Teacher Quality Standard I

Teachers demonstrate mastery of and pedagogical expertise in the content they teach. The elementary teacher is an expert in literacy and mathematics and is knowledgeable in all other content that he or she teaches (e.g., science, social studies, arts, physical education, or world languages). The secondary teacher has knowledge of literacy and mathematics and is an expert in his or her content endorsement area(s).

The key to distinguishing the knowledge base of teaching rests at the intersection of content and pedagogy.

—L. S. Shulman

> To teach all students according to today's standards, teachers need to understand subject matter deeply and flexibly so they can help students create useful cognitive maps, relate one idea to another, and address misconceptions. Teachers need to see how ideas connect across fields and to everyday life. This kind of understanding provides a foundation for pedagogical content knowledge that enables teachers to make ideas accessible to others. (Shulman, 1987)

Although Shulman's work dates back to the late 1980s, the importance of teacher content knowledge and pedagogical expertise has never been more important than it is now as teachers ensure students are college and career ready for the demands of the 21st century.

Element C: Math Teachers

Teachers demonstrate knowledge of mathematics and understand how to promote student development in numbers and operations, algebra, geometry and measurement, and data analysis and probability.

This section describes professional practices that should be demonstrated by Teachers responsible for teaching math.

The great book of nature can be read only by those who know the language in which it was written. . . and that language is mathematics.

—Galileo

Professional practices appearing under each element of the Rubric for Evaluating Colorado Teachers are cumulative. Therefore, for teachers of math to be proficient in demonstrating knowledge of mathematics and how to promote student development of mathematical concepts and skills, they must provide instruction that is a balance of conceptual understanding and procedural skills and is sequenced and appropriate for the age and grade of their students. Teachers also establish an environment where

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students are actively engaged in doing math that challenges their thinking, stimulates their curiosity, and encourages them to investigate further.

ACCOMPLISHED AND EXEMPLARY RATING LEVELS

The impact of a proficient implementation of the professional practices referenced in Element C, Teachers responsible for teaching math, will be students who are able to solve problems in a variety of ways. This will be demonstrated by their ability to explain their thinking to others and to their teacher. They will also be able to recognize their procedural errors and take steps to correct them.

PROFESSIONAL PRACTICES: STUDENTS:

- ***Solve problems in a variety of ways.***
- ***Demonstrate mathematical thinking by explaining their thinking to each other and to their teacher.***
- ***Recognize when they make procedural errors and take steps to correct them.***

Classroom Examples

Elementary mathematics: Students are working on the Colorado Academic Standard 1: Number Sense, Properties, and Operations, Grade Level Expectation 2—Number relationships can be used to solve addition and subtraction problems. ***(Implements lesson plans based on: Colorado Academic Standards)***

In a 1st-grade classroom, students are learning to solve addition and subtraction word problems. The teacher selects a problem that will have meaning for the students and also be one that they can solve. The teacher poses the problem through telling a story. The story is retold multiple times while the students use literacy strategies to make meaning, visualize the situation, act out the situation physically and/or using manipulatives, and retell the story. Multiple engagement strategies are included, providing opportunities for all students to engage in the problem. ***(Establishes an effective mathematics environment by: Challenging students to think deeply about the problems. Presents concepts: In sequence. In a manner appropriate to students' age and grade. Provides a balance of teaching for conceptual understanding and teaching for procedural fluency.)*** Once the students have had ample opportunity to make meaning of the problem and interpret the relationships, the teacher uses a mini-lesson to refine and formalize students' ideas about the situation. She uses visual models, such as drawings of the concrete representations, ten frames, bar models, and other part-whole models. ***(Models: Appropriate mathematical communication. A variety of mathematical practices.)*** The teacher follows up the lesson by having students work in pairs on additional problems in line with the gradual release model of instruction and moving towards independence.

High school mathematics: Students are working on Colorado Academic Standard 4: Shape, Dimension, and Geometric Relationships, Grade Level Expectation 1—Objects in the plane can be transformed, and

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those transformations can be described and analyzed mathematically. ***(Implements lesson plans based on: Colorado Academic Standards)***

Students are developing their verbal and written communication skills in order to compose proofs for geometric theorems. Routine warm-up activities have provided them with previous experience working with perimeter and area of simple geometric shapes and have helped them consolidate understanding and overcome common misconceptions. ***(Presents concepts in a sequence and in a manner appropriate to students' age and grade)***. Students also took a pre-assessment, and the teacher reviewed the results to provide questions that will help them refine their solutions throughout their upcoming lessons. ***(Helps students understand mathematics as a discipline.)*** Today's lesson requires students to engage in deep mathematical thinking by strategizing and collaborating with their peers. The teacher presents the learning targets: (1) understand the concept of length and area, and (2) construct examples and counterexamples to help justify or refute conjectures. She also refers to the Mathematical Practices highlighted in the day's lesson: MP2: Reason abstractly and quantitatively, and MP3: Construct viable arguments and critique the reasoning of others. ***(Focuses math instruction beyond: Recall of basic facts. Development of computation skills. Math as a series of rote procedures.)***

The teacher then poses a conjecture about whether equal areas are formed by the diagonals of a quadrilateral. Students develop their thoughts on whether the statement is always, sometimes, or never true. ***(Establishes an effective mathematics environment by: Challenging students to think deeply about the problems and posing questions that stimulate students' curiosity and encourage them to investigate further.)*** Once students have had time to develop and share their thoughts with a partner, the teacher brings the class together to discuss which quadrilaterals the students have worked with, their results, and their chosen methods for proving the conjecture. The students explain their ideas to each other using prompts, such as, "Josh thinks this statement is sometimes true. Susan why do you think John thinks this?" ***(Establishes an effective mathematics environment by: Requiring students to explain their solutions. Posing questions that stimulate students' curiosity and encourage them to investigate further. Actively engaging students in doing math.)***

For the second activity, the teacher provides each group two sample pieces of student work that focus on justifying/disproving the conjecture for two types of quadrilaterals: kites and parallelograms. She explains that the purpose of this activity is to consider what makes a good explanation and address any misconceptions in length and area***. (Models: A variety of mathematical practices. Provides a balance of teaching for conceptual understanding and procedural fluency.)*** Students work together to determine whether the student work made any incorrect justifications and to improve upon the solutions. The teacher monitors the students and their approaches and records some of their questions

on the board for a whole-class discussion. ***(Students demonstrate mathematical thinking by explaining their thinking to each other and to their teacher.)*** The students choose to work individually or in pairs on an additional conjecture with a real-world problem:

> *A family decides to put a pool in their backyard but also hopes it will not take up a large amount of space. They have a choice between a circular or quadrilateral pool. If both pools have the same perimeter, which pool will take up the larger area?* ***(Establishes an effective mathematics environment by: Using real-world examples for problems whenever possible.)***

In closing, students are given their pre-assessment and asked to review their original responses and think about what they have learned in this lesson and to improve their original responses***. (Students recognize when they make procedural errors and take steps to correct them).***

Note: This lesson and others are available on the Mathematics Assessment Project Website at http://map.mathshell.org/materials/lessons.php?taskid=212&subpage=concept. Where indicated throughout the site, materials are free to use and have been released under the Creative Commons Attribution.

Planning/Coaching Questions

- How will I ensure that math instruction focuses beyond recall of facts and rote procedures or steps?
- How will I model appropriate mathematical communication?
- What math vocabulary will be necessary for students to learn in order to communicate their thinking?
- Which mathematical practices will be incorporated into the lesson?
- How will I ask questions that build on students' understanding of the application of mathematical practices?
- How will I present concepts in a sequence that is appropriate to the students' age and grade?
- How will I help students understand mathematics as a discipline?
- How will I ensure my teaching balances conceptual understanding and procedural fluency?
- How will students be challenged to think deeply about the problems?
- How will students be required to explain their solutions?
- What questions will I ask to stimulate students' curiosity?
- How will I incorporate real-world examples connected to the learning objective?