In preparation for the 2018 review and revision of the Colorado Academic Standards, the Colorado Department of Education invited the public to submit comments and suggestions through an online feedback system that was open from October 2016 through April 2017. Below are the unedited comments and suggestions that were submitted for Mathematics, with the exception of the redaction of personally identifiable information. Each feedback item is prefaced with either Revise, Remove, Move, or Comment. Only those parts of the standards (Prepared Graduate Competencies, Grade Level Expectations, Evidence Outcomes, etc.) that received feedback are listed below, and as such, some PGCs/GLEs/EOs are missing from this document. The footnotes in the Colorado Academic Standards are also not included in this document. For more information on the Colorado Academic Standards and the 2018 review and revision, visit http://www.cde.state.co.us/standardsandinstruction.

Prepared Graduate Competencies

PGC: Apply transformation to numbers, shapes, functional representations, and data

Revise: I don't think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at an engineering level.

Comment: Keep the same.

Comment: Leave this as is.

Comment: Keep the same.

Comment: Clear to me

Comment: Keep the standards the same.

Comment: I know teachers still aren’t as familiar with these standards as they need to be to assess whether they are effective so overall in general I think the less changes you all make with this revision timeline the better. Give teachers a chance to let the updates and changes made play out and truly be implemented to see the impact on student learning instead of giving teachers a constantly moving target. I am making this comment within the math standards specifically as I know for a fact the teachers in our region are still not anywhere near knowing their standards well, unpacking them, trusting that their colleagues before and after are following them...etc. but it applies to all content areas, not just math.

Revise: Recognizing congruency is important, but transforming shapes isn't a real-world scenario.

Revise: The student will not be required to participate in service projects where the student must provide unpaid services to progressive organizations, the Democrat party, the Green party, etc. when applying transformation to numbers, shapes, functional representations, and data MAKING CITIZENS: HOW AMERICAN UNIVERSITIES TEACH CIVICS WITH CASE STUDIES OF THE UNIVERSITY OF COLORADO, BOULDER; COLORADO STATE UNIVERSITY; UNIVERSITY OF NORTHERN COLORADO; AND THE UNIVERSITY OF WYOMING The New Civics hopes to accomplish this by teaching students that a good citizen is a radical activist, and it puts political activism at the center of everything that students do in college, including academic study, extracurricular pursuits, and off-campus ventures. New Civics builds on “service-learning,” which is an effort to divert students from the classroom to vocational training as community activists. By rebranding itself as “civic engagement,” service earning succeeded in capturing nearly all the funding that formerly supported the old civics. In practice this means that instead of teaching college students the foundations of law, liberty, and self-government, colleges teach students how to organize protests, occupy buildings, and stage
demonstrations. These are indeed forms of "civic engagement," but they are far from being a genuine substitute for learning how to be a full participant in our republic https://www.nas.org/images/documents/NAS_makingCitizens_executiveSummary.pdf

**Comment:** The comment applies to ALL PGCs: The wording of the twelve prepared graduate competencies seem to shift from specific mathematical concepts (such as transformation) to ways of thinking (use critical thinking, communicate effective arguments). A stronger structure might be to base the PGCs on the Standards for Mathematical Practice and allow the specific mathematics to be addressed at the grade band where it is most relevant in creating a cohesive, comprehensive understanding of mathematics.

**PGC: Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency**

**Revise:** Who decides what is "basic". Is there a right method to choose? Is it wise to combine operations and algorithms into the same statement?

**Comment:** Keep the same.

**Comment:** Let this as it is.

**Comment:** Keep the same.

**Comment:** Keep the standards the same.

**Comment:** There seems to be an inability to memorize and retain information such as math facts in this generation. With technology available...why not teach them how to use a calculator and allow it, (once the understand the foundation of what multiplication, division, addition and subtraction is.)

**Revise:** There is far too much Khan Academy, Moby Max, etc used as homework. Studies have proven that the best way to learn math is to use a pencil and paper. Worksheets allow students to work through the problem and connect HOW to work through the problem instead of clicking on an answer. More time needs to be spend on math facts, particularly multiplication.

**PGC: Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking**

**Comment:** Keep the same.

**Comment:** Keep the same.

**Comment:** Keep the same.

**Comment:** Keep the standards the same.

**Comment:** Most Kids are still building foundational knowledge and not developmentally capable of doing this reasoning at age 10.

**Comment:** Proofs are important. They help teach students to think and reason logically and to back up their arguments with sound arguments.

**Revise:** The student will ensure that the student does not use papers which had an invalid methodology or which had been retracted when The student will ensure that the student does not use papers which had an invalid methodology or which had been retracted when communicating effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking From National Academy of Science, A growing body of evidence indicates that substantial percentages of published results in some fields are not reproducible, the report says, noting that this is a complex phenomenon and much remains to be learned. While a certain level of irreproducibility due to unknown variables or errors is a normal part of research, data falsification and detrimental research practices -- such as inappropriate use of statistics or after-the-fact fitting of hypotheses to previously collected data -- apparently also play a role. In addition, new forms of detrimental research practices are appearing, such as predatory journals that do little or no editorial review or quality
control of papers while charging authors substantial fees. And the number of retractions of journal articles has increased, with a significant percentage of those retractions due to research misconduct. The report cautions, however, that this increase does not necessarily indicate that the incidence of misconduct is increasing, as more-vigilant scrutiny by the community may be a contributing factor. http://www8.nationalacademies.org/onpinews/newsitem.aspx?RecordID=21896&utm_source=NASEM+News+and+Publications&utm_campaign=15d988f9b6-NAP_mail_new_2017.04.17&utm_medium=email&utm_term=0_96101de015-15d988f9b6-104332205&goal=0_96101de015-15d988f9b6-104332205&mc_cid=15d988f9b6&mc_eid=0f163c1e89 http://physics.nyu.edu/sokal/franklin_review_BtH.pdf Compounding Error: The Afterlife of Bad Science Authors Authors and affiliations Jaime A. Teixeira da Silva Email author Judit Dobránszki Judit Dobránszki, Institutes for Agricultural Research and Educational Farm of the University of Debrecen The failure to discover and correct errors in published scientific papers “poses significant risks for authors, editors, journals, and publishers” as well as for the wider academic pool and the public, and weakens reader and peer confidence in the credibility of scientists and their research. When errors in the published scientific literature are discovered they must be reported, and corrections made “quickly and completely,” urge Jaime A. Teixeira da Silva and Judit Dobránszki, who lay out the case for strengthening post-publication peer review. https://www.nas.org/articles/spring_2017_academic_questions http://link.springer.com/article/10.1007%2Fs12129-017-9621-0 Article: Highly cited retracted papers ABSTRACT: We examine the number of citations in 10 highly cited retracted papers, and compare their current pre- and post-citation values. We offer some possible explanations for the continued citation of these retracted papers, and point out some of the risks that may be involved for the communities that continue to cite them. In general, retracted papers should not be cited, but often there is fault with unclear publisher web-sites, the existence of pirate web-sites or sites that display copies of the unretracted version of the paper, or even the insistent citation of a retracted paper because the results remain valid, or because the authors (most likely) refuse to accept the retracted status of that paper, or continue to believe that the core findings of the study remain valid. Article · Mar 2017 · Scientometrics https://www.researchgate.net/profile/Judit_Dobranszki

PGC: Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

Revisit: I don’t think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at an engineering level.

Comment: Keep the same.

Comment: Keep the same.

Comment: Keep the same.

Comment: Keep the same.

PGC: Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Comment: Keep the same.

Comment: Keep the same.

Comment: Keep the same.

Comment: Keep the standards the same.

Comment: It is important that students can identify relationships, but memorizing names is not. They should be able to apply properties (identity, commutative, etc.) but don’t really need to have the names memorized.
PGC: Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Revise: Our pre-k through third grade teachers feel that patterns should be reinstated in CAS Math Standard Two as supported by experience, expertise, and recent research out of Vanderbilt University (March 2015). https://news.vanderbilt.edu/2015/03/18/common-core-math-should-include-pattern-abstraction/

Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the same.

Revise: Students will use models and exercises designed to calibrate judgment for making predictions. https://www.nytimes.com/2015/10/18/books/review/mindware-and-superforecasting.html?_r=0

PGC: Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Revise: I don't think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at an engineering level.

Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the same.

Comment: Keep the standards the same.

PGC: Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the same.

Comment: Keep the standards the same.

PGC: Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the same.

Comment: Keep the standards the same.

PGC: Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Revise: I don't think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at this level.

Comment: This is one of few competencies that reference algebra - it seems like there is over-representation of geometry and statistics in these graduate competencies as compared to the identified major content areas in the standards themselves. A look at alignment would be helpful.

Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the standards the same.

PGC: Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the standards the same.

PGC: Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Revise: The wording on this is good. However, Problem Solving and Critical Thinking are two of the most commonly referenced PWR skills that are missing in high school graduates. In talking with Employers, it is often identified that the missing piece is the ability to utilize problem solving and critical thinking skills in real world situations. Students don't know how to recognize what the problem is and then can't apply the models they learned to solve the problem. More real world experience and application is needed to help our students become PWR.

Comment: I am the [position redacted] of the State's Division of Housing. An analysis/report we had done last year, showed Black and Hispanic mortgage applicants making between 60 — 75k a year were denied at about twice the rate as non-Hispanic applicants in Colorado. https://drive.google.com/file/d/0B-vz6H4k45E5by1rWWxzbU9mN2c/view In talking with housing counselors, they report folks are ruining their credit in their 20s. I am not sure the time/place to start this conversation, so I submitting this comment. I am curious what curriculum exists around "life finances". Is there something already being provided in the last-mandatory-math-class about banking, credit, saving, and what is necessary to buy a home? I am not an educator, so I am interested in hearing what might work best to address this challenge. Thank you for your consideration. [name redacted]

Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the same.
Comment: Keep the standards the same.

Comment: Let’s get the foundations solid before we require a 10 year old brain to critically think.

Revise: The study should emphasis that there are psychological flaws in thinking. Thus one should seek to understand the psychological flaws in thinking and adjust the process to eliminate error to the extent possible. Students will use the questions at the end of the chapter question potential psychological flaws in thinking. http://www.nytimes.com/2011/11/27/books/review/thinking-fast-and-slow-by-daniel-kahneman-book-review.html Also, one should recognize when some models fail for specific purposes. People can be fooled by randomness. https://plus.maths.org/content/os/issue20/reviews/book1/index http://www.fooledbyrandomness.com/FatTails.html

Mathematics High School

Standard: 1. Number Sense, Properties, and Operations

GLE: 1. The complex number system includes real numbers and imaginary numbers

Remove: Not essential for post-secondary and career success. Only essential for those going into advanced mathematics later. No real-world context. No application for almost all students.
Evidence Outcome: Extend the properties of exponents to rational exponents. (CCSS: N-RN)
- Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.1 (CCSS: N-RN.1)
- Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS: N-RN.2)

Revise: I applaud our state's use of the Common Core Standards as a baseline for our standards and highly recommend we keep them as our foundation. I would be happy if we chose to add rigor above the current standards, but we must not reduce expectations below the current level if we want our students to be workforce ready. The Common Core allows us to tie into PWR Standards that have been set by business organizations such as the National Network of Business and Industry Associations and the Business Roundtable. Further, businesses have indicated that it is important to be able to know how graduates in one state compare to graduates in another when they are choosing where to locate their offices - the state with the best workforce almost always wins in these decisions. Finally, leveraging common standards across states is a fair approach to support families with school age children that move between states, such as those in our military.

Evidence Outcome: Extend the properties of exponents to rational exponents. (CCSS: N-RN)
- Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.1 (CCSS: N-RN.1)
- Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS: N-RN.2)

Remove: No real-world context. Not essential for almost all post-secondary and career options. Most students would never need to know it. Good to understand square-root and cube-root relationships especially as it relates to 2-d and 3-d measurements, but only a surface literacy of radical exponents is really important.

Evidence Outcome: Use properties of rational and irrational numbers. (CCSS: N-RN)
- Explain why the sum or product of two rational numbers is rational. (CCSS: N-RN.3)
- Explain why the sum of a rational number and an irrational number is irrational. (CCSS: N-RN.3)
- Explain why the product of a nonzero rational number and an irrational number is irrational. (CCSS: N-RN.3)

Remove: The expectation is that students use the properties but the bulleted statements are about explaining why a property is true rather than using the property

Evidence Outcome: Use properties of rational and irrational numbers. (CCSS: N-RN)
- Explain why the sum or product of two rational numbers is rational. (CCSS: N-RN.3)
- Explain why the sum of a rational number and an irrational number is irrational. (CCSS: N-RN.3)
- Explain why the product of a nonzero rational number and an irrational number is irrational. (CCSS: N-RN.3)

Remove: Not essential. Not necessary for post-secondary or career success. Almost all students will never need to know this. Eliminate.

Evidence Outcome: Perform arithmetic operations with complex numbers. (CCSS: N-CN)
- Define the complex number \( i \) such that \((i^2 = -1)\), and show that every complex number has the form \( a + bi \) where \( a \) and \( b \) are real numbers. (CCSS: N-CN.1)
- Use the relation \((i^2 = -1)\) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (CCSS: N-CN.2)

Remove: Students do not need to know this for post-secondary and career success unless in a limited strand of mathematics, physics, or engineering. Only applicable to higher math such as calculus. No real-world context. Eliminate.

Evidence Outcome: Use complex numbers in polynomial identities and equations. (CCSS: N-CN)
- Solve quadratic equations with real coefficients that have complex solutions. (CCSS: N-CN.7)
Remove: Eliminate. Standards should represent the fundamental core knowledge needed for post-secondary AND career success. Students can learn to be complex problem-solvers and rich thinkers without this standard that has no real-world context. Eliminate.

Inquiry Questions:

Revise: It is critical that all of our standards have identified 21st Century Skills and Readiness Competencies. I think it is great that the Math standards have them and feel they must be included in all of our standards. Students will not be successful in their transition to Post-secondary or the Workforce without the ability to apply what they have learned in a real-world setting. Relevance and Application is, therefore, critical to their success.

Remove: The number system in high school should focus on skills applicable to financial literacy, data analysis, statistics, making inferences and predictions, numbers in science and current events, numbers used computationally ("big data" and number crunching algorithms, coding). Not rational/irrational, complex and imaginary. This section does not represent 21st century mathematics. Please consider rewriting this section.

Relevance and Application:

Remove: No real-world context. Move into an elective course.

Nature Of:

Comment: I love the standards of mathematical practice. I argue that these can be taught using mathematics that is relevant and personal to what most high school students will actually experience during and after high school. Imaginary numbers, complex numbers, rational exponents, and properties of number sets shouldn't be the focus here. Focus should be on mathematics involved in interest / tax / discount / ledger and spreadsheet skills, computational analysis, predictions and numeric literacy in scientific and social studies writing.

GLE: 2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations

Evidence Outcome: Reason quantitatively and use units to solve problems (CCSS: N-Q)

- Use units as a way to understand problems and to guide the solution of multi-step problems. (CCSS: N-Q.1)
- Choose and interpret units consistently in formulas. (CCSS: N-Q.1)
- Choose and interpret the scale and the origin in graphs and data displays. (CCSS: N-Q.1)
- Define appropriate quantities for the purpose of descriptive modeling. (CCSS: N-Q.2)
- Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (CCSS: N-Q.3)
- Describe factors affecting take-home pay and calculate the impact (PFL)
- Design and use a budget, including income (net take-home pay) and expenses (mortgage, car loans, and living expenses) to demonstrate how living within your means is essential for a secure financial future (PFL)

Revise: Suggest adding "create meaningful data displays using computational algorithms, spreadsheets and/or other data analysis software". Making a meaningful data display goes beyond scale and origin - it can involve a design process in which students need to understand their audience and create a display that will persuade, convince or aid in understanding.

Relevance and Application:

Revise: If literacy in scientific notation is a high school standard, include it in the standards and not just in this "application" section. Students in sciences also learn about significant figures, unit conversions and relationships between units - the mathematical connections and context are strong here.
Standard: 2. Patterns, Functions, and Algebraic Structures

GLE: 1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables.

(PGC Feedback) Revise: I don't think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at an engineering level.

Revise: I don't think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at this level.

Revise: And as a function in computer code / programmable instructions. Functions have inputs and outputs which must be defined.

Evidence Outcome: Formulate the concept of a function and use function notation. (CCSS: F-IF)
- Explain that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range.1 (CCSS: F-IF.1)
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (CCSS: F-IF.2)
- Demonstrate that sequences are functions,2 sometimes defined recursively, whose domain is a subset of the integers. (CCSS: F-IF.3)

Revise: This mathematical concept should be taught in context and not in an isolated manner. Students should be able to define a reasonable range of inputs ("the domain") and the corresponding set of outputs that are reasonable ("the range") for a given situation. Students should be able to write and evaluate a formula for a real-world or contrived situation, using reasonable inputs in the domain. Students should be able to create an iterative or recursive sequence, especially using technology. The rest of this standard is not essential for career and post-secondary readiness.

Evidence Outcome: Interpret functions that arise in applications in terms of the context. (CCSS: F-IF)
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features3 given a verbal description of the relationship. * (CCSS: F-IF.4)
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.4 * (CCSS: F-IF.5)
- Calculate and interpret the average rate of change5 of a function over a specified interval. Estimate the rate of change from a graph.* (CCSS: F-IF.6)

Revise: This should all be done in the context of real world situations. If it can’t be, don’t teach it.

Evidence Outcome: Analyze functions using different representations. (CCSS: F-IF)
- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. * (CCSS: F-IF.7)
- Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS: F-IF.7a)
- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (CCSS: F-IF.7b)
- Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (CCSS: F-IF.7c)
- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (CCSS: F-IF.7e)
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (CCSS: F-IF.8)
- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (CCSS: F-IF.8a)
- Use the properties of exponents to interpret expressions for exponential functions. (CCSS: F-IF.8b)
- Compare properties of two functions each represented in a different way (algebraically, graphically,
numerically in tables, or by verbal descriptions). (CCSS: F-IF.9)

**Remove:** These are all rote skills involving almost no critical thinking or application. They do not teach students how to be problem-solvers. There is no connection to a real-world context. Not necessary for post-secondary or career success. Remove.

**Evidence Outcome:** Build a function that models a relationship between two quantities. (CCSS: F-BF)

- Write a function that describes a relationship between two quantities.* (CCSS: F-BF.1)
- Determine an explicit expression, a recursive process, or steps for calculation from a context. (CCSS: F-BF.1a)
- Combine standard function types using arithmetic operations. (CCSS: F-BF.1b)
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (CCSS: F-BF.2)

**Revise:** What is the asterisk for? "Given a real world situation, define inputs and outputs. Create a function rule that maps inputs to outputs. Model this function rule using the language of mathematics and technology, including computer code." Arithmetic and geometric sequences can also be modeled effectively with computer code. Using technology, students can easily identify if a geometric sequence converges or diverges and describe properties of geometric sequences. Arithmetic and geometric sequences should also be taught in the context of real-world situations such as personal finance, population growth or disease spread.

**Evidence Outcome:** Build new functions from existing functions. (CCSS: F-BF)

- Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k, 9 and find the value of k given the graphs. (CCSS: F-BF.3)
- Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- Find inverse functions. (CCSS: F-BF.4)

**Revise:** This is a rote skill mostly free of critical thinking and could boil down to memorizing rules. It’s a richer standard if students investigate and discover how to transform a graph or a behavior by modifying a function. How could I make the character speed up, slow down, jump higher or put less time in between jumps - for example. This planet actually orbits faster than what my model would predict - change the model to adjust. "Find Inverse Functions" can also be done in context. Instead of calculating the amount of money earned for “x” hours of work, find out how many hours of work are needed to make “d” dollars. None of this needs to be divorced from real world problem solving and engineering.

**Evidence Outcome:** Extend the domain of trigonometric functions using the unit circle. (CCSS: F-TF)

- Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: F-TF.1)
- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: F-TF.2)

**Remove:** These should not be requirements for all students.

**Evidence Outcome:** Extend the domain of trigonometric functions using the unit circle. (CCSS: F-TF)

- Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: F-TF.1)
- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: F-TF.2)

**Remove:** There are many different ways to measure angles - students should be able to describe what radians are, but this level of detail is not needed. Drop most of this in favor of "define radians".
Inquiry Questions:

Revise: These inquiry questions are not interesting or rich or contextualized or personalized. Surely there are better inquiries one could make in the area of functions. I would love to have some discussion around this topic.

Relevance and Application:

Revise: Keep digging for that relevance and application. "Preparation for calculus" shouldn’t be enough of a reason in the 21st century. Do these standards teach students to be creative problem solvers? If not, eliminate or revise. I believe most of these standards in the area of functions could be removed completely. Students should understand how to model a problem with inputs and outputs. They should be able to investigate when a model needs revision and how to revise it. Students should understand the reasonableness of inputs and outputs for a given situation. Students should be able to identify key points in a function relationship that represent important information - highest rate of infection, when a disease has reached its peak spread, when an epidemic can be said to be "over" etc. This maps to rates of change in a function. Keeping the focus on these points should be the goal. Most of these standards are devoid of context and not needed for further real-world problem solving or career success.

GLE: 2. Quantitative relationships in the real world can be modeled and solved using functions

Evidence Outcome: Construct and compare linear, quadratic, and exponential models and solve problems. (CCSS: F-LE)

- Distinguish between situations that can be modeled with linear functions and with exponential functions. (CCSS: F-LE.1)
- Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (CCSS: F-LE.1a)
- Identify situations in which one quantity changes at a constant rate per unit interval relative to another. (CCSS: F-LE.1b)
- Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (CCSS: F-LE.1c)
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.12 (CCSS: F-LE.2)
- Use graphs and tables to describe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (CCSS: F-LE.3)
- For exponential models, express as a logarithm the solution to \((ab^{ct} = d)\) where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. (CCSS: F-LE.4)

Revise: As it's written, this standard is not engaging and divorced from any real-world context. Could it be replaced with a set of essential learnings from physics and finance? Newton's laws, compound interest, population growth and more are exciting to learn and simulate, and they can actually be done with a real-world project. I have never seen a set of lessons that can teach these concepts in an interesting way. Kids hate them. They don't retain them if there's absolutely no reason to learn them other than taking a test and getting credit. We can do better here.

Evidence Outcome: Model periodic phenomena with trigonometric functions. (CCSS: F-TF)

- Choose the trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. * (CCSS: F-TF.5)

Revise: No. "Use trigonometric functions appropriately to simulate periodic phenomena" or "modify parameters in a simulation to fit a periodic curve" would be more real-world. Periodic phenomena are displayed in thousands of different ways, only one of which is a sinewave-shape on an xy curve. Students should be able to flexibly use trigonometric functions to model or better understand circular motion, elliptical motion, electrical pulses, electromagnetic waves and more. And even then, I argue for pushing this
standard into an "advanced" high school standard, as I think many career-ready high schoolers can do without it, but high schoolers looking for a STEM college degree and some certificates would need it.

Evidence Outcome: Model personal financial situations
- Analyze the impact of interest rates on a personal financial plan (PFL)
- Evaluate the costs and benefits of credit (PFL)
- Analyze various lending sources, services, and financial institutions (PFL)

Comment: Keep this one. It's great

Relevance and Application:
Comment: These connections are so specialized that it makes me think this whole section should be moved to science standards instead. Functions are VERY relevant to computer science but they are used more flexibly than the standards here. This whole section needs to be re-thought with an eye on relevance and context. The concepts should be generalized, creativity and inquiry based, and meaningful for 16 year old kids.

GLE: 3. Expressions can be represented in multiple, equivalent forms

(PGC Feedback) Revise: I don't think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at an engineering level.

Evidence Outcome: Interpret the structure of expressions. (CCSS: A-SSE)
- Interpret expressions that represent a quantity in terms of its context.* (CCSS: A-SSE.1)
- Interpret parts of an expression, such as terms, factors, and coefficients. (CCSS: A-SSE.1a)
- Interpret complicated expressions by viewing one or more of their parts as a single entity.13 (CCSS: A-SSE.1b)
- Use the structure of an expression to identify ways to rewrite it.14 (CCSS: A-SSE.2)

Comment: This standard makes sense.

Evidence Outcome: Write expressions in equivalent forms to solve problems. (CCSS: A-SSE)
- Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* (CCSS: A-SSE.3)
- Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS: A-SSE.3a)
- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS: A-SSE.3b)
- Use the properties of exponents to transform expressions for exponential functions.15 (CCSS: A-SSE.3c)
- Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. * (CCSS: A-SSE.4)

Revise: Eliminate the last bullet in b. This should not be for all students.

Evidence Outcome: Write expressions in equivalent forms to solve problems. (CCSS: A-SSE)
- Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* (CCSS: A-SSE.3)
- Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS: A-SSE.3a)
- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS: A-SSE.3b)
- Use the properties of exponents to transform expressions for exponential functions.15 (CCSS: A-SSE.3c)
- Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. * (CCSS: A-SSE.4)
**Remove:** Please remove this entire standard. It causes students so much stress and less than 5% of the kids will never do any of these again after high school and/or college math class.

Evidence Outcome: Perform arithmetic operations on polynomials. (CCSS: A-APR)
- Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (CCSS: A-APR.1)

**Remove:** Another standard that ought to be removed. Students should understand the basic idea of like terms but in a contextual situation. I don't want my 21st century digital-native students spending time adding and subtracting polynomials on paper. This standard is basically saying they should do that.

Evidence Outcome: Understand the relationship between zeros and factors of polynomials. (CCSS: A-APR)
- State and apply the Remainder Theorem.17 (CCSS: A-APR.2)
- Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (CCSS: A-APR.3)

**Remove:** Teenagers do not need to know this.

Evidence Outcome: Use polynomial identities to solve problems. (CCSS: A-APR)
- Prove polynomial identities18 and use them to describe numerical relationships. (CCSS: A-APR.4)

**Remove:** I'm an engineer and I don't know what this standard is. It is clearly not essential.

Evidence Outcome: Rewrite rational expressions. (CCSS: A-APR)
- I don't know what this means.

Evidence Outcome: Rewrite simple rational expressions in different forms.19 (CCSS: A-APR.6)
- The example is worse than the standard. I don't want my students to spend their time doing this.

Relevance and Application:
- Comment: If these are relevant to physics, construction or tech ed, make this more explicit in the standards - or just move them into physics, construction or tech ed.

**GLE: 4. Solutions to equations, inequalities and systems of equations are found using a variety of tools**

Evidence Outcome: Create equations that describe numbers or relationships. (CCSS: A-CED)
- Create equations and inequalities20 in one variable and use them to solve problems. (CCSS: A-CED.1)
- Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. (CCSS: A-CED.2)
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.21 (CCSS: A-CED.3)
- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.22 (CCSS: A-CED.4)

**Revise:** The fourth bullet point (rearrange formulas) can be a really important skill to teach. Graphing and visual modeling is SO different and context-dependent that I do not believe we should be constantly teaching math on the two-dimensional cartesian plane. Making a visual representation of a relationship or constraint is a creative process and it's time this subject moved in that direction.

**Standard: 3. Data Analysis, Statistics, and Probability**

**GLE: 1. Visual displays and summary statistics condense the information in data sets into usable knowledge**

(Recent Feedback) **Revise:** The standard of learning will use examples that can occur in life to make the concept concrete when they solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data. People can be lucky and think that they are smart and successful. They are fooled by randomness. We may listen to such people thinking them as having real insights. "Let us
use the Monte Carlo generator introduced earlier and construct a population of 10,000 fictional investment managers (the generator is not terribly necessary since we can use a coin, or even do plain algebra, but is considerably more illustrative -- and fun). Assume that they each have a perfectly fair game; each one has a 50% probability of making $10,000 at the end of the year, and a 50% probability of losing $10,000. Let us introduce an additional restriction; once a manager has a single bad year, he is thrown out of the sample, good-bye and have a nice life. Thus, we will operate like the legendary speculator George Soros who was said to tell his managers gathered in the room: “Half of you guys will be out by next year” (with an Eastern European accent). Like Soros, we have extremely high standards; we are only looking for managers with an unblemished record. We have no patience for low performers. The Monte Carlo generator will toss a coin; heads and the manager will make $10,000 over the year, tails and he loses $10,000. We run it for the first year. At the end of the year, we can expect 5,000 managers to be up $10,000 each and 5,000 to be down $10,000. Now we run the game a second year. Again, we can expect 2,500 managers to be up two years in a row; another year, 1,250; a fourth one, 625, a fifth, 313. We have now, simply a fair game, 313 managers who made money for five years in a row. Out of pure luck. Fooled by Randomness by Nassim Nicholas Taleb See page 154 https://www.fool.com/investing/value/2007/05/31/foolish-book-review-fooled-by-randomness.aspx

**Evidence Outcome: Summarize, represent, and interpret data on a single count or measurement variable. (CCSS: S-ID)**

- Represent data with plots on the real number line (dot plots, histograms, and box plots). (CCSS: S-ID.1)
- Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (CCSS: S-ID.2)
- Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (CCSS: S-ID.3)
- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and identify data sets for which such a procedure is not appropriate. (CCSS: S-ID.4)
- Use calculators, spreadsheets, and tables to estimate areas under the normal curve. (CCSS: S-ID.4)

**Comment:** I don't disagree with any of these standards but believe the last one (estimating areas under the normal curve) needs context (why would someone do this)? - this section focuses on the rote procedure of doing the statistical analysis and not enough on having students generate questions that these analyses can answer. Technology NEEDS to be part of this section. Mathematicians don't do this with paper and pencil. Nobody does. Include spreadsheets, Mathematica, Matlab, Desmos, hand-coding or whatever analysis packages make sense.

**Evidence Outcome: Summarize, represent, and interpret data on two categorical and quantitative variables. (CCSS: S-ID)**

- Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (CCSS: S-ID.5)
- Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (CCSS: S-ID.6)
- Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (CCSS: S-ID.6a)
- Informally assess the fit of a function by plotting and analyzing residuals. (CCSS: S-ID.6b)
- Fit a linear function for a scatter plot that suggests a linear association. (CCSS: S-ID.6c)

**Comment:** Just noting that all of these can be done with technology. And it would be worth exploring some function-fitting algorithms with students and having them create those algorithms with code. I doubt any mathematicians ever do this by hand anymore.

**Evidence Outcome: Interpret linear models. (CCSS: S-ID)**

- Interpret the slope2 and the intercept3 of a linear model in the context of the data. (CCSS: S-ID.7)
- Using technology, compute and interpret the correlation coefficient of a linear fit. (CCSS: S-ID.8)
• Distinguish between correlation and causation. (CCSS: S-ID.9)

Comment: I have never liked this standard because it’s divorced from any context, and the context would be key to driving learning. Students should be able to make predictions and models based on consistent rates, and also identify their own relevant inquiry questions such as "when will I run out of money?" or "on which day will the reservoir be empty?" or "how do we schedule a graduation ceremony for 625 high school students?" - then model, plan and solve these situations. Focusing on just slope and intercept doesn’t do this justice and robs students of rich thinking here.

Inquiry Questions:

Revise: The student should understand where sources of errors arise in the application of statistical techniques and understand psychological errors people make in understanding probabilities and statistics before experiencing events and the failure of the results of applications of techniques. The lessons should provide real world examples from life that students can connect to when 1. What makes data meaningful or actionable? The Signal and The Noise by Nate Silver see pages 42 to 44. Out of Sample event. There were a number of factors which lead up to the housing crisis. Among the mistakes, Moody’s estimated the extent to which mortgage defaults were correlated with one another by building a model from past data going back to 1980. The problem is that from 1980s through the mid-2000s , home prices were always steady or increasing. Under these circumstances, the assumption that one homeowner’s mortgage has little relationship to another’s was probably good enough. But noting in the past data would have described what happened when home prices began to decline in tandem. The housing collapse was an out-of-sample event. http://www.nytimes.com/2012/11/04/books/review/the-signal-and-the-noise-by-nate-silver.html 2. Why should attention be paid to an unexpected outcome? The student should learn to think for ones self and not use an appeal to authority. Quite often, the biggest names in the field make the worst predictions. This has an amazing bad forecast by a future Nobel Prize in Economics as one author and another author was to be the future head of the Office of the Management of the Budget under President Obama. Volume I, Issue 2 March 2002 © 2002 Fannie Mae. All rights reserved. 3900 Wisconsin Avenue, NW Washington, DC 20016-2892 Fannie Mae Papers is an occasional series on policy issues of interest to the housing community. PAPERS Implications of the New Fannie Mae and Freddie Mac Risk-based Capital Standard by Joseph E. Stiglitz, Jonathan M. Orszag and Peter R. Orszag The paper concludes that the probability of default by the GSEs is extremely small. Given this, the expected monetary costs of exposure to GSE insolvency are relatively small — even given very large levels of outstanding GSE debt and even assuming that the government would bear the cost of all GSE debt in the case of insolvency. For example, if the probability of the stress test conditions occurring is less than one in 500,000, and if the GSEs hold sufficient capital to withstand the stress test, the implication is that the expected cost to the government of providing an explicit government guarantee on $1 trillion in GSE debt is less than $2 million. To be sure, it is difficult to analyze extremely low-probability events, such as the one embodied in the stress test. Even if the analysis is off by an order of magnitude, however, the expected cost to the government is still very modest. http://online.wsj.com/public/resources/documents/stiglitzrisk.pdf When Long Term Capital Management (LTCM) collapsed, normal models using statistics like the Normal Curve failed. As Myron Scholes wrote about an extreme event with 10 standard deviation jump in a financial statistic. THE NEAR CRASH OF 1998 Crisis and Risk Management By MYRON S. SCHOLE S * This 20-basis-point change was a move of 10 standard deviations in the swap spread. After this date the volatility of the swap spread increased from 0.8 of a basis point per day to 8 basis points per day, and it remained high throughout 1999. http://www.andreimonov.com/Microstr_PhD/90020017.pdf Before the Housing Crisis of 2008, Nassim Nicholas Taleb wrote a book exploring...
problems people had in understanding the probability of extreme events and the psychological biases that prevented people from acting correctly based on probabilistic thinking. This is seen in a dismissive review of his book The Black Swan reviewed in 2007. The Black Swan by Nassim Nicholas Taleb https://www.theguardian.com/books/2007/may/12/society Reviewed before the financial crash 3. How can summary statistics or data displays be accurate but misleading? When talking about safe sex and the use of a condoms, various authoritative sources say that they are 98% reliable. We can use the binomial distribution to calculate the probability of "safe sex" after 25 sexual encounters. Page 536 Statistics: An Introductory Analysis 2nd Edition by Taro Yamane Pub. 1967 Probability (k=25 successes) = Combinatorial (n=25, k=25) x .98 to 25th power x .02 to the zero power. This equals Probability (k = 25 successes) = .6034 or 60.34 % chance of no pregnancy after 25 sexual encounters correctly using a condom. This is mathematically correct but misleading because a woman's chance of pregnancy only occurs a few days of the month when she is fertile. http://stattrek.com/probability-distributions/binomial.aspx Binomial Distribution and how to input into Excel Spreadsheet But term "safe sex" is misleading when speaking about sexually transmitted diseases. The CDC has no established rate for what level of protection a condom provides against sexually transmitted diseases. https://www.cdc.gov/condomeffectiveness/docs/condomfactsheetinbrief.pdf CDC Fact Sheet Correct and Consistent Use of Condoms See page 86 of Connected: How Your Friends' Friends' Friends' Affect Everything You Feel, Think, and Do Using data from a group of sexually active teenagers in Jefferson High School in Colorado Springs, the authors mapped out the network where an epidemic of STD infections occur. You can't determine your risk unless you have a birds eye view of the entire network. On page 87, they write, "As far as you know, you have merely have had sex with three partners, just like person A. Without such a complete view of the network, there is really no way for you to acquire that perspective. http://www.nytimes.com/2009/10/04/books/review/Stossel-t.html Connected; The Surprising Power of Our Social Networks and 3. How can summary statistics or data displays be accurate but misleading? A FIELD GUIDE TO LIES: Critical Thinking in the Information Age by Daniel J. Levitin See pages 20 - 22 on manipulating averages. https://www.kirkusreviews.com/book-reviews/daniel-j-levitin/a-field-guide-to-lies/ The Flaw of Averages: Why We Underestimate Risk in the Face of Uncertainty Paperback – March 26, 2012 by Sam L. Savage (Author), Jeff Danziger (Illustrator), Harry M. Markowitz (Foreword) T http://flawofaverages.com/ GLE: 2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions Evidence Outcome: Understand and evaluate random processes underlying statistical experiments. (CCSS: S-IC)

- Describe statistics as a process for making inferences about population parameters based on a random sample from that population. (CCSS: S-IC.1)
- Decide if a specified model is consistent with results from a given data-generating process.4 (CCSS: S-IC.2)

Comment: Is that really the definition of "statistics"? I don't like this standard. You can meet the first bullet point by having students write it in a notebook and regurgitate it on a test. What do statistics really tell us? There are different statistical analyses for different purposes and not all of them involve random sampling.

Evidence Outcome: Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (CCSS: S-IC)

- Identify the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (CCSS: S-IC.3)
- Use data from a sample survey to estimate a population mean or proportion. (CCSS: S-IC.4)
- Develop a margin of error through the use of simulation models for random sampling. (CCSS: S-IC.4)
- Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (CCSS: S-IC.5)
- Define and explain the meaning of significance, both statistical (using p-values) and practical (using effect size).
- Evaluate reports based on data. (CCSS: S-IC.6)

Remove: Remove p-values, this is ok for a stats class but is too much for Algebra II
Evidence Outcome: Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (CCSS: S-IC)

- Identify the purposes and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (CCSS: S-IC.3)
- Use data from a sample survey to estimate a population mean or proportion. (CCSS: S-IC.4)
- Develop a margin of error through the use of simulation models for random sampling. (CCSS: S-IC.4)
- Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (CCSS: S-IC.5)
- Define and explain the meaning of significance, both statistical (using p-values) and practical (using effect size).
- Evaluate reports based on data. (CCSS: S-IC.6)

Comment: Just a general comment that the relationships between science, math and social studies are too strong to ignore here. Students can demonstrate these in context by performing scientific experiments and social science surveys based on questions they generate, and using technology to interpret the results while adding to their repertoire of statistical tools. Students should also be able to evaluate reports about others' studies. They should be able to describe what story the statistics tell them and what open questions remain after analyzing the report. You hit on this in the very last bullet item. I'm challenged by this whole section because it almost points to a separate science/social studies/mathematics standards area which would be to conduct individual or team research studies and analyze the studies of others.

Inquiry Questions:

Comment: I like the inquiry questions here. The last two are very well phrased.

GLE: 3. Probability models outcomes for situations in which there is inherent randomness

Evidence Outcome: Understand independence and conditional probability and use them to interpret data. (CCSS: S-CP)

- Describe events as subsets of a sample space using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events. (CCSS: S-CP.1)
- Explain that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS: S-CP.2)
- Using the conditional probability of A given B as P(A and B)/P(B), interpret the independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (CCSS: S-CP.3)
- Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. (CCSS: S-CP.4)
- Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. (CCSS: S-CP.5)

Revise: CP-1, CP-2, and CP-3 are not essential skills for post-secondary and career success. Focus is on notation and rote skill. The last few bullet points are richer and more interesting. Students should be able to analyze probabilities in situations where events are independent or conditional. For the applications listed in the next column, expected value is an important concept and I don’t see it mentioned in the learning targets here?

Evidence Outcome: Use the rules of probability to compute probabilities of compound events in a uniform probability model. (CCSS: S-CP)

- Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. (CCSS: S-CP.6)
- Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. (CCSS: S-CP.7)
Revise: Include Permutations and Combinations

Evidence Outcome: Use the rules of probability to compute probabilities of compound events in a uniform probability model. (CCSS: S-CP)
- Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. (CCSS: S-CP.6)
- Apply the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), and interpret the answer in terms of the model. (CCSS: S-CP.7)

Revise: Please don’t ask them to memorize formulas. Don’t include a formula in a standard. This is a sure-fire way to encourage teaching by drill-and-test-and-forget. There is a bigger concept here which is that there are two ways to calculate probability: You can calculate the probability of an event occurring as a number from 0 to 1. Or you can calculate the probability of an event NOT occurring and subtract from 1. It takes critical thinking to figure out which method is the most straightforward. For example, calculating the probability of rolling five dice and getting at least one "6" is best done the second way, and solving that problem can teach you how to solve a problem such as "My friends and I bought 10 raffle tickets and 1000 were sold. There will be 50 winners. What is the probability that at least one of us is a winner?"

Standard: 4. Shape, Dimension, and Geometric Relationships

GLE: 1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically

Evidence Outcome: Make geometric constructions. (CCSS: G-CO)
- Make formal geometric constructions7 with a variety of tools and methods.8 (CCSS: G-CO.12)
- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (CCSS: G-CO.13)

Revise: This is too limited a view of geometry. Much of nature is fractal in nature. There are simple algorithms which could illustrate this and students could easily construct the shapes. Also, organic processes grow by iteration. Students need these insights.

GLE: 2. Concepts of similarity are foundational to geometry and its applications

(PGC Feedback) Revise: I don’t think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at this level.

Evidence Outcome: Understand similarity in terms of similarity transformations. (CCSS: G-SRT)
- Verify experimentally the properties of dilations given by a center and a scale factor. (CCSS: G-SRT.1)
- Show that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. (CCSS: G-SRT.1a)
- Show that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. (CCSS: G-SRT.1b)
- Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar. (CCSS: G-SRT.2)
- Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (CCSS: G-SRT.2)
- Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (CCSS: G-SRT.3)

Revise: There should be an additional evidence outcome that expects students to explain that as a space-shape is dilated its 1-dimensional measurements change by the scale factor, its 2-dimensional measurements change by the square of the scale factor, and its 3-dimensional measurements change by the cube of the scale factor.
GLE: 4. Attributes of two- and three-dimensional objects are measurable and can be quantified

(PGC Feedback) Revise: I don't think this should be at a college prep level. College prep and workforce ready are not the same thing. There are many careers that do not need proficiency of this measure at an engineering level.

Mathematics Eighth Grade

Standard: 2. Patterns, Functions, and Algebraic Structures

GLE: 1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically

Comment: In general, the mathematics at each grade level can be captured more coherently by keeping the related mathematics together, with big ideas clearly delineated in the structure of the document (perhaps as GLEs) followed by more specific explanation of the big idea within the EOs. The domains and cluster statements are often lost in the current structure.

Evidence Outcome: Compare two different proportional relationships represented in different ways.1 (CCSS: 8.EE.5)

Comment: In general, across the entire document, the footnoted examples should not be separated from the EO. Together, the statement and examples create a more robust and clear understanding of the EO.

Inquiry Questions:

Comment: The 21st Century Skills portion needs a major overhaul. Many of these entries appear forced, like they were an afterthought in the writing of the standards. The three categories don't make sense at the grain-size they are presented. For instance, Inquiry Questions, Relevance, and Application often don't apply at the level of a single EO, but rather at the level of the bigger mathematical idea. These statements have value, but not in the way they are currently represented.

Relevance and Application:

Move: #2 appears like it might fit better in standard 3 with the evidence outcomes on scatter plots. The evidence outcomes listed in this GLE are already referring to linear functions and would not need a line of best fit.

Nature Of:

Comment: In general throughout the document, mathematical modeling is lost. Although the asterisk is used to indicate standards that lend themselves to modeling opportunities, the right column of the document could be used to more explicitly name how modeling applies to the content of the page. This would serve to make the document more meaningful to teachers on a regular basis by helping them deepen their understanding of the mathematics by studying the document.

Standard: 4. Shape, Dimension, and Geometric Relationships

GLE: 1. Transformations of objects can be used to define the concepts of congruence and similarity

Evidence Outcome: Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. (CCSS: 8.G.4)

Revise: The language is a little unclear for evidence outcome e. It makes it sound like it could be any combination of the 4 transformations that produces similar figures, but the dilation is necessary to scale the figure. I think the evidence outcome should read, "Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations in combination with a dilation."
**Mathematics Sixth Grade**

**Standard: 2. Patterns, Functions, and Algebraic Structures**

GLE: 1. Algebraic expressions can be used to generalize properties of arithmetic

(PGC Feedback) **Revise:** I really do NOT want to revise this but wanted to provide a comment here and this wasn’t an option. The math standards have led to more students than ever taking algebra in 7th grade. These are the students who have had the CAS since they started school. I’m personally seeing so many students more advanced in math as they enter each new grade. The standards for math are working well and truly serving our kids.

**Mathematics Fifth Grade**

**Standard: 1. Number Sense, Properties, and Operations**

GLE: 1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

**Revise:** Please include all verbiage currently in footnotes related to each indicator directly with the description, and remove the footnotes. As footnotes this often critical information is overlooked.

GLE: 2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency

**Evidence Outcome:** Fluently multiply multi-digit whole numbers using standard algorithms. (CCSS: 5.NBT.5)

**Revise:** define the magnitude of numbers. Up to 3-digit by 2-digit would align with PARCC.

**Evidence Outcome:** Add, subtract, multiply, and divide decimals to hundredths. (CCSS: 5.NBT.7)

- Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 5.NBT.7)
- Relate strategies to a written method and explain the reasoning used. (CCSS: 5.NBT.7)

**Revise:** This says, "to hundredths." but it is unclear when this comes to multiplication if this refers to the products or factors. I would suggest we say "with products to the hundredths." PARCC does say multiply, "tenths with tenths, or tenths with hundredths." I think tenths with tenths is more reasonable.

**Mathematics Fourth Grade**

GLE: 1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

**Revise:** Please include all verbiage currently in footnotes related to each indicator directly with the description, and remove the footnotes. As footnotes this often critical information is overlooked.
Nature Of:

**Move:** Please create and additional "Mathematical Practices" standard that incorporates all of the science thinking and skills that are currently housed in the 'Nature of' section of the the standards.

**GLE: 2. Different models and representations can be used to compare fractional parts**

**Comment:** This seems to be the focus for most of the "state testing" and I'm not sure this is most important skills to master. Many students do not have a strong enough foundation in numeracy or are developmentally prepared to conquer what is required here.

**GLE: 3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency**

**Comment:** Because math is such an abstract concept, and the amount of foundational knowledge is so great......many kids at age 10 are not developmentally ready to master these skills.

Evidence Outcome: **Use the four operations with whole numbers to solve problems. (CCSS: 4.OA)**

- Interpret a multiplication equation as a comparison.13 (CCSS: 4.OA.1)
- Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.1)
- Multiply or divide to solve word problems involving multiplicative comparison.14 (CCSS: 4.OA.2)
- Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. (CCSS: 4.OA.3)
- Represent multistep word problems with equations using a variable to represent the unknown quantity. (CCSS: 4.OA.3)
- Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.3)
- Using the four operations analyze the relationship between choice and opportunity cost (PFL)

**Comment:** It seems today's student have a very hard time with retention of information. Getting kids to memorize facts is almost impossible and with technology available in our world a calculator is almost always within reach. We need to teach students the foundations of numeracy....but allow them to use tools (like calculators) to solve problems.

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**GLE: 1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time**

**Comment:** I wish there was a place to give a general overview of ALL standards. So I will do it here. Overall....it is just too much we expect kids to know with their background, developmental ability and time spent learning. We end up skimming over everything instead of giving kids enough practice time for it to solidify. They do many things badly, instead of some things well. Let us slow down....and get the basics in place before we expect them to be adults.

Evidence Outcome: **Use concepts of angle and measure angles. (CCSS: 4.MD)**

- Describe angles as geometric shapes that are formed wherever two rays share a common endpoint, and explain concepts of angle measurement.3 (CCSS: 4.MD.5)
- Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.6)
- Demonstrate that angle measure as additive.4 (CCSS: 4.MD.7)
- Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.5 (CCSS: 4.MD.7)

**Move:** Kids need more practice with measuring angles before they can find unknown angles... With the overwhelming amount of content, there is not enough time to get efficient at one thing before we bombard them with another
Mathematics Third Grade

Standard: 1. Number Sense, Properties, and Operations

GLE: 1. The whole number system describes place value relationships and forms the foundation for efficient algorithms

Revise: Please include all verbiage currently in footnotes related to each indicator directly with the description, and remove the footnotes. As footnotes this often critical information is overlooked.

Evidence Outcome: Use place value to read, write, count, compare, and represent numbers. (CCSS: 2.NBT)
• Represent the digits of a three-digit number as hundreds, tens, and ones. (CCSS: 2.NBT.1)
• Count within 1000. (CCSS: 2.NBT.2)
• Skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.2)
• Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS: 2.NBT.3)
• Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (CCSS: 2.NBT.4)

Revise: Students need additional work with number lines in representing values of quantities. They should estimate where a given number is on a number line given two endpoints 1-100 and 1-1000. They should be able to estimate the value of a given point displayed on a number line with endpoints given (1-100 and 1-1000).

Evidence Outcome: Use place value understanding and properties of operations to add and subtract. (CCSS: 2.NBT)
• Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.5)
• Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.6)
• Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.2 (CCSS: 2.NBT.7)
• Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS: 2.NBT.8)
• Explain why addition and subtraction strategies work, using place value and the properties of operations. (CCSS: 2.NBT.9)

Revise: Strategies listed need to include open/empty number line strategies. The current list implies heavily the need to learn the standard algorithm for addition and subtraction, and that was not the intent of the Common Core Standards.

Mathematics Second Grade

GLE: 1. The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms

Revise: Please include all verbiage currently in footnotes related to each indicator directly with the description, and remove the footnotes. As footnotes this often critical information is overlooked.

Evidence Outcome: Use place value to read, write, count, compare, and represent numbers. (CCSS: 2.NBT)
• Represent the digits of a three-digit number as hundreds, tens, and ones.1 (CCSS: 2.NBT.1)
• Count within 1000. (CCSS: 2.NBT.2)
• Skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.2)
• Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS: 2.NBT.3)
• Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (CCSS: 2.NBT.4)

Revise: Students need additional work with number lines in representing values of quantities. They should estimate where a given number is on a number line given two endpoints 1-100 and 1-1000. They should be able to estimate the value of a given point displayed on a number line with endpoints given (1-100 and 1-1000).

Evidence Outcome: Use place value understanding and properties of operations to add and subtract. (CCSS: 2.NBT)
• Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.5)
• Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.6)
• Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.2 (CCSS: 2.NBT.7)
• Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS: 2.NBT.8)
• Explain why addition and subtraction strategies work, using place value and the properties of operations. (CCSS: 2.NBT.9)

Revise: Strategies listed need to include open/empty number line strategies. The current list implies heavily the need to learn the standard algorithm for addition and subtraction, and that was not the intent of the Common Core Standards.
GLE: 2. Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency

Evidence Outcome: Fluently add and subtract within 20 using mental strategies. (CCSS: 2.OA.2)

Comment: Fluency standards should not be broken across K, 1, and 2 by range of number, but rather by the level of strategy used. See Level 1, 2, and 3 strategies in the Common Core progressions documents. Second Grade fluency should include Level 3 strategies and emphasize the use of anchoring to ten, near doubles, and other student invented strategies for deriving number combinations. Particular attention to subtraction strategies would also be helpful.

Evidence Outcome: Know from memory all sums of two one-digit numbers. (CCSS: 2.OA.2)

Revise: Fluency standards should not be broken across K, 1, and 2 by range of number, but rather by the level of strategy used. See Level 1, 2, and 3 strategies in the Common Core progressions documents. Second Grade fluency should include Level 3 strategies and emphasize that fluency is built from strong number relationships, not knowing from memory.

Standard: 3. Data Analysis, Statistics, and Probability

GLE: 1. Visual displays of data can be constructed in a variety of formats to solve problems

Evidence Outcome: Represent and interpret data. (CCSS: 2.MD)

• Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS: 2.MD.9)
• Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. (CCSS: 2.MD.10)
• Solve simple put together, take-apart, and compare problems using information presented in picture and bar graphs. (CCSS: 2.MD.10)

Comment: The first indicator for this Evidence Outcome is an activity, not a standard. What are students to learn from doing this activity? That should be the standard.

Standard: 4. Shape, Dimension, and Geometric Relationships

GLE: 2. Some attributes of objects are measurable and can be quantified using different tools

Evidence Outcome: Relate addition and subtraction to length. (CCSS: 2.MD)

• Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units1 and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.MD.5)
• Represent whole numbers as lengths from 0 on a number line2 diagram and represent whole-number sums and differences within 100 on a number line diagram. (CCSS: 2.MD.6)

Revise: This needs to be more explicitly tied to the number sense standard.

Mathematics First Grade

Standard: 1. Number Sense, Properties, and Operations

GLE: 1. The whole number system describes place value relationships within and beyond 100 and forms the foundation for efficient algorithms

Revise: Within this standard I believe that we should add an indicator related to the use of number lines. 1. Student is able to demonstrate how a number should be placed on a marked number line between 1-100.
2. Student understands that the spaces between all counting numbers on a number line are equal. Students are able to use a number line diagram to represent adding to and taking-from subtraction situations as movement on the number line.

Evidence Outcome: Count to 120 (CCSS: 1.NBT.1)
- Count starting at any number less than 120. (CCSS: 1.NBT.1)
- Within 120, read and write numerals and represent a number of objects with a written numeral. (CCSS: 1.NBT.1)

Revise: Please include all verbiage currently in footnotes related to each indicator directly with the description, and remove the footnotes. As footnotes this often critical information is overlooked.

Evidence Outcome: Count to 120 (CCSS: 1.NBT.1)
- Count starting at any number less than 120. (CCSS: 1.NBT.1)
- Within 120, read and write numerals and represent a number of objects with a written numeral. (CCSS: 1.NBT.1)

Revise: The standards need to include developing understanding of number lines as a representation of the number sequence. This should be linked to developing understanding of linear measurement to build the understanding of equal spacing and distance from zero.

Evidence Outcome: Use place value and properties of operations to add and subtract. (CCSS: 1.NBT)
- Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of ten, using concrete models or drawings, and/or the relationship between addition and subtraction. (CCSS: 1.NBT.4)
- Identify coins and find the value of a collection of two coins (PFL)
- Mentally find 10 more or 10 less than any two-digit number, without counting; explain the reasoning used. (CCSS: 1.NBT.5)
- Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 1.NBT.6)
- Relate addition and subtraction strategies to a written method and explain the reasoning used. (CCSS: 1.NBT.4 and 1.NBT.6)

Revise: Standards should include use of open/empty number lines as a strategy for developing addition and subtraction with larger numbers.

GLE: 2. Number relationships can be used to solve addition and subtraction problems

Evidence Outcome: Represent and solve problems involving addition and subtraction. (CCSS: 1.OA)
- Use addition and subtraction within 20 to solve word problems.2 (CCSS: 1.OA.1)
- Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.3 (CCSS: 1.OA.2)

Revise: Standards need to specify word problem structure types: Add To/Take From, Put Together/Take Apart, and Additive Comparison.

Evidence Outcome: Add and subtract within 20. (CCSS: 1.OA)
- Relate counting to addition and subtraction.6 (CCSS: 1.OA.5)
- Add and subtract within 20 using multiple strategies.7 (CCSS: 1.OA.6)
- Demonstrate fluency for addition and subtraction within 10. (CCSS: 1.OA.6)

Revise: Fluency standards should not be broken across K, 1, and 2 by range of number, but rather by the level of strategy used. See Level 1, 2, and 3 strategies in the Common Core progressions documents. First Grade level fluency should begin with Level 2 strategies and move into Level 3 strategies by the end of the year, and should include fluency with partitions of 10, 10 + a number, doubles, anchoring to 10 strategies, near doubles, and other student invented strategies by the end of the year.
Mathematics Kindergarten

Standard: 1. Number Sense, Properties, and Operations

GLE: 1. Whole numbers can be used to name, count, represent, and order quantity

Revise: All footnotes related to each indicator throughout the document should be included directly with the description and not put as footnotes where they are more often than not overlooked.

Evidence Outcome: Use number names and the count sequence. (CCSS: K.CC)
- Count to 100 by ones and by tens. (CCSS: K.CC.1)
- Count forward beginning from a given number within the known sequence.1 (CCSS: K.CC.2)
- Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20.2 (CCSS: K.CC.3)

Revise: This standard is lacking the beginning understanding of number lines. Students need to understand that the number sequence can be represented on a number line and that the values on the number line increase as the numbers move to the right and decrease as the numbers move to the left.

Evidence Outcome: Compare and instantly recognize numbers. (CCSS: K.CC)
- Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.5 (CCSS: K.CC.6)
- Compare two numbers between 1 and 10 presented as written numerals. (CCSS: K.CC.7)
- Identify small groups of objects fewer than five without counting

Revise: Before comparing numerals, students should compare quantities.

GLE: 2. Composing and decomposing quantity forms the foundation for addition and subtraction

Evidence Outcome: Model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings. (CCSS: K.OA)
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds,6 acting out situations, verbal explanations, expressions, or equations. (CCSS: K.OA.1)
- Solve addition and subtraction word problems, and add and subtract within 10.7 (CCSS: K.OA.2)
- Decompose numbers less than or equal to 10 into pairs in more than one way.8 (CCSS: K.OA.3)
- For any number from 1 to 9, find the number that makes 10 when added to the given number.9 (CCSS: K.OA.4)
- Use objects including coins and drawings to model addition and subtraction problems to 10 (PFL)

Revise: Colorado standards need to specify word problem types: Add To/Take From and Put Together/Take Apart Including coins in this standard is an artificial inclusion for PFL.

Evidence Outcome: Fluently add and subtract within 5. (CCSS: K.OA.5)

Revise: Fluency standards should not be broken across K, 1, and 2 by range of number, but rather by the level of strategy used. See Level 1, 2, and 3 strategies in the Common Core progressions documents. Kindergarten level fluency should be + and - 1 up to 20, partitions of all quantities to 5, 5 + a number up to 10.

Mathematics Preschool

GLE: 1. Quantities can be represented and counted

Evidence Outcome: Count and represent objects including coins to 10 (PFL)

Revise: "...including coins" is not clear. There is a difference between counting a number of object (say, cubes) and counting a number of coins of different values. Is the point here to simply count a set of object of any kind, and come up with a total of the number of objects in the set, or is the identification of a coin's value, or even its physical attributes, part of the intended mastery of this item?