlike denominators). Students should use their work in their prior learning experiences to solve these unlike denominator problems of tenths and hundredths.
<b>Generalization Connection(s):</b>	<p>Word problems and contexts involving joining and separating parts of the same (size) whole require the addition and subtraction of fractions</p> <p>To add and subtract mixed numbers with like denominators requires the use of properties of operations</p> <p>Place value (and its understanding) provides an efficient means to express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100</p>
<b>Teacher Resources:</b>	<p><a href="http://www.illustrativemathematics.org/illustrations/152">http://www.illustrativemathematics.org/illustrations/152</a> (story problem involving fractions and money)</p> <p><a href="http://www.k-5mathteachingresources.com/support-files/fraction-word-problems-like-denominator.pdf">http://www.k-5mathteachingresources.com/support-files/fraction-word-problems-like-denominator.pdf</a> (printable worksheet for fraction word problems)</p> <p><a href="https://www.teachervision.com/fractions/printable/31018.html">https://www.teachervision.com/fractions/printable/31018.html</a> (printable worksheet for addition word problems)</p> <p><a href="http://learnzillion.com/lessonsets/517-solve-word-problems-involving-addition-and-subtraction-of-fractions-with-like-denominators">http://learnzillion.com/lessonsets/517-solve-word-problems-involving-addition-and-subtraction-of-fractions-with-like-denominators</a> (videos about solving fraction word problems)</p>
<b>Student Resources:</b>	<p><a href="http://www.ixl.com/math/grade-5/add-and-subtract-fractions-with-like-denominators-word-problems">http://www.ixl.com/math/grade-5/add-and-subtract-fractions-with-like-denominators-word-problems</a> (practice for fraction word problems with like denominators)</p> <p><a href="http://mrnussbaum.com/fracword/">http://mrnussbaum.com/fracword/</a> (practice questions for fraction word problems with like denominators)</p>
<b>Assessment:</b>	<p>Students mastering the concept and skills of this lesson should be able to answer questions such as:</p> <p>How do you estimate an answer to a fractional word problem?</p> <p>When solving a word problem involving mixed numbers when is it beneficial to change the mixed numbers to fractions greater than one?</p>

**Colorado Teacher-Authored Sample Instructional Unit**

	Why is it important to check that fractions refer to the same whole before adding and subtracting them? When adding or subtracting mixed numbers does it matter if you work with the whole numbers and then the fractions or vice versa? How do you add or subtract tenths and hundredths? Why can't you just add the numerator of a tenth to a hundredth?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can solve word problems involving addition and subtraction of fractions that do not involve quantities greater than one
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="https://docs.google.com/viewer?a=v&amp;pid=sites&amp;srcid=Y25zdXNjaG9vbHMub3JnfGNuc3UtbWF0aC1jY3NzfGd4OjEyZkQDhkZGE0MzlhZQ">https://docs.google.com/viewer?a=v&amp;pid=sites&amp;srcid=Y25zdXNjaG9vbHMub3JnfGNuc3UtbWF0aC1jY3NzfGd4OjEyZkQDhkZGE0MzlhZQ</a> (Kansas Math Flip Book describes the importance of the same size whole on p. 34-36)	Students can create a presentation about the importance of adding and subtracting fractions with the same size whole for the class
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Add and subtract mixed numbers with like denominators</li> <li>• Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators</li> <li>• Express a fraction with denominator of 10 as an equivalent fraction with a denominator 100, and use this technique to add two fractions with respective denominators 10 and 100</li> </ul>	
<b>Critical Language:</b>	Addition, subtractions, sum, difference, decompose, compose, mixed number, improper fraction, fraction greater than one, equivalence, penny, dime, dollar	

<b>Learning Experience # 7</b>	
The teacher may model the collection of fraction data (e.g., how far each student in the class can jump) so that students can begin exploring ways to plot fractional data and use the data to solve addition and subtraction word problems (e.g., How much farther can Mary jump than Brian?). <i>Enactive:</i> Students can measure how far they jumped to the nearest $\frac{1}{8}$ of an inch. <i>Iconic:</i> Students can collect the data from the class to construct a line plot. <i>Symbolic:</i> Students can write word problems involving the addition and subtraction of fractions using information presented in the line plot and solve them.	
<b>Teacher Notes:</b>	Students can jump on a number line that is partitioned into eighths of an inch. The partitions do not need to be labeled with quantities but if they are labeled it is helpful to have them labeled in eighths (e.g., $\frac{1}{8}$ , $\frac{2}{8}$ ) rather than eighth, fourths, and halves. Students can repeat this learning experience measuring to the nearest tenth or hundredth of a meter.
<b>Generalization Connection(s):</b>	Word problems and contexts involving joining and separating parts of the same (size) whole require the addition and subtraction of fractions To add and subtract mixed numbers with like denominators requires the use of properties of operations

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Teacher Resources:</b>	<a href="http://www.math-aids.com/Measurement/Measuring_in_Inches.html">http://www.math-aids.com/Measurement/Measuring_in_Inches.html</a> (worksheets to practice measuring to the nearest eighth of an inch) <a href="http://dese.mo.gov/divimprove/assess/documents/asmt-sbac-math-gr4-sample-items.pdf">http://dese.mo.gov/divimprove/assess/documents/asmt-sbac-math-gr4-sample-items.pdf</a> (sample problem on p. 14) <a href="http://illuminations.nctm.org/LessonDetail.aspx?ID=L545">http://illuminations.nctm.org/LessonDetail.aspx?ID=L545</a> (lesson on measuring with fractions)	
<b>Student Resources:</b>	<a href="http://www.funbrain.com/measure/index.html">http://www.funbrain.com/measure/index.html</a> (measurement game)	
<b>Assessment:</b>	Students mastering the concept and skills of this lesson should be able to answer questions such as: How much farther is the longest jump than the shortest jump? How long would the shortest and longest jumps be if added together?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can measure to the nearest $\frac{1}{2}$
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can create word and solve word problems involving three or more fractions
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>) and solve problems involving addition and subtraction of fractions by using information presented in line plots</li> </ul>	
<b>Critical Language:</b>	Line plot, data, measure, mixed number, improper fraction, numerator, denominator	

<b>Learning Experience # 8</b>	
<p>The teacher may revisit skip counting games (e.g., Buzz or Zap) so that students can connect repeated addition with the concept of multiplication.</p> <p><i>Enactive:</i> Students can skip count by a fraction forwards saying Buzz whenever they land on a whole number.</p> <p><i>Iconic:</i> Students can represent on a number line skip counting by fraction a specified number of times (e.g., students might show 4 jumps on a number line of <math>\frac{1}{3}</math> by writing <math>\frac{1}{3}</math>, <math>\frac{2}{3}</math>, <math>\frac{2}{3}</math> (or 1), <math>\frac{4}{3}</math>).</p> <p><i>Symbolic:</i> Students can write a repeated addition and multiplication equation to represent their jumps (e.g., <math>\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}</math>; <math>4 \times \frac{1}{3} = \frac{4}{3}</math>) and explain the connection between the repeated addition and multiplication equations.</p>	
<b>Teacher Notes:</b>	<p>The teacher may want to begin with unit fractions. Students can develop an understanding that any fraction <math>\frac{a}{b}</math> can be written as <math>a</math> times the quantity <math>\frac{1}{b}</math> (e.g., <math>\frac{4}{3}</math> is equivalent to <math>4 \times \frac{1}{3}</math>). As students gain comfort with this idea they can be challenged to skip count by non-unit fractions and write corresponding multiplication equations including one using a unit fraction. For example, if a student skip counts by <math>\frac{2}{3}</math> four times they can write <math>4 \times \frac{2}{3} = \frac{8}{3}</math> and <math>8 \times \frac{1}{3} = \frac{8}{3}</math>. Students can explore the equivalence of these two expressions and a generalized strategy for finding the equivalent unit fraction multiplication expression through their work with skip counting and the number line model. This early understanding of fraction multiplication builds on students' early work with multiplication (e.g., number of groups <math>\times</math> group size) and the foundation for the fraction multiplication algorithm, which is developed in 5<sup>th</sup> grade.</p>

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Generalization Connection(s):</b>	The multiplication of a fraction ( $1/b$ ) by a whole number ( $a$ ) creates a fraction that is a multiple of the original fraction.	
<b>Teacher Resources:</b>	<a href="http://spoonful.com/family-fun/buzz">http://spoonful.com/family-fun/buzz</a> (Buzz using fractions) <a href="http://www.k-5mathteachingresources.com/support-files/models-for-fraction-multiplication-4nf4a.pdf">http://www.k-5mathteachingresources.com/support-files/models-for-fraction-multiplication-4nf4a.pdf</a> (fraction times a whole number multiplication game) <a href="http://learnzillion.com/lessons/2939-use-a-fraction-model-for-multiplication-of-fractions-and-whole-numbers">http://learnzillion.com/lessons/2939-use-a-fraction-model-for-multiplication-of-fractions-and-whole-numbers</a> (video describing multiplication of a fraction by a whole number)	
<b>Student Resources:</b>	<a href="https://www.splashmath.com/common-core-math/fourth-grade/number-and-operations-fractions/4b">https://www.splashmath.com/common-core-math/fourth-grade/number-and-operations-fractions/4b</a> (game to practice multiplication of a fraction by a whole number)	
<b>Assessment:</b>	Students mastering the concept and skills of this lesson should be able to answer questions such as: How is multiplication of whole number and fraction similar to multiplication of whole numbers? How many $1/5$ are in $10/3$ ? How can you write this relationship as a multiplication expression equal to $10/3$ ? How can you write the relationship of tenths to hundredths as a multiplication equation involving the quantities 10, $1/10$ and $1/100$ ? If you skip count by $2/3$ four times how many times would you need to skip count by $1/3$ to arrive at the same quantity? Why? How can you write this as two equivalent multiplication expressions? (e.g., $4 \times 2/3 = 8 \times 1/3$ ).	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://theworksheetsonline.com/numline.html">http://theworksheetsonline.com/numline.html</a> (fraction number lines)	Students can skip count by a fraction using a closed fraction number line (i.e., a pre-made number line with fractions)
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://learnzillion.com/lessons/2939-use-a-fraction-model-for-multiplication-of-fractions-and-whole-numbers">http://learnzillion.com/lessons/2939-use-a-fraction-model-for-multiplication-of-fractions-and-whole-numbers</a> (video describing multiplication of a fraction by a whole number)	Students can create a visual explanation of why $c \times a/b = ca/b$ (e.g., $4 \times 2/3 = 8/3$ )
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Understand a fraction <math>a/b</math> as a multiple of <math>1/b</math> and a multiple of <math>a/b</math> as a multiple of <math>1/b</math></li> <li>Multiply a fraction by a whole number and solve word problems involving multiplication of a fraction by a whole number</li> </ul>	
<b>Critical Language:</b>	Multiple, product, whole number, group size, number of groups, multiplication, repeated addition	

**Learning Experience # 9**

The teacher may provide word problems so that students can explore various models (e.g., number line, area model) to solve problems involving fractions multiplied by whole numbers.

*Enactive:* Students can use manipulatives to solve word problems (i.e. Susan is making chocolate chip cookies for the school bake sale. If each batch takes  $1/3$  cup of chocolate chips, how many cups will she need for 5 batches?).

*Iconic:* Students can draw visual representations (e.g., number line, area model) to solve word problems.

*Symbolic:* Students can write both repeated addition and multiplication expressions to represent word problems.

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Teacher Notes:</b>	This learning experience focuses on word problems related to the multiplication of whole number times a fraction as multiple copies of a fraction. The concept of multiplication of a fraction times a whole number (i.e., finding a fraction of a whole number) is dealt with in fifth grade using the concept of scaling.	
<b>Generalization Connection(s):</b>	The multiplication of a fraction ( $1/b$ ) by a whole number ( $a$ ) creates a fraction that is a multiple of the original fraction.	
<b>Teacher Resources:</b>	<a href="http://learnzillion.com/lessons/2845-solve-word-problems-involving-multiplying-a-fraction-and-a-whole-number-using-a-fraction-model">http://learnzillion.com/lessons/2845-solve-word-problems-involving-multiplying-a-fraction-and-a-whole-number-using-a-fraction-model</a> (video showing how to solve word problems involving fractions times a whole number) <a href="http://www.illustrativemathematics.org/illustrations/971">http://www.illustrativemathematics.org/illustrations/971</a> (assessment task involving fractions times a whole number word problems) <a href="http://www.k-5mathteachingresources.com/support-files/models-for-fraction-multiplication-4nf4a.pdf">http://www.k-5mathteachingresources.com/support-files/models-for-fraction-multiplication-4nf4a.pdf</a> (directions for Multiplication game)	
<b>Student Resources:</b>	<a href="http://www.ixl.com/math/grade-4/multiply-fractions-by-whole-numbers-word-problems">http://www.ixl.com/math/grade-4/multiply-fractions-by-whole-numbers-word-problems</a> (practice words problems involving fractions times a whole number)	
<b>Assessment:</b>	Students mastering the concept and skills of this lesson should be able to answer questions such as: How can I model the multiplication of a fraction by a whole number? What happens to the product when I multiply a fraction by a whole number?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can solve word problems involving products less than one and unit fractions
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	The teacher may provide expressions of whole numbers multiplied by fractions	Students can create word problems to represent the teacher provided expressions  Students can solve word problems involving fractions greater than one
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Understand a fraction <math>a/b</math> as a multiple of <math>1/b</math> and a multiple of <math>a/b</math> as a multiple of <math>1/b</math></li> <li>Multiply a fraction by a whole number and solve word problems involving multiplication of a fraction by a whole number</li> </ul>	
<b>Critical Language:</b>	Multiple, product, whole number, group size, number of groups, multiplication, repeated addition	