## Unit Title: Multiply, Divide \& Conquer

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BASED ON A CURRICULUM OVERVIEW SAMPLE AUTHORED BY

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This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacherauthors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.
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Colorado Teacher-Authored Sample Instructional Unit


| Unit Title | Multiply, Divide \& Conquer |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Focusing Lens(es) | Interpretation/Relationships | Standards and Grade <br> Level Expectations <br> Addressed in this Unit | MA10-GR.3-S.1-GLE.3 | 5 weeks |
| Inquiry Questions <br> (Engaging- <br> Debatable): | $\bullet$ How are multiplication and division related? (MA10-GR.3-S.1-GLE.3-IQ.1) |  |  |  |
| Unit Strands | Operations and Algebraic Thinking, Personal Financial Literacy |  |  |  |
| Concepts | Multiplication, division, equal groups, arrays, combinations, fair share, rate, scaling, area, unit conversions, addition, subtraction, rows, columns, times as <br> many, times fewer, unknown factor, inverse operations, substitution, models, properties of operations (commutative property, associative property, <br> distributive property, identity property, zero property), equations, arithmetic patterns |  |  |  |


| Generalizations | Guiding Questions |  |
| :---: | :---: | :---: |
| My students will Understand that... | Factual | Conceptual |
| Multiplication and division word problems can involve situations of equal groups, arrays, combinations, fair sharing, rate, scaling, area, and unit conversions (MA10-GR.3-S.1-GLE.3-EO.a.i, a.ii, a.iii) | What does the product (quotient) represent in a context? | How can you determine if a story problem represents a multiplication/division problem? |
| Compared with addition/subtraction, multiplication and/or division provide highly efficient means to solve equal-group story problems (MA10-GR.3-S.1-GLE.3-EO.c.i) | What does it mean to be efficient? <br> Why was multiplication invented? Why not just add? <br> (MA10-GR.3-S.1-GLE.3-IQ.3) <br> Why was division invented? Why not just subtract? <br> (MA10-GR.3-S.1-GLE.3-IQ.4) | Why is (insert a contextual problem with unequal groups) a multiplication or addition problem? |
| Division enables decision-making determinations regarding the size of groups or the number of groups in a given context (MA10-GR.3-S.1-GLE.3-EO.a.ii) | What does it mean to count how many are in a group versus the number of groups? <br> What are two types of equal group division problems? | How do equal problems lead to types of answers to division problems? |
| Arrays such that an array of m rows and n columns has nx m items can model multiplication and division word problems (MA10-GR.3-S.1-GLE.3-EO.a.iii) | What is an array? <br> What are rows and columns? <br> How can an array model be used to help solve a combination problem such as 3 pants and 2 shirts? | How is an array a model for multiplication? <br> How can an array model show the commutative property? <br> How are arrays connected to the concept of area? |

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The comparison of the size of a collection against the size of a group reflects multiplication and division problems related to the concept of "times as many" or "times fewer" (MA10-GR.3-S.1-GLE.3-EO.a)

Because multiplication and division are inverse operations, multiplication provides and effective means to solve division problems as unknown factor problems (MA10-GR.3-S.1-GLE.3-EO.b.ii)

Arithmetic patterns, justified by properties of operation, constitute strategies that can be used to multiply and divide (MA10-GR.3-S.1-GLE.3-EO.d.iv)

Fluency with multiplication and division facts results from multiple experiences with different models, representations, problem types, properties of operations and interrelationships among multiplication and division facts (MA10-GR.3-S.1-GLE.3-EO.c.i, c.ii)

What is the difference between comparing one group as n "times as many" than another group and comparing by stating how many more are in one group?

How can you use a multiplication or division fact to find a related fact?

What patterns do you notice in a multiplication table? Addition table?
How can three numbers be multiplied in any order to solve a multiplication problem (e.g., $2 \times 7 \times 5$ or $14 \times$ 5?)
How do arithmetic patterns help to build fluency with basic facts?

How can you use a multiplication or division fact to find a related fact? (MA10-GR.3-S.1-GLE.3-IQ.2)
How can strategies such as doubling, halving, skip counting, partitioning and reassembling help to develop fluency with basic multiplication facts?

How can comparing groups lead to multiplication and division problems?

How are multiplication and division related? (MA10 GR.3-S.1-GLE.3-IQ.1)
Why is division by zero undefined?

Why are zero and one special in multiplication?
Why do odd numbers times odd numbers result in odd numbers? Is there another way to get an odd number when multiplying?

Why is relying solely on rote memorization of facts limiting when learning more advanced mathematics?

## Key Knowledge and Skills: My students will...

What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.

- Solve word problems involving equal groups, measurement quantities, and arrays by using multiplication and division (MA10-GR.3-S.1-GLE.3-EO.a.iii)
- Represent multiplication and division story problems by using drawings and equations with a symbol for the unknown number (MA10-GR.3-S.1-GLE.3-EO.a.iii)
- Understand division as an unknown-factor problem (MA10-GR.3-S.1-GLE.3-EO.b.ii)
- Model strategies to achieve a personal financial goal using arithmetic operations (MA10-GR.3-S.1-GLE.3-EO.a.v)*
- Interpret whole-number products and quotients (MA10-GR.3-S.1-GLE.3-EO.a.i, a.ii)
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers (MA10-GR.3-S.1-GLE.3-EO.a.iv, b.ii)
- Apply properties of operations as strategies to multiply and divide (MA10-GR.3-S.1-GLE.3-EO.d)
- Identify and explain patterns in arithmetic (including patterns in addition and multiplication tables) using properties of operations (MA10-GR.3-S.1-GLE.3-EO.d)
- Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division (MA10-GR.3-S.1-GLE.3-EO.C)

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Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.
EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: "Mark Twain exposes the hypocrisy of slavery through the use of satire."

A student in $\qquad$ can demonstrate the ability to apply and comprehend critical language through the following statement(s):
Academic Vocabulary: Fair share, represent, equal, groups, model, solve, interpret, rows, columns, times as many, times fewer, unknown

Technical Vocabulary:

If 24 apples are arranged into 4 equal rows, I know that 6 apples will be in each row because $6 \times 4$ equals 24 , and 24/4 equals 6.

Academic Vocabulary: Fair share, represent, equal, groups, model, solve, interpret, rows, columns, times as many, times fewer, unknown

Product, quotient, whole number, operation, multiplication, division, pattern, equation, strategy, relationship, unknown factor, properties, array, combinations, area, addition, subtraction, factor, inverse operations,

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| Unit Description: | This unit focuses on an introduction to multiplication and division concepts. The students begin with exploring items that come in equal groups and the connections between how equal groups, repeated addition, and skip counting relate to multiplication. Students explore multiplication and find factors in a geometric context through the construction of rectangular arrays and use these skills to problem solve. Students then use arrays and number strings to demonstrate their understanding of the Associative and Distributive properties of multiplication. The culmination of this unit explores patterns in the multiplication chart in order to build fluency of multiplication facts by the end of third grade. |
| :---: | :---: |
| Unit Generalizations |  |
| Key Generalization: | Compared with addition/subtraction, multiplication and/or division provide highly efficient means to solve equal-group story problems |
| Supporting Generalizations: | Multiplication and division word problems can involve situations of equal groups, arrays, combinations, fair sharing, rate, scaling, area, and unit conversions |
|  | Division enables decision-making determinations regarding the size of groups or the number of groups in a given context |
|  | Arrays such that an array of m rows and n columns has $\mathrm{n} \times \mathrm{m}$ items can model multiplication and division word problems |
|  | The comparison of the size of a collection against the size of a group reflects multiplication and division problems related to the concept of "times as many" or "times fewer" |
|  | Because multiplication and division are inverse operations, multiplication provides and effective means to solve division problems as unknown factor problems |
|  | Arithmetic patterns, justified by properties of operation, constitute strategies that can be used to multiply and divide |
|  | Fluency with multiplication and division facts results from multiple experiences with different models, representations, problem types, properties of operations and interrelationships among multiplication and division facts |

## Performance Assessment: The capstone/summative assessment for this unit.

## Claims:

(Key generalization(s) to be mastered and demonstrated through the capstone assessment.)

## Stimulus Material:

(Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)

Compared with addition/subtraction, multiplication and/or division provide highly efficient means to solve equal-group story problems.

A county fair board has hired you to create the layout of the livestock barn for the fair this year. When designing your barn you need to think about the following:

- 36 rabbits are part of the fair and 3 fit in each cage
- 24 chickens are part of the fair and 4 fit in each cage
- The fair only has 9 cages for turkeys and each cage only holds 2 turkeys
- The fair only has 8 pens for the goats and each pens holds exactly 4 goats
- The fair wants a 5 by 6 array of pens set aside for the sheep each pen can hold 2 sheep
- The fair provides 20 yards of fence to tie up cattle each steer needs 3 yards
- The fair is allowing 28 horses to be held in an array of stalls

Determine the best arrangement of the cages and pens for each type of animal by creating arrays of the pens and cages.

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## Product/Evidence: <br> (Expected product from students)

## Differentiation

(Multiple modes for student expression)

As part of the presentation to the county fair board students will need to create a visual model of the livestock barn. The visual model should show the arrangements of the animals in arrays. Students will also need to provide a written description about the decisions they made when creating the barn design. The written description should include how students found unknown information (e.g., how many total of each type of animal, how many pens/cages for each animal, how many animals in each pen/cage).
Teacher note: This unit was created in a rural community in Colorado. The performance task reflects a context relevant to the students in this community. Teachers may modify the context as needed for urban and suburban environments.

Students can be given different numbers of animals from the county fair board to differentiate or extend the task.
Students can be given additional types of situations such as measurement problems.
Students can create the visual model of their barn using graph paper or on the computer
Students can write up their explanations using visuals and sentence frames provided by the teachers or in a formal report form.

## Texts for independent reading or for class read aloud to support the content

Informational/Non-Fiction
What Comes in 2's, 3 's \& 4's? by Susan Aker (Lexlie level 470)

## Fiction

One Hundred Hungry Ants by Elinor Pinczes (Lexlie level 650) A Remainder of One by Elinor Pinczes (Lexlie level 570) Each Orange Had 8 Slices by Paul Giganti Jr. (Lexlie level 400) Two of Everything: A Chinese Folktale by Lily Toy Hong (Lexlie level 540) The Best of Times: Math Strategies that Multiply by Greg Tang (Lexlie level 130) The Grapes of Math by Greg Tang (Lexlie level 350+) The Doorbell Rang by Pat Hutchins (Lexlie level 340) Divide and Ride by Stuart Murphy and George Ulrich (Lexile level 660)

## Ongoing Discipline-Specific Learning Experiences

1 Description: Think/work like a mathematician Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others

Choose and explain a strategy for solving multiplication and division problems and identify and describe a flaw in the reasoning of others

| Teacher <br> Resources: | http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm <br> (lesson plans contains exemplars that could be replicated for students to critique the <br> reasoning of others) |
| :--- | :--- |
| Student <br> Resources: | N/A |
| Assessment: | Students choose a strategy for solving multiplication and division problems and explain why <br> their strategy led to a correct response. Careful attention should be paid to precise use of <br> vocabulary and symbols. Periodically throughout the unit, students should be provided <br> with flawed solutions and asked to interpret, critique and correct the flaw. |


| 2. | Description: | Think/work like a mathematician Engaging in the practice of modeling the solution to real world problems | Teacher Resources: | https://www.sites.google.com/a/cmpso.org/caccss-resources/k-8-modeling-task-force/k-8-modeling-resources (examples of modeling problems and resources for teachers on teaching and scoring them) <br> https://sites.google.com/a/bryantschools.org/math-common-core-resource-site/home-1/3rd-grade/3-oa-3 (modeling problems) |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Student <br> Resources: | http://www.internet4classrooms.com/common core/use multiplication division within 100 solve operations algebraic thinking third 3rd grade math mathematics.htm (online multiplication and division words problems for students) |
|  | Skills: | Model real world problems by using stated assumptions, mapping relationships with appropriate models, analyze relationships to draw conclusions, interpret results in relation to context, justify and defend the model, and reflect on whether results make sense | Assessment: | Modeling Problems <br> Students use arrays, number lines, bar models, and equations to represent and analyze relationships of real world two-step problems to draw conclusions and interpret results in relation to the context of the problem. |
| 3. | Description: | Mathematicians fluently multiply and divide within 100 and add and subtract within 20 | Teacher Resources: | http://investigations.terc.edu/library/Games 23.cfm\#a multiplication (list of multiplication <br> fluency websites for students) <br> http://www.playkidsgames.com/games/mathfact/mathFact.htm (fluency for all four operations, creates reports on a student) |
|  |  |  | Student <br> Resources: | $\underline{\text { http://www.sheppardsoftware.com/math.htm (fluency for all four operations) }}$ |
|  | Skills: | Multiply, divide, add and subtract based on properties of operations and arithmetic patterns | Assessment: | Fluency Problems <br> Students practice building fluency with multiplication and division facts towards the end of this unit and throughout the rest of this year. <br> Students can also continue to strengthen the fluency of addition and subtraction facts begun in $2^{\text {nd }}$ grade throughout this unit, which will support their work with multi-digit addition and subtraction in the next unit. |

## Prior Knowledge and Experiences

Student knowledge of addition and subtraction will support their learning in this unit; however, these ideas are reinforced throughout. Students should be encouraged to skip count rather than count by one, if they are still struggling with addition and subtraction facts, to encourage multiplicative reasoning. Familiarity with skip counting and repeated addition is beneficial prior to teaching this unit. Students may begin the unit with an understanding of multiplication as repeated addition but a richer interpretation of multiplication is the goal of this unit (e.g., equal groups, arrays, times-as-many). Student exposure to hundreds charts will also provide a strong foundation for finding patterns in the beginning of the unit.

## Learning Experience \# 1

The teacher may provide visuals (e.g., magazines, newspapers, internet) of things that come in groups so that students can brainstorm examples of equal groupings from the real world.
Enactive: Students can find or take pictures of items in equal groups.
Iconic: Students can draw pictures of items in equal groups.
Symbolic: Students can represent their drawings with numeric expressions (e.g., 5+5+5 or $5 \times 3$ ).

| Teacher Notes: | This lesson is a pre-assessment of students' familiarity with equal groups and arrays representing multiplication. The remainder of the unit will build upon the students' current knowledge. |  |
| :---: | :---: | :---: |
| Generalization Connection(s): | Multiplication and division word problems can involve situations of equal groups, arrays, combinations, fair sharing, rate, scaling, area, and unit conversions <br> Compared with addition/subtraction, multiplication and/or division provide highly efficient means to solve equal-group story problems <br> Arrays such that an array of $m$ rows and $n$ columns has $n \times m$ items can model multiplication and division word problems |  |
| Teacher Resources: | http://www.canstockphoto.com/ (website with images such as ice cube trays, octopus, wheels on a bike) What comes in 2's, 3's \& 4's? by Suzanne Aker |  |
| Student Resources: | N/A |  |
| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: <br> Can you have a group of one? (e.g., noses) <br> Why is it important to have equal groups? <br> What is an example of an item that comes in groups of different quantities (e.g., eggs come in 6, 12 and 18)? |  |
| Differentiation: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
| (Multiple means for students to access content and multiple modes for student to express understanding.) | http://www.canstockphoto.com/ (images of equal and unequal groups) http://www.brainpopjr.com/math/multiplicationanddivision/ makingequalgroups/preview.weml (movie about equal groups from BrainPop) | Students can determine if an image shows equal or unequal groups <br> Students can verbally describe what they learned about equal groups and multiplication |
| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | N/A | N/A |
| Key Knowledge and Skills: | - Solve word problems involving equal groups, measurement quantities, and arrays by using multiplication and division <br> - Represent multiplication and division story problems by using drawings and equations with a symbol for the unknown number |  |
| Critical Language: | Equal groups |  |

## Learning Experience \# 2

## The teacher may provide directions for playing a multiplication game (e.g., circles and stars) so that students can begin to connect multiplication to equal groups.

Enactive: Students can roll a number cube to determine the number of groups to create (e.g., draw circles) and then roll a number cube to determine the number of items (e.g., beans) to place in each group.
Iconic: Students can roll a number cube to determine the number of groups to create (e.g., draw circles) and then roll a number cube to determine the number of items (e.g., stars) to draw in each group.
Symbolic: Students can write equations representing the quantity in their drawings using both repeated addition and multiplication (e.g., $6+6+6=3 \times 6$ ).

| Teacher Notes: | Throughout the start of this unit it is important to be consistent with the multiplication equations students write (e.g., number of groups X group size). This consistency provides students with a strong foundation for understanding multiplication and avoids creating misconceptions. The concept of commutativity takes time to develop and should not be rushed. This learning experience provides an opportunity to reinforce the concept of the equal sign as a balance between quantities of the same size (e.g., $6+6+6=3 \times 6$ ). The focus is on connecting equal groups and repeated addition to the concept of multiplication rather than finding the total. |
| :---: | :---: |
| Generalization Connection(s): | Multiplication and division word problems can involve situations of equal groups, arrays, combinations, fair sharing, rate, scaling, area, and unit conversions <br> Compared with addition/subtraction, multiplication and/or division provide highly efficient means to solve equal-group story problems |
| Teacher Resources: | http://teacher.scholastic.com/products/dothemath/pdfs/Sample Teacher Guide Pt1.pdf (Marilyn Burns's Circles and Stars plan) http://www.brainpopjr.com/math/multiplicationanddivision/makingequalgroups/preview.weml (equal groups movie from BrainPop) http://www.k-5mathteachingresources.com/3rd-grade-number-activities.html (multiplication/division array cards, Doorbell Rang, <br> other texts to use for multiplication and division, task cards, tied to each standard) <br> http://exchange.smarttech.com/search.html?q=egg+carton+multiplication\&subject=All+subjects\&grade=All+grades\&region=en US (multiplication array number stories Slide 4 ) <br> http://www.nea.org/tools/tips/Egg-Carton-Multiplication.html (egg carton and bean game) <br> Amanda Bean's Amazing Dream by Cindy Neuschwander |
| Student Resources: | ```http://kidscraftzone.com/post/Egg-Carton-Multiplication.aspx (student friendly site for using egg carton to represent equal groupings) https://grade3commoncoremath.wikispaces.hcpss.org/Math+Vocabulary (CCSS 3rd grade math vocabulary word wall)``` |
| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: <br> What does it mean to be efficient? <br> How do you know that $6+6+6=3 \times 6$ ? <br> Why was multiplication invented? Why not just add? <br> How can you prove that you number sentence matches your picture? <br> Why do the groups need to be equal to use multiplication? <br> What does it mean to have one group? What does it look like in a picture and in a number sentence? <br> What does it mean to be efficient? <br> How can you count equal groups efficiently? |

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| Differentiation: <br> (Multiple means for students to access content and multiple modes for student to express understanding.) | Access (Resources and/or Process) | Expression (Products and/or Performance) |
| :---: | :---: | :---: |
|  | http://www.esc16.net/users/0001/i3/13 2012/HANDOUTS/Q uestioning\%20as\%20a\%20Strategy/blank-cubetemplate.pdf (blank template for creating number cubes to provide number cubes with the only 2,3 , and 4 ) <br> The teacher may provide students with a partially completed graphic organizer- <br> http://wvde.state.wv.us/strategybank/FrayerModel.html (Frayer Model graphic organizer for vocabulary) | Students can create the enactive, iconic and symbolic using an altered number cube <br> Students can use a graphic organizer to complete the symbolic equations <br> Students can create a graphic organizer using the Frayer model to define words such as factor and multiplication |
| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | http://www.esc16.net/users/0001/i3/13 2012/HANDOUTS/Q uestioning\%20as\%20a\%20Strategy/blank-cubetemplate.pdf (blank template for creating number cubes to provide number cubes with the larger numbers) <br> The teacher may provide students with a predetermined amount of beans (e.g., 24 or 48) | Students can create the enactive, iconic and symbolic using an altered number cube <br> Students can equally group a predetermined amount of beans in as many ways as possible <br> Student can represent the visual groupings as multiplication equations with products |
| Key Knowledge and Skills: | - Solve word problems involving equal groups, measurement quantities, and arrays by using multiplication and division <br> - Represent multiplication and division story problems by using drawings and equations with a symbol for the unknown number <br> - Interpret whole-number products and quotients <br> - Solve word problems involving equal groups, measurement quantities, and arrays by using multiplication and division <br> - Represent multiplication and division story problems by using drawings and equations with a symbol for the unknown number |  |
| Critical Language: | Number of groups, equal, group size, number cubes, roll, factors, equal groups, skip count, equation |  |
| Learning Experience \# 3 |  |  |
| The teacher may provide a hundreds chart (authors recommend using a zero to 99 chart) so that students can begin exploring the connection between skip counting, equal groups and multiplication. <br> Enactive: Students can play a skip count game for $2,3,5$, and 10 (e.g., Buzz or Zap, http://spoonful.com/family-fun/buzz). <br> Iconic: Students can shade in different hundreds charts showing the multiples of $2,3,5$, and 10. <br> Symbolic: Students can write repeated addition and multiplication equations for shaded numbers on their hundreds chart, (e.g., $5+5+5=15 ; 3 \times 5=15$ ). |  |  |
| Teacher Notes: | Students should continue to see the first number in the multiplication problem as the number of groups and the second as the size of a group. For example, 5 times 2 is five groups or jumps on the hundreds chart of two while 2 times 5 is two groups or jumps on the hundreds chart of five. It is not necessary for them to formalize commutative property at this time. Students may notice the answers are the same but may not be able to explain why till later in the unit. Using the 0 to 99 chart (rather than 1 to 100) emphasizes the grouping of the multiplies of ten and multiplication by zero. |  |

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| Generalization Connection(s): | Arithmetic patterns, justified by properties of operation, constitute strategies that can be used to multiply and divide |  |
| :---: | :---: | :---: |
| Teacher Resources: | http://www.superteacherworksheets.com/hundredschart/99-chart-filled.pdf (0-99 chart) http://spoonful.com/family-fun/buzz (Buzz game instructions) |  |
| Student Resources: | http://resources.woodlands-junior.kent.sch.uk/maths/interactive/numbers.htm (spooky sequence games to reinforce skip counting) |  |
| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: What patterns do you see in each chart and why? <br> Why would you shade zero on each chart? <br> Why are 0 and 1 special in multiplication? <br> Which numbers are shaded in for more than one chart and why? <br> How can you find a product to a multiplication problem using a hundreds chart? <br> How is it different to find $5 \times 2$ versus $2 \times 5$ ? |  |
| Differentiation: <br> (Multiple means for students to access content and multiple modes for student to express understanding.) | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | http://www.abcya.com/one hundred number chart game.h $t \mathrm{tm}$ (visual of image of multiples of two on a hundreds chart) | Students can complete the missing numbers on a hundreds chart and orally describe the pattern of twos |
| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | ```http://www.superteacherworksheets.com/hundredschart/99- chart-filled.pdf (0-99 charts with shaded multiples of 2, 3, 5,10s)``` | Students can use their $2,3,5$, and 10 charts and write predictions regarding which charts have all the multiples of 4,6 and 8 shaded and explain why |
| Key Knowledge and Skills: | - Identify and explain patterns in arithmetic (including patterns in addition and multiplication tables) using properties of operations |  |
| Critical Language: | Multiples, product, skip counting, hundreds chart, multiplication |  |
| Learning Experience \# 4 |  |  |
| The teacher may use a children's story (e.g., The Doorbell Rang by Pat Hutchins) to introduce the concept of fair share (group size unknown) so that students can explore dividing a quantity into equal groups and its connection to multiplication. <br> Enactive: Students can be given twenty-four objects to share among 1, 2, 3, 4, 6, 8, 12, and 24 people (i.e., make equal groups). <br> Iconic: Students can draw the groups created for each fair share situation and circle each group to emphasize the number of equal groups in relation to the group size. Symbolic: Student can write both the division and multiplication problem represented by each drawing (e.g., $24 \div 2=12$ and $2 \mathrm{X} 12=24$ ). |  |  |
| Teacher Notes: | This is the first learning experience introducing the concept of division. It is helpful to introduce only fair share (group size unknown) division contexts in this learning experience. The concept of the number of groups unknown for division is developed later in the unit. It is important to continue to emphasize what each number of a multiplication and division equation means in relation to a context and stay consistent with multiplication problems representing number of groups X group size to avoid confusion in students. |  |

## Colorado Teacher-Authored Sample Instructional Unit

| Generalization Connection(s): | Multiplication and division word problems can involve situations of equal groups, arrays, combinations, fair sharing, rate, scaling, area, and unit conversions <br> Because multiplication and division are inverse operations, multiplication provides and effective means to solve division problems as unknown factor problems <br> Division enables decision-making determinations regarding the size of groups or the number of groups in a given context |  |
| :---: | :---: | :---: |
| Teacher Resources: | The Doorbell Rang by Pat Hutchins <br> Spunky Monkeys on Parade by Stuart J. Murphy <br> http://www.k-5mathteachingresources.com/3rd-grade-number-activities.html (multiplication extensions and texts) <br> http://www.k-5mathteachingresources.com/support-files/countingcollections.pdf (counting collections) <br> http://investigations.terc.edu/library/components/childrenslit3-5 2ed.pdf (list of children's literature related to investigations math) <br> http://exchange.smarttech.com/search.html?q=fair+shares+ (Smartboard activities for fair share activities) |  |
| Student Resources: | N/A |  |
| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: What strategy do you use to solve fair share problems? <br> How do know when you have shared fairly? <br> How are multiplication and division related? <br> When you write out the division equation for a fair share problem, what does the answer tell you? |  |
| Differentiation: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
| (Multiple means for students to access content and multiple modes for student to express understanding.) | The teacher may provide students with graphic organizers that provide descriptors underneath the multiplication and division equations (i.e., labeling "groups" and "number of items in the group") | Student can complete the graphic organizers with appropriate quantities |
| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | N/A | Students may create fair share problems with larger numbers (i.e., greater than 24) and explore numbers that cannot be shared fairly |
| Key Knowledge and Skills: | - Solve word problems involving equal groups, measurement quantities, and arrays by using multiplication and division <br> - Represent multiplication and division story problems by using drawings and equations with a symbol for the unknown number <br> - Determine the unknown whole number in a multiplication or division equation relating three whole numbers <br> - Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division |  |
| Critical Language: | Fair share, products, equal groups |  |

## Learning Experience \# 5

The teacher may provide word problems where the number of groups is unknown (e.g., how many times do you need to jump two feet to reach ten feet) to motivate the use of a number line model so that students can develop additional strategies for solving multiplication and division problems.
Enactive: Students can solve number of groups unknown division problems by jumping on a student size number line.
Iconic: Students can draw jumps on a number line and count the number of jumps to determine the number of groups
Symbolic: Student can create a division and multiplication equation based on their number line representations (e.g., $10 \div 2=5 ; 5 \times 2=10$ ).

| Teacher Notes: | This learning experience introduces the second type of division. Students should begin to notice that the answer for these types of <br> problems is the number of groups. By remaining consistent with the structure of the multiplication problems (number of groups X <br> group size) students can notice the difference in location of the quotient and divisor in the original multiplication problem in <br> comparison to the fair share problems. |
| :--- | :--- |
| Generalization Connection(s): | Because multiplication and division are inverse operations, multiplication provides and effective means to solve division problems as <br> unknown factor problems <br> Compared with addition/subtraction, multiplication and/or division provide highly efficient means to solve equal-group story <br> problems <br> The comparison of the size of a collection against the size of a group reflects multiplication and division problems related to the <br> concept of "times as many" or "times fewer" <br> Division enables decision-making determinations regarding the size of groups or the number of groups in a given context |
| Teacher Resources: | https://www.singaporemath.com/v/vspfiles/assets/images/sp emft 1.6.pdf ("group size unknown" and "number of groups <br> unknown" division problems) <br> http://langfordmath.com/ECEMath/Multiplication/DivModelsSolutions.html ("group size unknown" and "number of groups <br> unknown" division problems) <br> http://www.k-5mathteachingresources.com/empty-number-line.html (explanation of open number line) |
| Student Resources: | https://grade3commoncoremath.wikispaces.hcpss.org/Assessing+3.OA.8 (assessment and performance tasks) <br> http://exchange.smarttech.com/search.html?q=multiplication+word+problems\&subject=All+subjects\&grade=All+grades\&region=en <br> US (SMART board activities for multiplication word problems) |
| Assessment: | Divide and Ride by Stuart Murphy and George Ulrich |
| Students mastering the concept and skills of this lesson should be able to answer questions such as: |  |
| What strategies are you using to solve this problem? |  |
| What is the important information needed to solve this problem? |  |
| How do you determine that you have the correct amount of groups? |  |
| When you write out the division equation for a "number of groups unknown" problem, what does the answer tell you? |  |
| What are the differences and similarities between the two types of division? |  |

## Colorado Teacher-Authored Sample Instructional Unit

| Differentiation: <br> (Multiple means for students to access content and multiple modes for student to express understanding.) | Access (Resources and/or Process) | Expression (Products and/or Performance) |
| :---: | :---: | :---: |
|  | The teacher may provide the students with closed number lines <br> http://www.math-aids.com/Number Lines/ (create your own number lines) | N/A |
| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | N/A | Students can create a context for writing two division problems (one "group size unknown" and one "number of groups unknown") |
| Key Knowledge and Skills: | - Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding |  |
| Critical Language: | Multiplication, division, open number line |  |
| Learning Experience \# 6 |  |  |
| The teacher may provide several representations of a multiplication problem (e.g., equation, words problems, visuals) so that students can deepen their understanding of multiplication by matching the corresponding representations. |  |  |
| Generalization Connection(s): | Because multiplication and division are inverse operations, multiplication provides and effective means to solve division problems as unknown factor problems <br> Division enables decision-making determinations regarding the size of groups or the number of groups in a given context |  |
| Teacher Resources: | http://www.jennyray.net/uploads/1/2/9/7/12975776/3rd grade multiplication.pdf (formative assessment plans with "matching" card game that reflects multiple representations of multiplication) |  |
| Student Resources: | N/A |  |
| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: How are multiplication and division related? <br> How do your pictures and words represent the equation? <br> What does the product (quotient) represent in the context? |  |
| Differentiation: <br> (Multiple means for students to access content and multiple modes for student to express understanding.) | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | Teachers may provide students with a blank 3 flap flip book. http://www.readwritethink.org/classroom-resources/student-interactives/flip-book-30054.html (interactive flip book creator) | Students can create a flip book for one (teacher provided) multiplication equation that presents a drawing that shows equal groups, a repeated addition equation for the drawing, and a connected word problem |

## Colorado Teacher-Authored Sample Instructional Unit

| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
| :---: | :---: | :---: |
|  | Teachers may provide students with a blank 3 flap flip book. http://www.readwritethink.org/classroom-resources/student-interactives/flip-book-30054.html (interactive flip book creator) | Students can create a flip book for (teacher provided) division equations (one "group size unknown" and one "number of groups unknown")that presents two drawings and two connected word problem |
| Key Knowledge and Skills: | - Solve word problems involving equal groups, measurement quantities, and arrays by using multiplication and division <br> - Represent multiplication and division story problems by using drawings and equations with a symbol for the unknown number <br> - Determine the unknown whole number in a multiplication or division equation relating three whole numbers <br> - Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding |  |
| Critical Language: | Number sentence, equation, equal groups, Inverse operations, division |  |
|  |  |  |
| Learning Experience \# 7 |  |  |
| The teacher may use a children's story (e.g., One Hundred Hungry Ants by Elinor Pinczes) involving array connect the concept of arrays to multiplication and division. <br> Enactive: Students can arrange 12 manipulatives to demonstrate the different ways they can be arranged in equal rows. Iconic: Students can draw representations of the different arrangements of equal rows. <br> Symbolic: Students can label their representations with equations showing repeated addition, multiplication, and division. |  |  |
| Teacher Notes: | This is the first learning experience connecting arrays to students developing understanding of multiplication and division. Students often struggle to coordinate row and column structure in an array. It may be helpful for students to connect their work with equal groups to the concept of row and columns. For instance, students might see $m$ rows and $n$ columns as $n \mathrm{Xm}$ and also as n groups of $X \mathrm{~m}$ sized groups. |  |
| Generalization Connection(s): | Multiplication and division word problems can involve situations of equal groups, arrays, combinations, fair sharing, rate, scaling, area, and unit conversions <br> Arrays such that an array of $m$ rows and $n$ columns has $n \times m$ items can model multiplication and division word problems Because multiplication and division are inverse operations, multiplication provides and effective means to solve division problems as unknown factor problems <br> Fluency with multiplication and division facts results from multiple experiences with different models, representations, problem types, properties of operations and interrelationships among multiplication and division facts |  |
| Teacher Resources: | http://www.k-5mathteachingresources.com/support-files/x5x10wordproblems.pdf (Number story array problems) <br> http://www.k-5mathteachingresources.com/support-files/arraypicturecards.pdf (array pictures) <br> http://www.k-5mathteachingresources.com/3rd-grade-number-activities.html <br> (resources for teaching arrays and connections to literature) |  |
| Student Resources: | N/A |  |

## Colorado Teacher-Authored Sample Instructional Unit

| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: How are multiplication and division related? <br> What is an array? What are rows and columns? <br> How is an array a model for multiplication? <br> How does your equation explain an array? <br> How does an array help organize quantities into equal groups? |  |
| :---: | :---: | :---: |
| Differentiation: <br> (Multiple means for students to access content and multiple modes for student to express understanding.) | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | http://www.guided-math.com/2012/01/math-game-racing- <br> rectangles.html (game that reinforces concepts of arrays) http://highered.mcgraw- <br> hill.com/sites/0072532947/student view0/grid and dot paper.html (grid paper templates) | Students can play the game to reinforce conceptual understanding of arrays <br> Students may use grid paper to create arrays |
| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | N/A | Students can create row and column statements for the arrays they have created |
| Key Knowledge and Skills: | - Interpret whole-number products and quotients <br> - Solve word problems involving equal groups, measurement quantities, and arrays by using multiplication and division <br> - Represent multiplication and division story problems by using drawings and equations with a symbol for the unknown number <br> - Determine the unknown whole number in a multiplication or division equation relating three whole numbers <br> - Understand division as an unknown-factor problem |  |
| Critical Language: | Equation, array, value, compose |  |

## Learning Experience \# 8

## The teacher may provide grid paper and a bag of 36 square tiles so that students can use their knowledge of arrays to find all the arrays formed by 36 squares.

Enactive: Students can arrange color tiles to form rectangular arrays of 36 tiles.
Iconic: Students can show the arrangements on grid paper by shading in rectangular arrays.
Symbolic: Students can label their arrays with a multiplication equation (e.g., columns X rows).

## Generalization Connection(s):

Because multiplication and division are inverse operations, multiplication provides and effective means to solve division problems as
unknown factor problems
Arithmetic patterns, justified by properties of operation, constitute strategies that can be used to multiply and divide
Fluency with multiplication and division facts results from multiple experiences with different models, representations, problem
types, properties of operations and interrelationships among multiplication and division facts

## Colorado Teacher-Authored Sample Instructional Unit

| Teacher Resources: | http://www.pinterest.com/pin/129478558008865853/ (Marilyn Burns candy box-pinterest)http://teach-and-tech.blogspot.com/2012/01/candy-box-array-math.html (candy box array)http://highered.mcgraw-hill.com/sites/0072532947/student view0/grid and dot paper.html (grid paper templates) |  |
| :---: | :---: | :---: |
| Student Resources: | One Hundred Hungry Ants by Elinor Pinczes |  |
| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: How many different ways can you arrange (36) tiles to form rectangular arrays? |  |
| Differentiation: <br> (Multiple means for students to access content and multiple modes for student to express understanding.) | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | N/A | Students can use a smaller number of tiles to create the arrays |
| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | N/A | Students can create corresponding division problems for their arrays |
| Key Knowledge and Skills: | - Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division <br> - Model strategies to achieve a personal financial goal using arithmetic operations <br> - Understand division as an unknown-factor problem |  |
| Critical Language: | Rectangular arrays |  |

## Learning Experience \# 9

## The teacher may provide grid paper and scissors so that students can find all the possible row and column structures for each

 number from 1 to 10.Enactive/Iconic: Students can find the arrays for each number from 1 to 10 on graph paper and cut out them out.
Symbolic: Students can write the multiplication expression inside each array and compare arrays with same numbers (e.g., $5 \times 2$ and $2 \times 5$ ).

| Teacher Notes: | This learning experience lays the foundation for the Commutative Property (i.e., 3 rows of 5 and 5 rows of 3 can be seen from the same rectangle by rotating 90 degrees). |
| :---: | :---: |
| Generalization Connection(s): | Arithmetic patterns, justified by properties of operation, constitute strategies that can be used to multiply and divide Fluency with multiplication and division facts results from multiple experiences with different models, representations, problem types, properties of operations and interrelationships among multiplication and division facts <br> Arrays such that an array of $m$ rows and $n$ columns has $n \times m$ items can model multiplication and division word problems |
| Teacher Resources: | http://www.k-5mathteachingresources.com/3rd-grade-number-activities.html (multiplication extensions and texts) <br> http://www.doe.virginia.gov/testing/solsearch/sol/math/3/mess 3-20ab 2.pdf (commutative property) <br> http://www.risd.k12.nm.us/assessment evaluation/ImprovSBAscores/3rdgrade/Commutative\%20property\%20of\%20multiplication. <br> pdf (commutative property) <br> http://www.teacherspayteachers.com/Product/Commutative-Property-of-Multiplication-BOOK-good-read-aloud-170550 (teacher <br> created book for read aloud) <br> http://highered.mcgraw-hill.com/sites/0072532947/student view0/grid and dot paper.html (grid paper templates) |

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## Learning Experience \# 10

The teacher may utilize the arrays in the previous learning experience so that students can begin to explore the concept of the distributive property by composing new arrays.
Enactive/Iconic: Students can fit together arrays with the same number of row or columns to create new rectangles.
Symbolic: Students can write equations to represent their composed arrays (e.g., $3 \times 5=(2 \times 5)+(1 \times 5))$.

| Teacher Notes: |
| :--- |
| Generalization Connection(s): |
|  |
|  |
|  |

Students are developing an understanding of the distributive property of multiplication by composing arrays. The composition of arrays can be used to support the development of multiplication facts by building unknown facts from known facts.

Multiplication and division word problems can involve situations of equal groups, arrays, combinations, fair sharing, rate, scaling, area, and unit conversions
Arithmetic patterns, justified by properties of operation, constitute strategies that can be used to multiply and divide
Fluency with multiplication and division facts results from multiple experiences with different models, representations, problem types, properties of operations and interrelationships among multiplication and division facts

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| Teacher Resources: | http://www.k-5mathteachingresources.com/3rd-grade-number-activities.html (multiplication extensions and texts) <br> http://www.mathsisfun.com/definitions/distributive-law.html (laws of multiplication illustrated for students) <br> http://www.nsa.gov/academia/ files/collected learning/elementary/arithmetic/add the spread.pdf (extension lessons for distributive property) <br> http://highered.mcgraw-hill.com/sites/0072532947/student view0/grid and dot paper.html (grid paper templates) http://exchange.smarttech.com/details.html?id=7309ef15-d6f7-41fa-9ae1-7ce228af39ba (SMARTboard presentation on the properties of multiplication) |  |
| :---: | :---: | :---: |
| Student Resources: | http://www.k-5mathteachingresources.com/support-files/Split-a-Factor.pdf (student center activity for splitting factors) |  |
| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: <br> How would you record your new arrays as an equation? <br> Why might it be helpful to break apart arrays when solving multiplication problems? <br> How does composing arrays help you relate addition and multiplication? |  |
| Differentiation: <br> (Multiple means for students to access content and multiple modes for student to express understanding.) | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | Teacher may provide students with smaller arrays | Students can work with partners to construct new arrays |
| Extensions for depth and complexity: | Access (Resources and/or Process) | Expression (Products and/or Performance) |
|  | N/A | Students can be challenged to find the largest array they can create with their collection of arrays |
| Key Knowledge and Skills: | - Identify and explain patterns in arithmetic (including patterns in addition and multiplication tables) using properties of operations <br> - Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division <br> - Interpret whole-number products and quotients |  |
| Critical Language: | Factors, products, array, multiplication, addition, Commutative Property, Distributive Property, Associative Property |  |

## Learning Experience \# 11

The teacher may have students use the arrays created in the previous two learning experiences so that student can begin constructing their own multiplication chart.
Iconic/Symbolic: Students can use their arrays and composition of arrays to fill in the multiplication chart.

## Teacher Notes:

A multiplication chart is an array model. If the squares from the arrays students create are the same size as the multiplication chart they can place them directly on the chart otherwise they can use their arrays to find the same number of rows and columns on the chart and find the product. After students fill in the chart using their arrays, they may also have other answers on the chart that are known products for them which can be added. Students can use this chart as a starting point for gaining fluency. By working with just a portion of the chart at a time and using known facts to learn unknown facts students build fluency throughout the school year while continuing to develop an understanding of multiplication and division. As students work to add facts to their tool box it is helpful to reinforce both the multiplication and division relationship and their connection to known facts by drawing composed arrays.

## Colorado Teacher-Authored Sample Instructional Unit

| Generalization Connection(s): | Multiplication and division word problems can involve situations of equal groups, arrays, combinations, fair sharing, rate, scaling, area, and unit conversions <br> Division enables decision-making determinations regarding the size of groups or the number of groups in a given context Arrays such that an array of $m$ rows and $n$ columns has $n \times m$ items can model multiplication and division word problems Because multiplication and division are inverse operations, multiplication provides and effective means to solve division problems as unknown factor problems <br> Arithmetic patterns, justified by properties of operation, constitute strategies that can be used to multiply and divide Fluency with multiplication and division facts results from multiple experiences with different models, representations, problem types, properties of operations and interrelationships among multiplication and division facts |
| :---: | :---: |
| Teacher Resources: | http://www.k-5mathteachingresources.com/3rd-grade-number-activities.html (multiplication extensions and texts) <br> http://www.mathsisfun.com/definitions/distributive-law.html (laws of multiplication illustrated for students) <br> http://exchange.smarttech.com/details.html?id=7309ef15-d6f7-41fa-9ae1-7ce228af39ba (SMARTboard presentation on the properties of multiplication) <br> http://www.superteacherworksheets.com/multiplication/multiplication-chart-blank-9 TZBFZ.pdf (0-9 blank multiplication chart) http://www.teacherspayteachers.com/Browse/Search:multiplication+tables (printable multiplication chart) |
| Student Resources: | http.//www.k-Smathteachingresources.com/support-fles/Splita-Factor.pdf (student center activity for |
| Assessment: | Students mastering the concept and skills of this lesson should be able to answer questions such as: What relationships are you seeing between the arrays and the multiplication chart? <br> Do you notice any relationships between the facts you know fluently and those you still need to learn? If you move the array around the chart, does it produce the same product every time? |
| Differentiation | Access (Resources and/or Process) ${ }^{\text {a }}$ ( Expression (Products and/or Performance) |
| (Multiple means for students to access content and multiple modes for student to express understanding.) | N/A Students can use the arrays from Learning Experience \#9 (arrays for <br> $\# 1-10) ~ t o ~ f i l l ~ i n ~ a s ~ m u c h ~ o f ~ t h e ~ c h a r t ~ a s ~ p o s s i b l e ~$ |
| Extensions for depth and complexity: | Access (Resources and/or Process) ${ }^{\text {a }}$ ( Expression (Products and/or Performance) |
|  | N/A $\quad$Students can complete the chart by decomposing and composing <br> arrays created in Learning Experiences \#9 and \#10 |
| Key Knowledge and Skills: | - Interpret whole-number products and quotients <br> - Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division <br> - Identify and explain patterns in arithmetic (including patterns in addition and multiplication tables) using properties of operations |
| Critical Language: | Factors, products, array, multiplication, fluency, commutative property, distributive property, associative property |


[^0]:    *Denotes connection to Personal Financial Literacy (PFL)

