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COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Colorado Academic
STANDARDS

Sixth Grade

Mathematics

Mathematics

Colorado Academic Standards in Mathematics and The Common Core State Standards for Mathematics

On December 10, 2009, the Colorado State Board of Education adopted the revised Mathematics Academic Standards, along with academic standards in nine other content areas, creating Colorado's first fully aligned preschool through high school academic expectations. Developed by a broad spectrum of Coloradans representing Pre-K and K-12 education, higher education, and business, utilizing the best national and international exemplars, the intention of these standards is to prepare Colorado schoolchildren for achievement at each grade level, and ultimately, for successful performance in postsecondary institutions and/or the workforce.

Concurrent to the revision of the Colorado standards was the Common Core State Standards (CCSS) initiative, whose process and purpose significantly overlapped with that of the Colorado Academic Standards. Led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), these standards present a national perspective on academic expectations for students, Kindergarten through High School in the United States.

Upon the release of the Common Core State Standards for Mathematics on June 2, 2010, the Colorado Department of Education began a gap analysis process to determine the degree to which the expectations of the Colorado Academic Standards aligned with the Common Core. The independent analysis proved a nearly 95% alignment between the two sets of standards. On August 2, 2010, the Colorado State Board of Education adopted the Common Core State Standards, and requested the integration of the Common Core State Standards and the Colorado Academic Standards.

In partnership with the dedicated members of the Colorado Standards Revision Subcommittee in Mathematics, this document represents the integration of the combined academic content of both sets of standards, maintaining the unique aspects of the Colorado Academic Standards, which include personal financial literacy, 21st century skills, school readiness competencies, postsecondary and workforce readiness competencies, and preschool expectations. The result is a world-class set of standards that are greater than the sum of their parts.

The Colorado Department of Education encourages you to review the Common Core State Standards and the extensive appendices at www.corestandards.org. While all the expectations of the Common Core State Standards are embedded and **coded with CCSS:** in this document, additional information on the development and the intentions behind the Common Core State Standards can be found on the website.

Colorado Academic Standards
Mathematics Standards

"Pure mathematics is, in its way, the poetry of logical ideas."
Albert Einstein

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"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."
Dr. Michael Brown, Nobel Prize Laureate

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In the 21st century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how 21st century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The "Continuum of State Standards Definitions" section that follows shows the hierarchical order of the standards components. The "Standards Template" section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

Prepared Graduate Competencies: The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Standard: The topical organization of an academic content area.

High School Expectations: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. *What do students need to know in high school?*

Grade Level Expectations: The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

Evidence Outcomes: The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

21st Century Skills and Readiness Competencies: Includes the following:

- ***Inquiry Questions:***
  Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

- ***Relevance and Application:***
  Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

- ***Nature of the Discipline:***
  The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Continuum of State Standards Definitions

Prepared Graduate Competency
Prepared Graduate Competencies are the P-12 concepts and skills that all students leaving the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Standards
Standards are the topical organization of an academic content area.

P-8

Grade Level Expectations
Expectations articulate, at each grade level, the knowledge and skills of a standard that indicates a student is making progress toward high school.

What do students need to know?

Evidence Outcomes
Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.

How do we know that a student can do it?

21st Century and PWR Skills
Inquiry Questions:
Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
Relevance and Application:
Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
Nature of the Discipline:
The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

High School

High School Expectations
Expectations articulate the knowledge and skills of a standard that indicates a student is making progress toward being a prepared graduate.

What do students need to know?

Evidence Outcomes
Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.

How do we know that a student can do it?

21st Century and PWR Skills
Inquiry Questions:
Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
Relevance and Application:
Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
Nature of the Discipline:
The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

STANDARDS TEMPLATE

Content Area: NAME OF CONTENT AREA
Standard: The topical organization of an academic content area.

Prepared Graduates:

- The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

High School and Grade Level Expectations

Concepts and skills students master:

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

What do students need to know?

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: Evidence outcomes are the indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*	**Inquiry Questions:** Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation. **Relevance and Application:** Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context. **Nature of the Discipline:** The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Colorado Academic Standards
Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Modeling Across the Standards
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Standards for Mathematical Practice
from
The Common Core State Standards for Mathematics

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 *equals the well remembered* $7 \times 5 + 7 \times 3$*, in preparation for learning about the distributive property. In the expression* $x^2 + 9x + 14$*, older students can see the 14 as* 2×7 *and the 9 as* $2 + 7$*. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or* *as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.*

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Sixth Grade	
1. Number Sense, Properties, and Operations	1. Quantities can be expressed and compared using ratios and rates 2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency 3. In the real number system, rational numbers have a unique location on the number line and in space
2. Patterns, Functions, and Algebraic Structures	1. Algebraic expressions can be used to generalize properties of arithmetic 2. Variables are used to represent unknown quantities within equations and inequalities
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Objects in space and their parts and attributes can be measured and analyzed

From the Common State Standards for Mathematics, Pages 39-40

Mathematics | Grade 6
In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios,

and they use equations (such as 3x = y) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

21st Century Skills and Readiness Competencies in Mathematics

Mathematics in Colorado's description of 21st century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21st century skills, as follows:

Critical Thinking and Reasoning
Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

Information Literacy
The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

Collaboration
Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

Self-Direction
Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

Invention
Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

Colorado's Description for School Readiness
(Adopted by the State Board of Education, December 2008)
School readiness describes both the preparedness of a child to engage in and benefit from learning experiences, and the ability of a school to meet the needs of all students enrolled in publicly funded preschools or kindergartens. School readiness is enhanced when schools, families, and community service providers work collaboratively to ensure that every child is ready for higher levels of learning in academic content.

Colorado's Description of Postsecondary and Workforce Readiness
(Adopted by the State Board of Education, June 2009)
Postsecondary and workforce readiness describes the knowledge, skills, and behaviors essential for high school graduates to be prepared to enter college and the workforce and to compete in the global economy. The description assumes students have developed consistent intellectual growth throughout their high school career as a result of academic work that is increasingly challenging, engaging, and coherent. Postsecondary education and workforce readiness assumes that students are ready and able to demonstrate the following without the need for remediation: Critical thinking and problem-solving; finding and using information/information technology; creativity and innovation; global and cultural awareness; civic responsibility; work ethic; personal responsibility; communication; and collaboration.

How These Skills and Competencies are Embedded in the Revised Standards
Three themes are used to describe these important skills and competencies and are interwoven throughout the standards: *inquiry questions; relevance and application; and the nature of each discipline.* These competencies should not be thought of stand-alone concepts, but should be integrated throughout the curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without the content to think about, it is equally impossible for students to understand the content of a discipline without grappling with complex questions and the investigation of topics.

Inquiry Questions – Inquiry is a multifaceted process requiring students to think and pursue understanding. Inquiry demands that students (a) engage in an active observation and questioning process; (b) investigate to gather evidence; (c) formulate explanations based on evidence; (d) communicate and justify explanations, and; (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

Relevance and Application – The hallmark of learning a discipline is the ability to apply the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, developing, adapting, and refining solutions for the betterment of society. The application of a discipline, including how technology assists or accelerates the work, enables students to more fully appreciate how the mastery of the grade level expectation matters after formal schooling is complete.

Nature of Discipline – The unique advantage of a discipline is the perspective it gives the mind to see the world and situations differently. The characteristics and viewpoint one keeps as a result of mastering the grade level expectation is the nature of the discipline retained in the mind's eye.

1.Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Quantities can be expressed and compared using ratios and rates

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.[1] (CCSS: 6.RP.1) b. Apply the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.[2] (CCSS: 6.RP.2) c. Use ratio and rate reasoning to solve real-world and mathematical problems.[3] (CCSS: 6.RP.3) i. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. (CCSS: 6.RP.3a) ii. Use tables to compare ratios. (CCSS: 6.RP.3a) iii. Solve unit rate problems including those involving unit pricing and constant speed.[4] (CCSS: 6.RP.3b) iv. Find a percent of a quantity as a rate per 100.[5] (CCSS: 6.RP.3c) v. Solve problems involving finding the whole, given a part and the percent. (CCSS: 6.RP.3c) vi. Use common fractions and percents to calculate parts of whole numbers in problem situations including comparisons of savings rates at different financial institutions (PFL) vii. Express the comparison of two whole number quantities using differences, part-to-part ratios, and part-to-whole ratios in real contexts, including investing and saving (PFL) viii. Use ratio reasoning to convert measurement units.[6] (CCSS: 6.RP.3d)	**Inquiry Questions:** 1. How are ratios different from fractions? 2. What is the difference between quantity and number? **Relevance and Application:** 1. Knowledge of ratios and rates allows sound decision-making in daily life such as determining best values when shopping, creating mixtures, adjusting recipes, calculating car mileage, using speed to determine travel time, or making saving and investing decisions. 2. Ratios and rates are used to solve important problems in science, business, and politics. For example developing more fuel-efficient vehicles, understanding voter registration and voter turnout in elections, or finding more cost-effective suppliers. 3. Rates and ratios are used in mechanical devices such as bicycle gears, car transmissions, and clocks. **Nature of Mathematics:** 1. Mathematicians develop simple procedures to express complex mathematical concepts. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians reason abstractly and quantitatively. (MP)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Fluently divide multi-digit numbers using standard algorithms. (CCSS: 6.NS.2) b. Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithms for each operation. (CCSS: 6.NS.3) c. Find the greatest common factor of two whole numbers less than or equal to 100. (CCSS: 6.NS.4) d. Find the least common multiple of two whole numbers less than or equal to 12. (CCSS: 6.NS.4) e. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.[7] (CCSS: 6.NS.4) f. Interpret and model quotients of fractions through the creation of story contexts.[8] (CCSS: 6.NS.1) g. Compute quotients of fractions.[9] (CCSS: 6.NS.1) h. Solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.[10] (CCSS: 6.NS.1)	**Inquiry Questions:** 1. Why might estimation be better than an exact answer? 2. How do operations with fractions and decimals compare to operations with whole numbers? **Relevance and Application:** 1. Rational numbers are an essential component of mathematics. Understanding fractions, decimals, and percentages is the basis for probability, proportions, measurement, money, algebra, and geometry. **Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians look for and make use of structure. (MP)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

3. In the real number system, rational numbers have a unique location on the number line and in space

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain why positive and negative numbers are used together to describe quantities having opposite directions or values.[11] (CCSS: 6.NS.5) i. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (CCSS: 6.NS.5) b. Use number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates.[12] (CCSS: 6.NS.6) i. Describe a rational number as a point on the number line. (CCSS: 6.NS.6) ii. Use opposite signs of numbers to indicate locations on opposite sides of 0 on the number line. (CCSS: 6.NS.6a) iii. Identify that the opposite of the opposite of a number is the number itself.[13] (CCSS: 6.NS.6a) iv. Explain when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (CCSS: 6.NS.6b) v. Find and position integers and other rational numbers on a horizontal or vertical number line diagram. (CCSS: 6.NS.6c) vi. Find and position pairs of integers and other rational numbers on a coordinate plane. (CCSS: 6.NS.6c) c. Order and find absolute value of rational numbers. (CCSS: 6.NS.7) i. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.[14] (CCSS: 6.NS.7a) ii. Write, interpret, and explain statements of order for rational numbers in real-world contexts.[15] (CCSS: 6.NS.7b) iii. Define the absolute value of a rational number as its distance from 0 on the number line and interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.[16] (CCSS: 6.NS.7c) iv. Distinguish comparisons of absolute value from statements about order.[17] (CCSS: 6.NS.7d) d. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane including the use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS: 6.NS.8)	**Inquiry Questions:** 1. Why are there negative numbers? 2. How do we compare and contrast numbers? 3. Are there more rational numbers than integers? **Relevance and Application:** 1. Communication and collaboration with others is more efficient and accurate using rational numbers. For example, negotiating the price of an automobile, sharing results of a scientific experiment with the public, and planning a party with friends. 2. Negative numbers can be used to represent quantities less than zero or quantities with an associated direction such as debt, elevations below sea level, low temperatures, moving backward in time, or an object slowing down **Nature of Mathematics:** 1. Mathematicians use their understanding of relationships among numbers and the rules of number systems to create models of a wide variety of situations. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians attend to precision. (MP)

Standard: 1. Number Sense, Properties, and Operations
Sixth Grade

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[1] For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." (CCSS: 6.RP.1)
[2] For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (CCSS: 6.RP.2)
[3] e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (CCSS: 6.RP.3)
[4] For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? (CCSS: 6.RP.3b)
[5] e.g., 30% of a quantity means 30/100 times the quantity. (CCSS: 6.RP.3c)
[6] manipulate and transform units appropriately when multiplying or dividing quantities. (CCSS: 6.RP.3d)
[7] For example, express 36 + 8 as 4 (9 + 2). (CCSS: 6.NS.4)
[8] For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (CCSS: 6.NS.1)
[9] In general, (a/b) ÷ (c/d) = ad/bc.). (CCSS: 6.NS.1)
[10] How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? (CCSS: 6.NS.1)
[11] e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). (CCSS: 6.NS.5)
[12] Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. (CCSS: 6.NS.6)
[13] e.g., −(−3) = 3, and that 0 is its own opposite. (CCSS: 6.NS.6a)
[14] For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right. (CCSS: 6.NS.7a)
[15] For example, write −3 °C > −7 °C to express the fact that −3 °C is warmer than −7 °C. (CCSS: 6.NS.7b)
[16] For example, for an account balance of −30 dollars, write |−30| = 30 to describe the size of the debt in dollars. (CCSS: 6.NS.7c)
[17] For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars. (CCSS: 6.NS.7d)

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Algebraic expressions can be used to generalize properties of arithmetic

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Write and evaluate numerical expressions involving whole-number exponents. (CCSS: 6.EE.1) b. Write, read, and evaluate expressions in which letters stand for numbers. (CCSS: 6.EE.2) i. Write expressions that record operations with numbers and with letters standing for numbers.[1] (CCSS: 6.EE.2a) ii. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) and describe one or more parts of an expression as a single entity.[2] (CCSS: 6.EE.2b) iii. Evaluate expressions at specific values of their variables including expressions that arise from formulas used in real-world problems.[3] (CCSS: 6.EE.2c) iv. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). (CCSS: 6.EE.2c) c. Apply the properties of operations to generate equivalent expressions.[4] (CCSS: 6.EE.3) d. Identify when two expressions are equivalent.[5] (CCSS: 6.EE.4)	**Inquiry Questions:** 1. If we didn't have variables, what would we use? 2. What purposes do variable expressions serve? 3. What are some advantages to being able to describe a pattern using variables? 4. Why does the order of operations exist? 5. What other tasks/processes require the use of a strict order of steps? **Relevance and Application:** 1. The simplification of algebraic expressions allows one to communicate mathematics efficiently for use in a variety of contexts. 2. Using algebraic expressions we can efficiently expand and describe patterns in spreadsheets or other technologies. **Nature of Mathematics:** 1. Mathematics can be used to show that things that seem complex can be broken into simple patterns and relationships. 2. Mathematics can be expressed in a variety of formats. 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians look for and make use of structure. (MP) 5. Mathematicians look for and express regularity in repeated reasoning. (MP)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Variables are used to represent unknown quantities within equations and inequalities

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
a. Describe solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? (CCSS: 6.EE.5)
b. Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS: 6.EE.5)
c. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. (CCSS: 6.EE.6)
  i. Recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (CCSS: 6.EE.6)
d. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (CCSS: 6.EE.7)
e. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. (CCSS: 6.EE.8)
f. Show that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS: 6.EE.8)
g. Represent and analyze quantitative relationships between dependent and independent variables. (CCSS: 6.EE)
  i. Use variables to represent two quantities in a real-world problem that change in relationship to one another. (CCSS: 6.EE.9)
  ii. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (CCSS: 6.EE.9)
  iii. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.[6] (CCSS: 6.EE.9)
	Inquiry Questions:
	1. Do all equations have exactly one unique solution? Why?
	2. How can you determine if a variable is independent or dependent?
	
	Relevance and Application:
	1. Variables allow communication of big ideas with very few symbols. For example, d = r * t is a simple way of showing the relationship between the distance one travels and the rate of speed and time traveled, and $C = \pi d$ expresses the relationship between circumference and diameter of a circle.
	2. Variables show what parts of an expression may change compared to those parts that are fixed or constant. For example, the price of an item may be fixed in an expression, but the number of items purchased may change.
	
	Nature of Mathematics:
	1. Mathematicians use graphs and equations to represent relationships among variables. They use multiple representations to gain insights into the relationships between variables.
	2. Mathematicians can think both forward and backward through a problem. An equation is like the end of a story about what happened to a variable. By reading the story backward, and undoing each step, mathematicians can find the value of the variable.
	3. Mathematicians model with mathematics. (MP)

[1] For example, express the calculation “Subtract y from 5” as 5 – y. (CCSS: 6.EE.2a)
[2] For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. (CCSS: 6.EE.2b)
[3] For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = 1/2. (CCSS: 6.EE.2c)
[4] For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. (CCSS: 6.EE.3)
[5] i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. Reason about and solve one-variable equations and inequalities. (CCSS: 6.EE.4)
[6] For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time. (CCSS: 6.EE.9)

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.[1] (CCSS: 6.SP.1) b. Demonstrate that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (CCSS: 6.SP.2) c. Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (CCSS: 6.SP.3) d. Summarize and describe distributions. (CCSS: 6.SP) i. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (CCSS: 6.SP.4) ii. Summarize numerical data sets in relation to their context. (CCSS: 6.SP.5) 1. Report the number of observations. (CCSS: 6.SP.5a) 2. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. (CCSS: 6.SP.5b) 3. Give quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (CCSS: 6.SP.5c) 4. Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS: 6.SP.5d)	**Inquiry Questions:** 1. Why are there so many ways to describe data? 2. When is one data display better than another? 3. When is one statistical measure better than another? 4. What makes a good statistical question? **Relevance and Application:** 1. Comprehension of how to analyze and interpret data allows better understanding of large and complex systems such as analyzing employment data to better understand our economy, or analyzing achievement data to better understand our education system. 2. Different data analysis tools enable the efficient communication of large amounts of information such as listing all the student scores on a state test versus using a box plot to show the distribution of the scores. **Nature of Mathematics:** 1. Mathematicians leverage strategic displays to reveal data. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)

[1] For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages. (CCSS: 6.SP.1)

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Objects in space and their parts and attributes can be measured and analyzed

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can a. Develop and apply formulas and procedures for area of plane figures i. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. (CCSS: 6.G.1) ii. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.1) b. Develop and apply formulas and procedures for volume of regular prisms. i. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths. (CCSS: 6.G.2) ii. Show that volume is the same as multiplying the edge lengths of a rectangular prism. (CCSS: 6.G.2) iii. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS: 6.G.2) c. Draw polygons in the coordinate plan to solve real-world and mathematical problems. (CCSS: 6.G.3) i. Draw polygons in the coordinate plane given coordinates for the vertices. ii. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. (CCSS: 6.G.3) d. Develop and apply formulas and procedures for the surface area. i. Represent three-dimensional figures using nets made up of rectangles and triangles. (CCSS: 6.G.4) ii. Use nets to find the surface area of figures. (CCSS: 6.G.4) iii. Apply techniques for finding surface area in the context of solving real-world and mathematical problems. (CCSS: 6.G.4)	**Inquiry Questions:** 1. Can two shapes have the same volume but different surface areas? Why? 2. Can two figures have the same surface area but different volumes? Why? 3. What does area tell you about a figure? 4. What properties affect the area of figures? **Relevance and Application:** 1. Knowledge of how to find the areas of different shapes helps do projects in the home and community. For example how to use the correct amount of fertilizer in a garden, buy the correct amount of paint, or buy the right amount of material for a construction project. 2. The application of area measurement of different shapes aids with everyday tasks such as buying carpeting, determining watershed by a center pivot irrigation system, finding the number of gallons of paint needed to paint a room, decomposing a floor plan, or designing landscapes. **Nature of Mathematics:** 1. Mathematicians realize that measurement always involves a certain degree of error. 2. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians reason abstractly and quantitatively. (MP)

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