MATHEMATICS
High School

Introduction

The standards for high school mathematics are designed to provide students opportunities to learn rigorous, relevant mathematics for all four years of high school. High school mathematics should empower students to expand professional opportunities, to understand and critique the world, and to experience wonder, joy, and beauty (NCTM, 2018).

A standard is a “clear, measurable learning target for what a student should know or be able to do” (C.R.S. 22-7-1003(22)). Three major features organize the high school mathematics standards:

- The **mathematical domain(s)** of a standards' content,
- The **mathematical and statistical process(es)** students are most likely to engage in for a standard, and
- The **mathematical pathway(s)** that channel students' interests into a plan for high school and postsecondary success.

The organization of the high school mathematics standards into these three major features (domain, process, and pathway) is achieved through the use of labels, marked with a 🏷. Each standard is labeled with one or more items from each of the three major features. Clicking the label (when enabled on a web page) will filter the standards to show only those labeled with the selected label.

Mathematical Domains

Colorado's high school mathematics standards are labeled with at least one of four domains: algebra, functions, statistics, and geometry.

Algebra (🏷 Algebra), as a high school topic of study, explores the arithmetic of expressions involving unknown or variable quantities. It is also a set of methods for identifying those unknowns and describing relationships between variables. **Prepared graduates** in the algebra domain should be able to demonstrate:

- Solving equations and inequalities, including those that require creativity to isolate a variable of interest, and
- Flexible and appropriate use of solution techniques for linear equations/inequalities, non-linear equations/inequalities, and systems of equations/inequalities.

Functions (🏷 Function) describe special relationships between quantities. Understanding and working with functions requires not only algebraic skills for working with equations, tables, and graphs, but also a qualitative understanding of these relationships as mathematical objects. **Prepared graduates** in the functions domain should be able to demonstrate:

- Recognition of common function types and the mathematical behavior of those functions as they are interpreted and transformed, and
- Skill with multiple technologies to understand and apply functions to model and make sense of real-world phenomena.

Statistics (Statistics) involves understanding, explaining, and quantifying data, and using data to make predictions and informed decisions. Statistical literacy is a necessity in a data-rich world. Facility with large data sets, study designs, and statistical inference is a gateway to emerging career opportunities. **Prepared graduates** in the statistics domain should be able to demonstrate:

- Critical literacy with statistical data, data displays, and claims that describe and predict things about our world, and
- Skill with the tools and techniques of descriptive and basic inferential statistics to make claims and predictions with data of their own.

Geometry (Geometry) is the study of space, shape, size, and position. At the high school level, geometric measurement of increasingly complex two- and three-dimensional shapes is used to model the world and develop concepts in calculus. Geometry also provides a rich context for mathematical reasoning and proving. **Prepared graduates** in the geometry domain should be able to demonstrate:

- Application of geometric measurement to model shapes, surfaces, and volumes in the world, and
- Reasoning with the attributes of shapes to prove claims about those shapes, particularly as those shapes are subjected to geometric transformations.

Mathematical and Statistical Processes and the Colorado Essential Skills

Student success in mathematics relies as much on mastery and confidence in a set of mathematical practices as it does on learning any particular content. To describe these practices, these standards are labeled with NCTM's *Mathematical and Statistical Processes* (2024), which are 11 statements sorted into four groups:

Modeling and Using Tools and Representations

- Model: Model with mathematics and statistics.
- Contextualize: Decontextualize and recontextualize mathematical and statistical situations.
- Tools: Use appropriate tools, including technology, strategically.
- Representations: Use representations to examine multiple mathematical and statistical points of view.

Explaining, Reasoning, and Proving

- Conjecture: Conjecture and reason inductively and deductively.
- Communicate: Construct viable arguments and critique the reasoning of others.

Seeing, Describing, and Generalizing Structure

- Structure: Look for and make use of structure.
- Generalize: Look for and express regularity in repeated reasoning.

Habits of a Productive Mathematical and Statistical Thinker

- Sensemaking: Make sense of problems and persevere in solving them.
- Precision: Attend to precision in mathematical and statistical language and processes.
- Tinker: Tinker productively with mathematical and statistical ideas and problems.

The Mathematical and Statistical Processes are a math-specific reflection of the *Colorado Essential Skills*, which are a set of durable, forward-looking skills that span all of Colorado's content areas and grade levels. As described in statute, the Colorado Essential Skills "require a student to develop and demonstrate creativity and innovation skills; critical-thinking and problem-solving skills; communication and collaboration skills; social and cultural awareness; civic engagement; initiative and self-direction; flexibility; productivity and accountability; character and leadership; the ability to use information and communications technologies to find, evaluate, create, and communicate information; and other skills critical to preparing students for the twenty-first-century workforce and for active citizenship" (C.R.S. 22-7-1005)

The high school mathematics standards provide opportunities to integrate the Colorado Essential Skills (CES) and the Mathematical and Statistical Processes (MSP), especially in the following ways:

- *Creativity and innovation skills* (CES) are *habits of a productive mathematical thinker* (MSP), such as when someone makes sense of solution strategies to authentic problems when no prescribed solution process is given or known.
- *Critical thinking and problem-solving skills* (CES) are central to the study of mathematics and its application, especially in how mathematical thinkers use *models, tools, and representations* (MSP) and how they *see, describe, and generalize structure* (MSP).
- *Communication and collaboration skills* (CES) are needed when *explaining, reasoning, and proving* (MSP), especially as mathematics is integrated into new fields, used across diverse teams, and becomes part of our ability to be literate consumers of media and data.

Prepared graduates will engage in mathematical and statistical processes across lessons, units, and courses, and as they do, they learn how to participate in mathematical communities of practice. Although some content will lend itself more to some processes than others, students need repeated and intentional opportunities to engage with all 11 processes, and to make explicit connections between their activities and their development of the processes and the Colorado Essential Skills.

Mathematics Pathways

A significant amount of high school mathematics content is **foundational** (Foundational) for every student. Learning this content prepares each student to pursue a wide range of opportunities. Learning foundational content should be the focus of every student in their first 4-6 semesters of high school. In the final 2-4 semesters of high school, students should focus on learning advanced mathematics that aligns with their interests and post-secondary plans. These standards include advanced content in three pathways: quantitative reasoning, statistics, and advanced algebra/calculus.

The **Quantitative Reasoning Pathway** (Quantitative Reasoning Pathway) offers opportunities for students who wish to connect mathematical concepts with practical applications. Students pursuing this pathway are expected to justify their reasoning and evaluate quantitative arguments. The quantitative reasoning pathway reflects the growing demand for mathematical skills across all fields of work and the need for citizens to make sense of data displays, statistical studies, and their findings.

The **Statistics Pathway** (Statistics Pathway) offers opportunities for students who wish to develop skills for making sense of an increasingly data-driven world. Students pursuing this pathway are expected to address complexity and uncertainty by designing statistical studies and analyzing authentic, multivariable data. The statistics pathway reflects the growing demand for workers who can use modern tools and techniques to analyze data and make informed decisions.

The **Advanced Algebra/Calculus Pathway** (Advanced Algebra/Calculus Pathway) offers opportunities for students who wish to develop theoretical and applied knowledge about functions, algebraic structures, rates of change, accumulation, and geometry. Students pursuing this pathway are expected to be fluent in algebraic methods and concepts. The advanced algebra/calculus pathway reflects the knowledge needed in many business fields or a career in science, technology, engineering, or mathematics (STEM).

Prepared graduates will be college- and career-ready by completing one or more of the three advanced pathways. This means they are prepared to successfully earn credit in one or more of Colorado's college-level, Guaranteed Transfer (GT) courses, either in high school through dual/concurrent enrollment or upon entering college.

Draft High School Mathematics Standards

HS.1. Understanding Expressions and Mathematical Objects

Expressions are foundational mathematical objects that represent quantities and relationships. In high school, students deepen their understanding of expressions as components of equations, inequalities, and functions, and use these distinctions to clarify how different algebraic representations convey meaning. Through this study of structure and representation, students develop greater precision, flexibility, and sensemaking that support advanced mathematical pathways.

1. Recognize that expressions are components of equations and inequalities. Algebra Structure Precision Foundational
2. Articulate that an expression is a quantity while equations and inequalities are relationships of expressions, and demonstrate through proper use of these representations. Algebra Communicate Structure Foundational
3. Rewrite expressions to reveal information about the expression. Algebra Structure Tinker Foundational
4. Rewrite expressions involving radicals and rational exponents using the properties of exponents. Extend a pattern to demonstrate how the meaning of rational exponents follows from extending the properties of integer exponents to rational exponents and related radicals. Algebra Structure Generalize Foundational
5. Recognize that polynomials form a system analogous to the integers. Apply this understanding to add, subtract, and multiply polynomials. Algebra Precision Advanced Algebra/Calculus Pathway
6. Know and apply the Remainder Theorem to polynomials of degree less than 5. Algebra Precision Advanced Algebra/Calculus Pathway

7. Know and apply the Binomial Theorem for the expansion of n powers of x and y for a positive integer n, with coefficients determined by Pascal's Triangle. Algebra Sensemaking Precision Advanced Algebra/Calculus Pathway
8. Rewrite simple rational expressions in different forms using inspection, division, and/or with technology, when appropriate. Algebra Tools Structure Advanced Algebra/Calculus Pathway
9. Add, subtract, multiply, and divide rational expressions. Algebra Structure Precision Advanced Algebra/Calculus Pathway
10. Analyze expressions that represent real-world quantities, explaining what each term, factor, or component represents in the situation. Algebra Contextualize Precision Quantitative Reasoning Pathway
11. Articulate how equivalent expressions describe the same quantitative relationship in different ways. Algebra Conjecture Sensemaking Quantitative Reasoning Pathway
12. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Algebra Structure Precision Advanced Algebra/Calculus Pathway

HS.2. Connecting to Functions

Arithmetic and geometric sequences help students connect ordered pairs to functions as unified objects with distinct behaviors. Explicit links are made between arithmetic sequences and linear functions, and between geometric sequences and exponential functions. Students make explicit connections between sequences and their corresponding functions to deepen functional thinking.

1. Investigate arithmetic sequences with and without technology and represent related patterns with an expression. Algebra Tools Generalize Foundational
2. Articulate how arithmetic sequences are related to linear functions. Describe how arithmetic sequences are discrete and linear functions are continuous. Algebra Function Communicate Generalize Foundational
3. Investigate geometric sequences with and without technology and represent related patterns with an expression. Algebra Tools Generalize Foundational
4. Articulate how geometric sequences are related to exponential functions. Describe how geometric sequences are discrete and exponential functions are continuous. Algebra Function Communicate Generalize Foundational
5. Derive the formula for the sum of the first n terms of an arithmetic series. Algebra Conjecture Generalize Quantitative Reasoning Pathway Advanced Algebra/Calculus Pathway
6. Derive the formula for the sum of a finite geometric series when the common ratio is not 1. Algebra Conjecture Generalize Quantitative Reasoning Pathway Advanced Algebra/Calculus Pathway
7. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Algebra Communicate Generalize Advanced Algebra/Calculus Pathway

HS.3. Making Meaning of Solutions and Solving Methods

Students build on previous learning with solving equations and inequalities to understand the meaning of what solutions represent and how they relate to algebraic expressions, graphs, and contextual situations. They interpret solution sets in relation to the quantities and relationships being represented and use this understanding to choose and justify appropriate solving methods. These ideas extend to quadratic equations, including those with non-real solutions, and interpreting how these solutions relate to features of graphs.

1. Describe relationships and make connections between algebraic representations, the related graph, and its solution set for equations, inequalities, and systems. Algebra Communicate Sensemaking Foundational
2. Solve quadratic equations using multiple methods. Methods should include graphing and identifying x-intercepts by hand and with technology, factoring, and using the Quadratic Formula. Make connections between the equation and the solution(s) and explore when real solutions exist. Algebra Tools Sensemaking Foundational
3. Solve linear equations and inequalities in one variable by graphing with and without technology to make connections between the equation or inequality and the solution(s). Algebra Representations Sensemaking Foundational
4. Solve systems of linear equations algebraically and graphically, solving simple cases by hand and/or using technology when appropriate. Articulate that a solution, when it exists, is represented by the points of intersection on a graph or is an ordered pair that makes both equations true. Algebra Tools Representations Foundational
5. Solve systems consisting of a linear equation and a nonlinear equation in two variables algebraically or graphically, using technology when appropriate. Articulate that a solution, when it exists, is represented by the points of intersection on a graph or is an ordered pair that makes both equations true. Algebra Tools Representations Foundational
6. Identify the number of solutions to a system of linear equations without solving. Algebra Conjecture Structure Foundational
7. Solve systems of linear inequalities graphically, solving by hand and using technology when appropriate. Articulate that a solution, when it exists, is represented by the region of intersection that makes both inequalities true. Algebra Tools Representations Foundational
8. Determine the total number of roots of a polynomial using the Fundamental Theorem of Algebra. Algebra Communicate Generalize Advanced Algebra/Calculus Pathway
9. Identify zeros of polynomials when suitable factorizations are available. Use the zeros to construct a rough graph of the function defined by the polynomial. Algebra Representations Structure Advanced Algebra/Calculus Pathway
10. Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Algebra Sensemaking Precision Advanced Algebra/Calculus Pathway

HS.4. Modeling Authentic Contexts with Equations, Inequalities, and Systems

Expressions, equations, and inequalities can be used to analyze and make predictions about relationships and changing quantities. They support mathematical modeling in authentic problem contexts involving linear, quadratic, and exponential situations, as well as vectors. Students create and interpret models, make assumptions explicit, and communicate results in relation to the context.

1. Create linear, quadratic, and exponential equations and linear inequalities and use them to solve problems in authentic contexts. Algebra Model Contextualize Foundational
2. Model problems in authentic contexts by describing and using constraints with equations, inequalities, and systems. Interpret solution sets as viable or nonviable based on the context. Algebra Model Contextualize Foundational
3. Apply algebraic properties to rewrite formulas to highlight a quantity of interest in an authentic modeling context. Algebra Model Structure Foundational
4. Justify when lines are parallel or perpendicular using slope relationships and apply this reasoning to create equations that model particular conditions or contexts. Algebra Model Contextualize Foundational

HS.5. Describing Function Relationships

Functions describe special relationships between quantities through two complementary perspectives: correspondence, where each input maps to exactly one output, and covariation, where changes in one quantity drive changes in another. Understanding both perspectives deepens students' grasp of how functions model real-world phenomena. Students think flexibly about functions by considering both how inputs map to outputs and how quantities change together.

1. Explain that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. Use the notation $y = f(x)$ to mean that y is a function of x. Function Communicate Sensemaking Foundational
2. Explain that a function is a co-varying relationship between quantities that change together. For $y = f(x)$, describe how the values of x and y change together. Function Communicate Sensemaking Foundational
3. Model authentic contexts with functions and describe the correspondence and covariation using the context. Function Model Contextualize Foundational
4. Use technology to visualize functions and explore how changes in one quantity affect the other. Function Tools Conjecture Foundational
5. Model authentic situations with functions, using technology when appropriate. Identify and interpret relationships between quantities in terms of correspondence and covariation. Function Tools Contextualize Foundational

HS.6. Representing Functions in Different Ways

Functions can be represented in multiple forms: equations, graphs, tables, diagrams, and verbal descriptions. Each form offers unique ways to visualize, describe, and solve problems. Different representations of the same function reveal different features and insights about the relationship between variables. Students select and fluently move between different representations of functions to deepen understanding and solve varied problems.

1. Represent functions using equations, graphs, tables, diagrams, and/or verbal descriptions. Describe relationships between different representations of the same function. Function Representations Communicate Foundational
2. Given one representation for a function, create one or more different representations of that function. Function Representations Foundational

3. Interpret graphs to show how two changing quantities relate on a coordinate plane, and articulate that different-looking graphs can represent the same function. Function Generalize Foundational
4. Represent real-world relationships using function notation, evaluate these functions for specific inputs in their domains, and interpret statements that use function notation in terms of their contexts. Function Contextualize Precision Foundational
5. Use technology to manipulate parameters to observe effects on correspondence and covariation across representations of functions. Function Contextualize Tools Foundational
6. Given a situation modeled by contextual data, use technology to visualize the data, explore different potential functions to model the data, analyze the potential function models, and determine the best model to represent the data. Justify the selected function model. Function Tools Tinker Quantitative Reasoning Pathway Statistics Pathway

HS.7. Comparing Within and Across Function Families

Functions can be rewritten in equivalent symbolic forms using properties of equality to reveal and explain specific function features visible in graphical representations. Functions are organized into families, where functions in the same family share the same relationship between variables, including foundational families like linear, exponential, and quadratic. Students rewrite functions in different forms and classify functions by family to understand how symbolic forms connect to contextual interpretations and graphs.

1. Describe linear, exponential, and quadratic functions in terms of correspondence in their input and output values. Function Communicate Foundational
2. Describe linear, exponential, and quadratic functions in terms of covariation in their input and output values. Function Sensemaking Foundational
3. Describe absolute value, logarithmic, polynomial, rational, periodic, piecewise defined, radical, and trigonometric functions in terms of correspondence in their input and output values. Function Structure Advanced Algebra/Calculus Pathway
4. Describe absolute value, logarithmic, polynomial, rational, periodic, piecewise defined, radical, and trigonometric functions in terms of covariation in their input and output values. Function Structure Advanced Algebra/Calculus Pathway
5. Describe linear, exponential, and quadratic functions in terms of the absolute change in the input values and in the output values. Function Communicate Precision Foundational
6. Describe the rates of change of linear, exponential, and quadratic functions. Function Communicate Precision Foundational
7. Calculate and interpret average rates of change for linear, exponential, and quadratic functions. Function Precision Foundational
8. Calculate and interpret average rates of change for absolute value, logarithmic, polynomial, rational, periodic, piecewise defined, radical, and trigonometric functions. Function Precision Advanced Algebra/Calculus Pathway
9. Identify and connect key features of linear, exponential, and quadratic functions using equations, graphs, tables, diagrams, and verbal descriptions. Key features include intercepts, intervals where the function is increasing, decreasing, positive or negative; relative maxima and

minima; and symmetries. Make explicit connections between representations of the same function. Function Structure Foundational

10. Identify and connect key features of absolute value, logarithmic, polynomial, rational, periodic, piecewise defined, radical, and trigonometric functions using equations, graphs, tables, diagrams, and verbal descriptions. Key features include intercepts, intervals where the function is increasing, decreasing, positive or negative; relative maxima and minima; symmetries; periodicity; and end behavior. Make explicit connections between representations of the same function. Function Structure Advanced Algebra/Calculus Pathway
11. Make generalizations about the form and behavior of linear, exponential, and quadratic functions. Function Generalize Foundational
12. Make generalizations about the form and behavior of absolute value, logarithmic, polynomial, rational, periodic, piecewise defined, radical, and trigonometric functions. Function Structure Generalize Advanced Algebra/Calculus Pathway
13. Fit and compare models to data, using technology. Interpret parameters and discuss which model best captures observed trends and why. Function Tools Quantitative Reasoning Pathway Statistics Pathway
14. Determine and represent which function family best models a situation given by a set of data using technology. Function Statistics Tools Structure Quantitative Reasoning Pathway Statistics Pathway

HS.8. Transforming Functions to Create New Functions

Transforming and combining functions connects algebraic changes to geometric changes on a graph, helping students understand what is preserved and what changes under different transformations. Through shifts, reflections, stretches, and compressions, students track how points move and develop intuition about function behavior. Students apply transformations, explore compositions and inverses, and create new functions from existing ones.

1. Describe the relationship between a function and its inverse and explain that the range of the function is the domain of its inverse, and the domain of the function is the range of its inverse. Function Communicate Foundational
2. For linear, exponential, and quadratic functions, identify the effect on the graph of $y = f(x)$ by substituting $f(x)$ with $af(x)$, $f(bx)$, $f(x + c)$, $f(x) + d$, and/or combinations each for specific values of a, b, c, d, both positive and negative. Determine the value of a, b, c, and/or d given the graphs. Function Model Representations Foundational
3. For absolute value, logarithmic, polynomial, rational, periodic, piecewise defined, radical, and/or trigonometric functions, identify the effect on the graph of $y = f(x)$ by substituting $f(x)$ with $af(x)$, $f(bx)$, $f(x + c)$, $f(x) + d$, and/or combinations each for specific values of a, b, c, d, both positive and negative. Determine the value of a, b, c, and/or d given the graphs. Function Model Representations Advanced Algebra/Calculus Pathway
4. Determine how points on the graph of a function change under transformations. Describe how shifts, reflections, stretches, and compressions affect the graph and the coordinates of points of interest. Identify which properties of the function are preserved and which are changed. Function Communicate Foundational
5. Use arithmetic operations and/or function composition to combine functions to form a new function. Given $f(x)$ and $g(x)$, determine $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \times g(x)$, $f(x) \div$

$g(x)$, and $f(g(x))$. Use technology to represent these function combinations. Function Tools Representations Advanced Algebra/Calculus Pathway

6. Given a combination of functions, identify possible component parts by hand and with technology when appropriate. Function Tools Representations Advanced Algebra/Calculus Pathway
7. Use technology to demonstrate the relationship between inverse functions and their compositions. Function Tools Advanced Algebra/Calculus Pathway
8. Use technology to transform a functional model of an authentic context in response to a change in conditions. Function Contextualize Tools Quantitative Reasoning Pathway Statistics Pathway Advanced Algebra/Calculus Pathway

HS.9. Organizing, Summarizing, and Interpreting Data

Data can be collected, displayed, summarized, and interpreted to identify patterns, relationships, and departures from expectations. Categorical data are described using relative frequencies, comparative language, and features of visual representations. Quantitative data are characterized by the shape of the distribution, measures of center and spread, and unusual features. Students compare distributions using both numerical summaries and visual displays to draw meaningful conclusions from data.

1. Classify data that arises from a context as univariate or bivariate and classify whether the variable(s) associated with a set of data are categorical or quantitative (discrete or continuous). Statistics Contextualize Precision Foundational
2. Create visualizations to organize data according to its classification, using appropriate technology. Statistics Tools Representations Foundational
3. Summarize the characteristics of a data display in context using comparative language and appropriate statistics, calculated using technology. Statistics Tools Precision Foundational
4. Justify data-based conclusions about one or two sets of data, using a display or summary statistics as evidence Statistics Communicate Precision Foundational
5. Calculate the percentile and standard score for an individual data point as it compares to a larger group and interpret those in context. Statistics Contextualize Precision Foundational
6. Model a context with a normal distribution using the mean and standard deviation. Interpret the model to solve problems related to the relative position of data points using the empirical rule and describe the results in context. Statistics Model Contextualize Foundational
7. Model a context with a normal or binomial distribution using the mean and standard deviation. With technology, interpret the model to solve problems related to the relative position of any data points, using calculations appropriate to the distribution, and describe the results in context. Statistics Model Contextualize Statistics Pathway
8. Justify data-based conclusions about multiple sets of data using a display or summary statistics as evidence. Statistics Communicate Precision Statistics Pathway

HS.10. Modeling Bivariate Data

Analyzing relationships between two variables allows students to formalize associations, compare groups, and interpret trends across contexts. Students consider when an observed association may or may not suggest a causal relationship, recognizing the importance of study design. Models created from bivariate data support prediction and interpretation, and students evaluate these models by examining their implications, limitations, and the error associated with their predictions.

1. Represent the relationship between two quantitative variables with a linear regression model, using appropriate technology. Interpret key features of the regression model, including the correlation coefficient, in context. Statistics Model Contextualize Foundational
2. Calculate and interpret predicted values and residuals from a linear regression. Explain restrictions on the domain that produce reliable predictions, and explain how extrapolation makes predictions less reliable. Statistics Representations Generalize Foundational
3. For a non-linear data set, use technology to choose an appropriate model. Statistics Tools Tinker Foundational
4. Describe the limitations of applying a regression model to a set of data. Identify potential confounding variables in context. Statistics Representations Sensemaking Foundational
5. Differentiate between correlation and causation. Statistics Communicate Precision Foundational
6. Create and interpret residual plots to formally assess the appropriateness of a linear model. Statistics Tools Generalize Statistics Pathway

HS.11. Drawing Inferences

Statistical inference enables students to draw conclusions about a population by using data from a sample. By integrating ideas from simulation and probability, students describe the confidence they can place in an inference and quantify the potential error in their conclusions. Students learn to evaluate claims, recognize the role of variability, and understand both the power and the limitations of making generalizations from sample data.

1. Articulate what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation. Statistics Communicate Tinker Foundational
2. Employ simulation models for random sampling to estimate a margin of error, determine approximate sampling distributions, and informally evaluate statistical significance. Statistics Tools Sensemaking Foundational
3. Analyze and use statistical evidence to evaluate results and to answer statistical investigative questions about one population parameter or the difference between two population parameters. Justify statistical reasoning and results in written and visual formats. Statistics Conjecture Communicate Foundational
4. Describe a sampling distribution for a population parameter by identifying the mean and standard deviation that would be generated by repeated samples of size n. Articulate how the size of the sample impacts the variability in the sampling distribution of a sample statistic. Statistics Tools Structure Statistics Pathway
5. Represent the distribution of sample averages as a normal distribution by applying the central limit theorem. Statistics Representations Generalize Statistics Pathway
6. Calculate and interpret p-values to evaluate results and to answer statistical questions about a population parameter. Statistics Tools Precision Statistics Pathway

7. Interpret a confidence level as a success rate for capturing a population parameter. Calculate and interpret a confidence interval as an estimate of a population parameter. Statistics Tools Generalize Statistics Pathway
8. Justify a claim using a decision based on a significance test or a confidence interval. Statistics Communicate Sensemaking Statistics Pathway

HS.12. Designing Studies

Surveys, observational studies, and experiments are common methods of data collection that students encounter in many aspects of daily life. Understanding the differences among these designs and the role of randomization in their planning and execution is essential for interpreting results and evaluating the strength of the evidence. Through this work, students develop the critical thinking needed to assess how data are generated and what conclusions can be reliably drawn.

1. Distinguish between sample surveys, experiments, and observational studies. Choose an appropriate data-collection method to answer an investigative question of interest. Statistics Model Sensemaking Foundational
2. Articulate what constitutes good practice in designing a sample survey, an experiment, and an observational study. Statistics Model Communicate Foundational
3. Identify potential sources of bias and confounding variables in a study. Explain the implications of bias and confounding variables in the interpretation of the results. Statistics Communicate Precision Foundational
4. Explain that the purpose of randomly assigning subjects to experimental treatment groups is to make the treatment groups as similar to each other as possible. Statistics Communicate Generalize Foundational
5. Describe the role of randomization in relation to the scope and validity of statistical inferences. Statistics Contextualize Communicate Foundational
6. Design a completely randomized experiment in context. Statistics Model Tinker Statistics Pathway

HS.13. Understanding Bias, Precision, and Sampling

The conditions under which data are collected play a critical role in the conclusions that can be drawn from them. Randomization in sampling helps minimize bias and produce samples that more accurately represent the population. When evaluating the use of statistics in media or public reports, students consider the study design, data collection methods, analytical approaches, and the appropriateness of the resulting summaries and conclusions.

1. Distinguish between sampling methods. Choose a method to accurately represent a population. Statistics Model Sensemaking Foundational
2. Explain how bias may occur in sampling, yielding results that are not representative of the population of interest. Statistics Communicate Precision Foundational
3. Articulate how the size of a sample impacts the variability in a sample statistic. Statistics Model Communicate Foundational
4. Explain that the purpose of randomly selecting subjects from populations is to make the samples as similar to the population as possible. Statistics Communicate Generalize Foundational

5. Design a procedure to generate a simple random sample in context. Statistics Model Tinker Statistics Pathway

HS.14. Reasoning with Chance and Probability

Probability deepens students' understanding of randomness, chance, and uncertainty. Through the study of independence, conditional probability, and compound events, students learn to quantify randomness and assess risk in real-life situations. These concepts form a foundation for statistical reasoning, supporting students in understanding variability, making inferences from data, and drawing conclusions in the presence of uncertainty.

1. Determine whether events are independent. Justify independence of events using Venn diagrams and two-way tables. Statistics Conjecture Sensemaking Foundational
2. Informally explain the concepts of independence and conditional probability in context. Statistics Contextualize Communicate Foundational
3. Calculate probabilities of compound events and conditional events using probability models, including organized lists, Venn diagrams, tree diagrams, two-way tables, or rules of probability. Statistics Tools Representations Foundational
4. Assess decisions that involve risk using probability concepts to justify a conclusion. Statistics Conjecture Sensemaking Foundational
5. Simulate random phenomena using simple probability tools and explain how the results of a simulation should approach the theoretical probability for a large number of trials. Statistics Contextualize Tools Foundational
6. Define a random variable for a quantity of interest by assigning a numerical value to each outcome in a sample space, along with a corresponding probability. Create an appropriate visual for the probability distribution. Statistics Model Precision Statistics Pathway
7. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution in context. Statistics Contextualize Generalize Statistics Pathway
8. Calculate theoretical probabilities using a probability distribution for a random variable defined for a sample space. Statistics Model Precision Statistics Pathway
9. Create a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically. Statistics Model Structure Statistics Pathway
10. Analyze decisions and strategies that use random variables and expected values to justify a conclusion. Statistics Conjecture Sensemaking Statistics Pathway

HS.15. Transforming Shapes

Geometric transformations allow students to analyze how figures change and what attributes are preserved, supporting descriptions of symmetry and congruence. By examining how a figure can be mapped onto itself or another figure, students deepen their understanding of geometric relationships. Transformations also connect geometry with algebra through the lens of functions, as students analyze how rules for shifts, reflections, and dilations correspond to movements of points and figures in the coordinate plane.

1. Represent transformations in the plane using tools such as transparencies, tracing paper, and geometry software, and as functions that take points in the plane as inputs and give other points as outputs. Geometry Tools Communicate Foundational

2. Develop precise definitions of rotations, reflections, and translations in terms of the relationship between the image and preimage. For example, for the definition of a rotation: Points connecting the preimage and image of a rotation lie on concentric circles around the center of rotation. Geometry Conjecture Precision Foundational
3. Use geometric descriptions of rigid motions to draw the transformed figure using appropriate tools such as graph paper, tracing paper, or geometry software, and predict the effect of a given rigid motion on a given figure with justification. Geometry Tools Conjecture Foundational
4. Given two figures, define congruence in terms of rigid motions by identifying a sequence of transformations that maps a preimage to an image. Geometry Communicate Generalize Precision Foundational

HS.16. Justifying Geometric Relationships

Constructing and exploring geometric figures with dynamic technology allows students to investigate how assumptions, constraints, and definitions shape geometric relationships. As students test conjectures and examine which properties depend on or remain independent of those assumptions, they deepen their understanding of geometric structure. This process supports the development of reasoning and justification, preparing students to explain and validate geometric ideas with clarity and precision.

1. Apply definitions of rotations, reflections, and translations to justify triangle congruence and similarity theorems in authentic contexts. Geometry Conjecture Communicate Foundational
2. Perform geometric constructions with geometry software to explore relationships between lines, segments, angles, triangles and parallelograms. Geometry Tools Communicate Structure Foundational
3. Justify and apply theorems of line relationships, angles, triangles, and parallelograms to solve problems in authentic contexts. Geometry Communicate Structure Foundational
4. Describe and represent structures and patterns between inscribed angles, radii, and chords. Relationships should include the relationships between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. Geometry Structure Generalize Advanced Algebra/Calculus Pathway
5. Define the radian measure of an angle and use a radian to calculate arc length and area of a sector. Geometry Sensemaking Precision Advanced Algebra/Calculus Pathway

HS.17. Reasoning About Congruence and Similarity

Reasoning about congruence involves demonstrating that one figure can be mapped onto another through a rigid motion, or a sequence of translations, rotations, reflections, or glide reflections. Reasoning about similarity requires identifying a dilation, or a combination of a dilation with rigid motions, that maps one figure onto another. Through these transformation- based definitions, students develop a coherent understanding of congruence and similarity and use them to justify geometric relationships.

1. Use the definition of congruence in terms of rigid motions to show that two figures are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Geometry Conjecture Communicate Foundational
2. Communicate geometric arguments about lines and angles using proofs and logical reasoning. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Geometry Conjecture Communicate Foundational
3. Communicate geometric arguments about triangles using proofs and logical reasoning. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Geometry Conjecture Communicate Foundational
4. Communicate geometric arguments about quadrilaterals using proofs and logical reasoning. Properties and theorems may include: relationship of the diagonals of a kite, opposite sides of a parallelogram are congruent, opposite angles of a parallelogram are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Geometry Tools Conjecture Communicate Foundational
5. Show that two figures are similar by using similarity transformations. Geometry Conjecture Communicate Generalize Foundational

HS.18. Analyzing Triangles via Similarity and Trigonometry

Triangle similarity focuses the concept of similarity transformations on a single, fundamental shape, allowing students to develop proportional relationships that connect directly to real-world situations. Building on these ideas, trigonometry provides powerful tools for analyzing triangles and modeling physical phenomena. Together, similarity and trigonometry form a foundation for solving contextual problems and interpreting geometric relationships in the world around us.

1. Communicate geometric arguments about triangle similarity theorems using proofs, logical reasoning, and geometry software. Properties and theorems include: use the properties of similarity transformations to establish the AA criterion for two triangles to be similar, a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. Geometry Tools Communicate Foundational
2. Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Geometry Generalize Precision Foundational
3. Model and solve right-triangle problems in authentic contexts using Pythagorean Theorem and trigonometric ratios. Geometry Model Contextualize Foundational
4. Calculate unknown measurements in triangles by applying the Law of Sines and the Law of Cosines. Geometry Contextualize Foundational
5. Explain and use the relationship between the sine and cosine of complementary angles. Geometry Structure Generalize Advanced Algebra/Calculus Pathway

HS.19. Measuring

Geometry models the physical world by analyzing shapes and their measurable attributes. Students learn to simplify complex objects into manageable parts; calculate lengths, areas, and volumes; and recombine these parts to make sense of the whole. Measurement provides essential tools for understanding shape and size, allowing students to make sense of real-world objects and interpret the quantities that describe them.

1. Write the equation of a circle in standard form using the center of the circle and the radius. May use technology to graph and identify the center of a circle and/or radius using the distance formula or Pythagorean Theorem. Geometry Tools Sensemaking Foundational
2. Calculate perimeters of polygons and areas of triangles and rectangles using coordinates and the distance formula. Geometry Sensemaking Foundational
3. Explain the relationship among formulas for the perimeters, areas, and volumes of two- and three-dimensional figures. Figures include circles, polygons, cylinders, prisms, cones, pyramids, and spheres. Geometry Generalize Sensemaking Foundational
4. Solve tasks using area formulas for triangles, parallelograms, trapezoids, regular polygons, and circles in authentic contexts. Geometry Model Sensemaking Foundational
5. Solve tasks using surface area and volume formulas for cylinders, pyramids, cones, and spheres in authentic contexts. Geometry Model Sensemaking Foundational
6. Explain the change in area and volume under dilation transformations. Meaning, when scale factor k is applied to a figure, area scales by k^2 and volume scales by k^3. Geometry Generalize Sensemaking Precision Foundational
7. Identify the shapes of two-dimensional cross-sections of three-dimensional objects and visualize solids generated by rotating two-dimensional figures using physical models or technology. Geometry Structure Sensemaking Foundational
8. Use geometric shapes, their measures, and their properties to describe real-world objects, and solve related authentic modeling tasks. Geometry Model Contextualize Sensemaking Foundational
9. Apply concepts of density based on area and volume in authentic modeling situations. Geometry Model Contextualize Quantitative Reasoning Pathway