Balanced Assessment

ADVANCED HIGH SCHOOL ASSESSMENT

PACKAGE 2

Balanced Assessment for the Mathematics Curriculum

BERKELEY ■ HARVARD ■ MICHIGAN STATE ■ SHELL CENTRE

Dale Seymour Publications®

Expanded Table of Contents *

Long Tasks	Task Type	Circumstances of Performance
1. Survey Says	30-minute evaluation task; applied power in a nonroutine adult context	individual response, or in pairs
2. World Oil Consumption	30-minute interpretation and prediction task; applied power in a nonroutine adult context	individual written response
3. Disc-Ness	45-minute definition task; nonroutine mathematical concepts and connections; open middle	in pairs, or small groups
4. Don't Fence Me In	45-minute problem; pure mathematics; nonroutine context; open middle	individual response, or in pairs
5. Garages and Phones	45-minute representation and interpretation task; applied power; nonroutine context from adult life	individual response, or in pairs
6. MasterMind	45-minute problem; applied power in a nonroutine game context	individual response, or in pairs
7. Telephone Service	60-minute evaluation and recommendation task; applied power in a nonroutine context from adult life	individual response, or in pairs
8. All Aboard!	45-minute evaluation and recommendation task; applied power in a nonroutine context from adult life	individual response, or in pairs
9. Cheetah's Lunch	60-minute evaluation and recommendation task; applied power in a nonroutine context from adult life	individual response, or in pairs

* For explanations of terms that may be unfamiliar, see the Glossary and the *Dimensions of Balance* table in the Introduction.

Advanced High School Package 2

Mathematical Content	Mathematical Processes
Data, Statistics, and Probability: interpret given data; represent it graphically; explain the reasoning	interpretation; graphical representation; communication
Data, Statistics, and Probability, with Function: interpret given data; calculate rates of increase; make predictions	interpretation; manipulation and estimation; explanation
Patterns, Functions, and Algebra, in a Geometric context: construct an algebraic definition of a geometric concept	formulation of the concept; transformation; interpretation and evaluation of trial definitions
Geometry, Space, and Shape: accessible regions; speed, time, and distance	formulation; geometrical construction
Patterns, Functions, and Algebra: graphical representation; linear and piecewise constant functions	formulation of a graphical representation of a function, described in words; interpretation and evaluation
Data, Statistics, and Probability: compute the combinations of different arrangements	formulation and computation combinations; suggest generalizations
Patterns, Functions, and Algebra, with Data: interpret given data to formulate functions; represent graphically; interpret and recommend	formulation; representation; interpretation; evaluation; recommendation
Patterns, Functions, and Algebra, with Data: interpret given data to formulate functions; represent graphically; interpret	formulation; representation; interpretation; evaluation
Patterns, Functions, and Algebra, with Data: interpret given data to formulate functions; represent graphically, or symbolically; interpret	formulation; representation; interpretation; evaluation

Expanded Table of Contents

Short Tasks	Task Type	Circumstances of Performance
10. L to Scale	15-minute problem; applied power in an adult context; some nonroutine mathematical connections	individual response, or in pairs
11. Multifigures	15-minute problem; pure mathematics; nonroutine context	individual response, or in pairs
12. Ostrich and Seahorse	15-minute problem; applied power in a nonroutine adult context	individual response, or in pairs
13. Toilet Graph	15-minute problem; applied power in a nonroutine context from student life	individual response, or in pairs
14. Birthday Card	25-minute problem; pure mathematics; nonroutine context; open middle	individual response, or in pairs

Advanced High School Package 2

Mathematical Content	Mathematical Processes
Geometry, Space, and Shape: scaling of drawing; area and perimeter	interpretation of drawing; transformation of scale
Geometry, Space, and Shape: scaling of drawing; recognizing and performing linear scaling of perimeter, quadratic scaling of area	interpretation of drawing; transformation of scale
Geometry, Space, and Shape: linear scaling of drawing to real situation; relating lengths	measurement of drawing; transformation of scale; judging appropriate precision
Patterns, Functions, and Algebra: graphical representation—sketching graph of everyday situation	formulation of the practical situation; representation as a line graph
Patterns, Functions, and Algebra: analysis of a given algorithm; symbolic representation of verbal algorithm	interpretation; symbolic representation

Student D

Disc-Ness

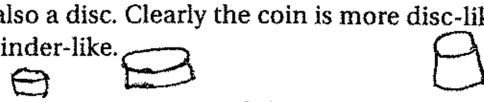
	w	h
Quarter	1"	$\frac{7}{8}$"
Tuna can	3"	$1\frac{1}{2}$"
Soup can	$2\frac{1}{3}$"	4"
Tube	$1\frac{3}{4}$"	10"
Straw	$\frac{1}{4}$"	8"
Spaghetti	$\frac{1}{16}$"	$8\frac{1}{2}$"

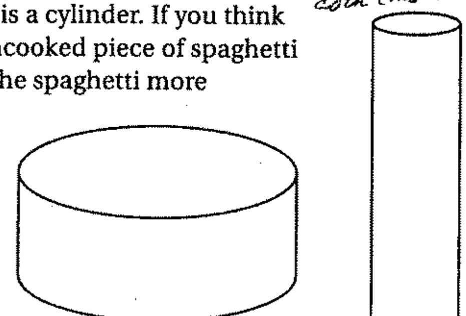
This problem gives you the chance to

- *choose the most and least disc-like objects based on their actual measurements*
- *devise an algebraic measure for the disc-ness of any object*
- *apply this measure to other objects that are cylindrical, and devise alternative ways of measuring this geometric property*

Width matters more than height so the width should matter much more than height, but height should still be a factor. In discness it is the soup can, tuna can then coin (most)

A coin is a disc, and an uncooked piece of spaghetti is a cylinder. If you think about it, however, a coin is also a cylinder and an uncooked piece of spaghetti is also a disc. Clearly the coin is more disc-like and the spaghetti more cylinder-like.





1. Given a coin, a tuna fish can, and a soup can: Devise a definition of disc-ness that allows you to say which object is the most disc-like and which is the least disc-like.

2. Given a mailing tube, a straw, and a piece of uncooked spaghetti: Use your definition of disc-ness to determine which object is the most disc-like and which is the least. the mailing tube would have the most discness, than the straw and least would be the uncooked spaghetti.

3. Write a formula (or algorithm or algebraic sentence) that expresses your measure of disc-ness. You may introduce any labels and definitions and use any mathematical language that you think is necessary.
   disc-ness: $4w \div \frac{1}{2}h$

4. Make any measurements you need and calculate a numerical value of disc-ness for each of the six items. mailing tube = .35 straw = .0625 uncooked spaghetti = .015 tuna can = 16 soup can = 1.25 coin = 64

5. How would you change your answers to these questions if you were asked to write a formula for cylinder-ness rather than disc-ness?
   Our answers would have changed because we would have taken the height of the object into effect much more than width instead of the other way around, so it would be $4h \div \frac{w}{2}$

Task 4

Overview

Demonstrate an understanding of relationships between distance, time, and speed.

Don't Fence Me In

Long Task

Task Description

The intent of this task is to probe the understanding of relationships between distance, time, and speed.

Assumed Mathematical Background

Students should be able to derive conclusions from visual and numerical data.

Core Elements of Performance

- demonstrate an understanding of the relationships involved
- graphically depict the interdependence of the relationships and the constraints involved

Circumstances

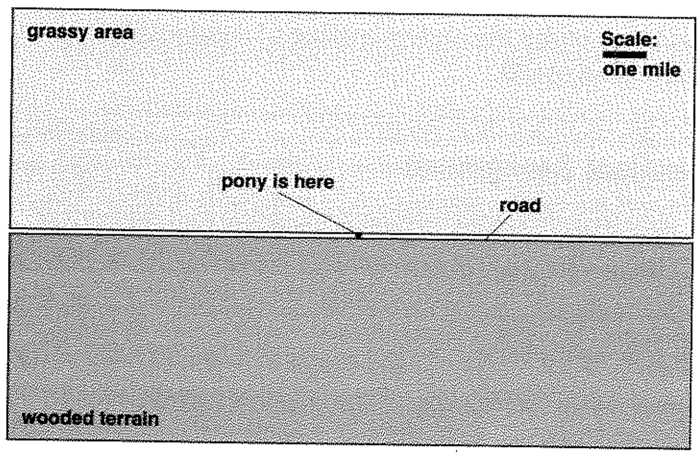
Grouping:	Students may work individually or in pairs.
Materials:	rulers
Estimated time:	45 minutes

Name Date

Don't Fence Me In

This problem gives you the chance to

- *demonstrate your understanding of the relationships between distance, time, and speed*
- *draw a picture that accurately depicts the described situation*



A pony is standing still on a long straight road. The road surface is such that the pony can move along the road at 5 miles per hour.

On one side of the road is a grassy area through which the pony can travel at 4 miles per hour.

On the other side of the road is wooded terrain through which the pony can travel at 3 miles per hour.

Suppose you want to erect a fence that would enclose all of the area that the pony can reach in one hour. (Ignore the width of the road.) Show where the fence would go if:

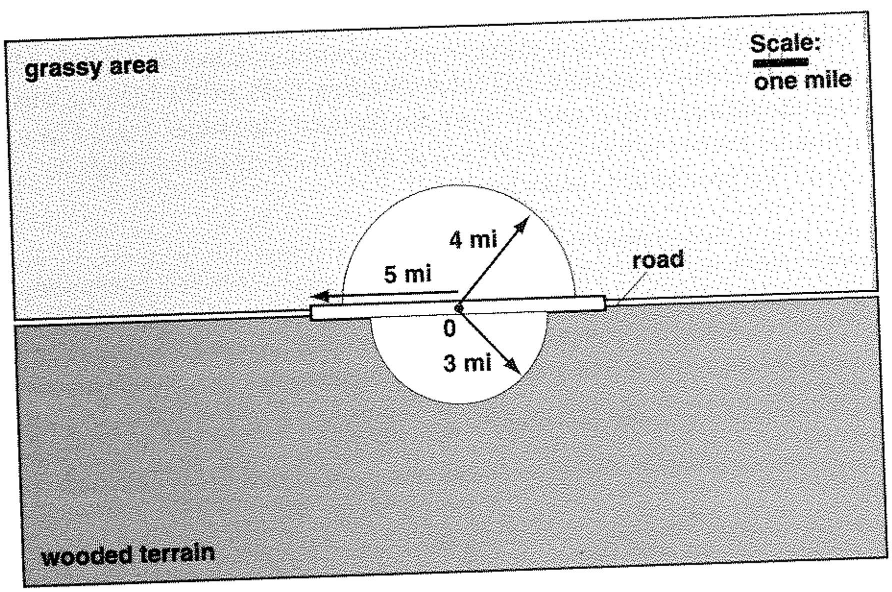
1. Once the pony starts in a given direction it does not change direction. (Show fence with a solid line.)
2. The pony moves freely. (Show fence with a dotted line.)

In each case explain your answer.

Task 4

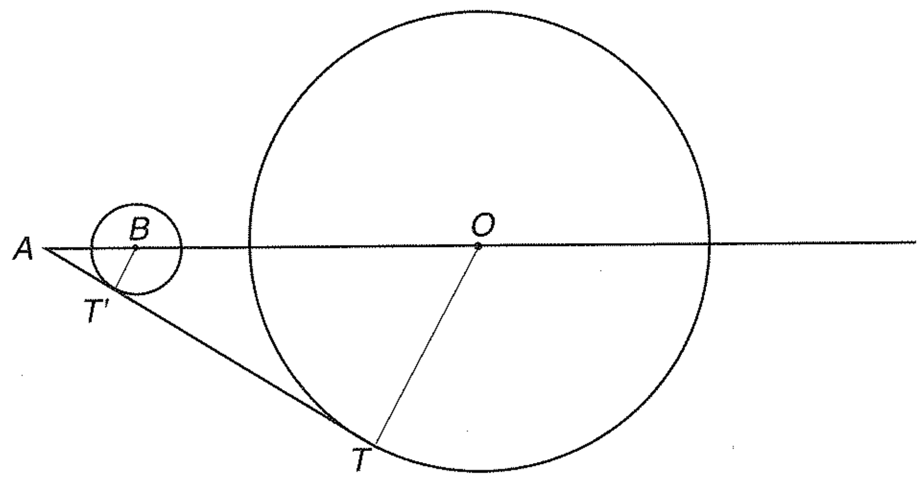
A Sample Solution

1. The pony's speed is constant on either side of the road, so starting from a specified point and going in one direction (along a straight path), the distance covered in a specified amount of time would be the same in any direction. This means that the pony can reach all the points on one side of the road that are equidistant from the point where it starts. That is a semicircle. In the field, the radius of the semicircle would be 4 miles, in the wooded terrain, the radius would be 3 miles, and on the road, it would be 5 miles. Since drawing a circle on a straight line is not possible, the "circle" is actually only a segment, centered at O and extending 5 miles in either direction.

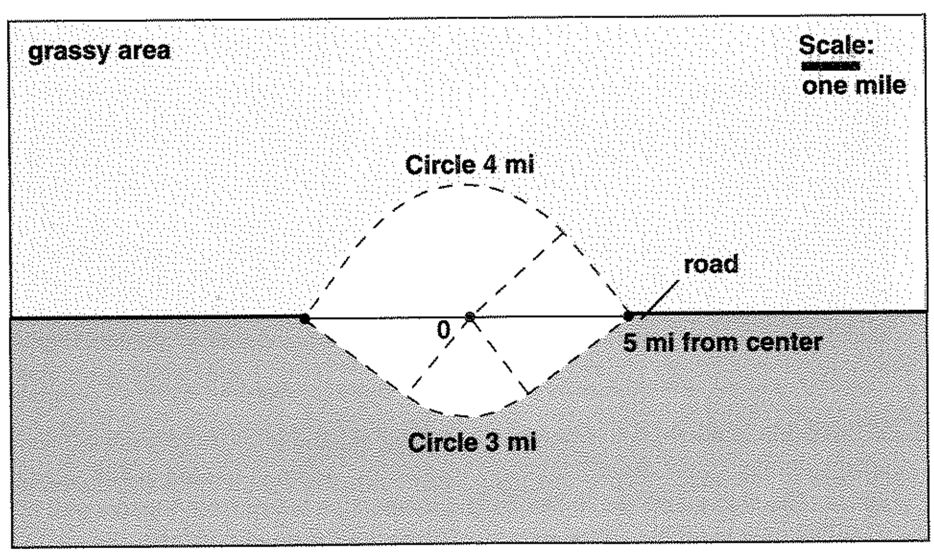


2. If the pony is allowed to change directions once it is in the woods, it cannot cover more than 3 miles in one hour, so it would stay within the circle. The same applies to the field. However, it is possible for the pony to follow the road for some period of time and then get off into the woods. Then it would have a few minutes left and it can cover the area inside a small semicircle as shown on the next page.

Task 4



The radius of the small circle depends entirely on the position of the center. In fact, the radius is exactly $\frac{3}{5}$ of the distance left to cover to point A. So, $|BT'| = \frac{3}{5}|BA|$, but the same ratio applies to the large semicircle $|OT| = \frac{3}{5}|OA|$. Since the angles between tangents and corresponding radii are 90 degrees, this means the triangles ABT' and AOT are similar and the points T and T' are on one line no matter where point B (where the pony gets off the road) is located. The picture now looks like this:



This is the final figure.

(Note: the circles and lines are not drawn with the same scale as the one given in the picture, but the relative lengths are represented accurately.)

Using this Task

Task 4

Extensions

Imagine a circular road with a radius of 5 miles, on which you can travel at a speed of 5 miles/hour. Inside this circular road is a wooded terrain, through which you can travel at 3 miles/hour; outside the circular road is a grassy area through which you can travel at 4 miles/hour.

What is the shape of the area reachable in one hour?

Task 4

Characterizing Performance

This section offers a characterization of student responses and provides indications of the ways in which the students were successful or unsuccessful in engaging with and completing the task. The descriptions are keyed to the *Core Elements of Performance.* Our global descriptions of student work range from "The student needs significant instruction" to "The student's work meets the essential demands of the task." Samples of student work that exemplify these descriptions of performance are included below, accompanied by commentary on central aspects of each student's response. These sample responses are *representative;* they may not mirror the global description of performance in all respects, being weaker in some and stronger in others.

The characterization of student responses for this task is based on these *Core Elements of Performance:*

1. Demonstrate an understanding of the relationships involved.
2. Graphically depict the interdependence of the relationships and the constraints involved.

Descriptions of Student Work

The student needs significant instruction.

The student is unable to graphically depict semicircular regions of different radii.

Student A

This student has only been able to transfer the given information to the picture, and to draw straight-line paths for these rates of travel.

The student needs some instruction.

The student is able to identify semicircular regions of different radii in question 1, but not 2. The fence boundary may not be completely connected or clearly defined.

Student B

This student has developed an appropriate scale and has drawn the correct semicircles for question 1, but seems unclear as to what the actual boundary of the fence should look like.

The student's work needs some revision.

The student recognizes that semicircles must be the basis of the constructions, and that nonstraight paths off the road are not important in describing the region. An adequately labeled diagram indicating the region and a clear fence or boundary is provided, but there may be little verbal explanation of the arguments and constructions.

Student C

This student has drawn a correct diagram for question 1 and the fence boundary is clearly delineated. The explanation of the construction needs to be expanded, and an answer needs to be provided for question 2.

The student's work meets the essential demands of the task.

In question 2, the student identifies the smaller semicircles along the road, and recognizes the need to include them. The relationship between the radii of the small circles and the distance from their centers to the origin or to the end of the path is determined. Multiple diagrams demonstrating the constructions and argumentation are provided, with separate answers to both questions.

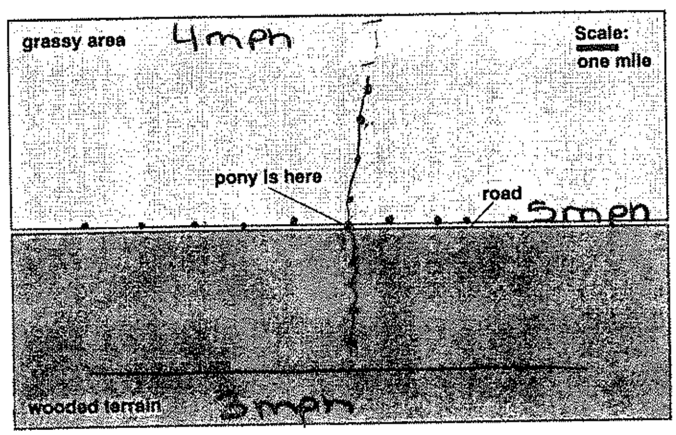
No student examples at this level.

Student A

Don't Fence Me In

This problem gives you the chance to

- *demonstrate your understanding of the relationships between distance, time, and speed*
- *draw a picture that accurately depicts the described situation*



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Suppose you want to erect a fence that would enclose all of the area that the pony can reach in one hour. (Ignore the width of the road.) Show where the fence would go if:

1. Once the pony starts in a given direction it does not change direction. (Show fence with a solid line.)

2. The pony moves freely. (Show fence with a dotted line.)

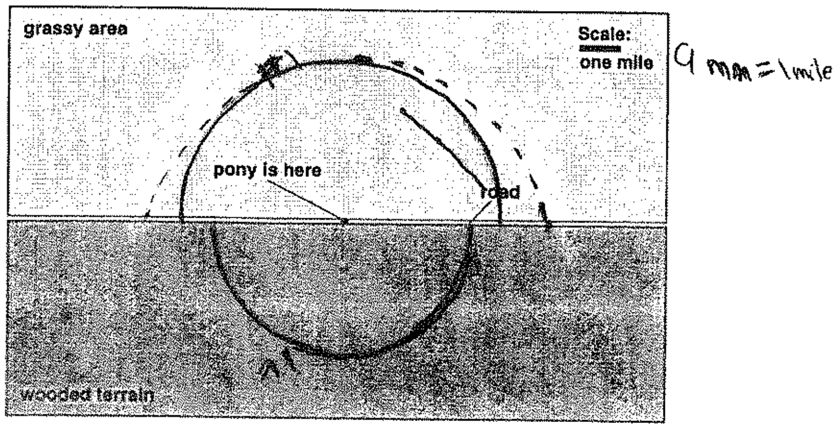
In each case explain your answer.

Student B

Don't Fence Me In

This problem gives you the chance to

- *demonstrate your understanding of the relationships between distance, time, and speed*
- *draw a picture that accurately depicts the described situation*



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In each case explain your answer.

Student C

Don't Fence Me In

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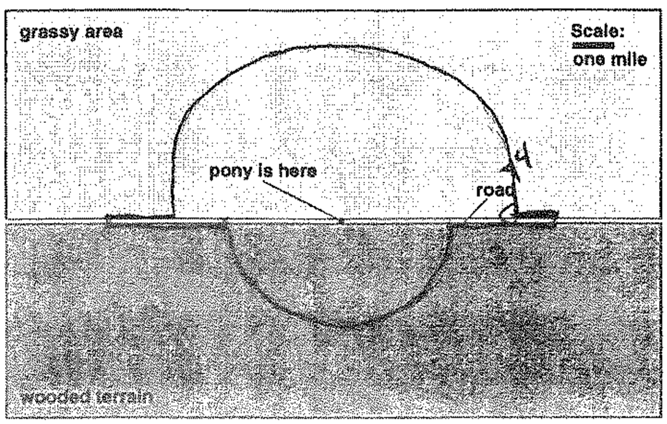
- *demonstrate your understanding of the relationships between distance, time, and speed*
- *draw a picture that accurately depicts the described situation*



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On the other side of the road is wooded terrain through which the pony can travel at 3 miles per hour.

Suppose you want to erect a fence that would enclose all of the area that the pony can reach in one hour. (Ignore the width of the road.) Show where the fence would go if:

because of how fast the pony can travel in certain areas, is how the fence was designed anywhere in these semicircles are possible paths for the pony.

1. Once the pony starts in a given direction it does not change direction. (Show fence with a solid line.)

2. The pony moves freely. (Show fence with a dotted line.)

?

In each case explain your answer.

Task 5 Overview

Garages and Phones

Demonstrate an understanding of the attributes and the differences between linear and piece-wise constant functions.

Long Task

Task Description

The intent of this task is to probe student understanding of the attributes and the differences between linear and piece-wise constant ("step" or "greatest integer") functions.

Assumed Mathematical Background

Students who have completed an Algebra I course should be able to do questions 1 and 2. Pre-calculus students should be able to do question 3.

Core Elements of Performance

- plot both linear and piece-wise constant functions
- discuss the significance of the difference between a linear function and a piece-wise function
- treat open and closed intervals on piece-wise functions properly

Circumstances

Grouping:	Students may work individually or in pairs.
Materials:	grid (provided) and rulers
Estimated time:	45 minutes