This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.

DATE POSTED: NOVEMBER 7, 2014
<table>
<thead>
<tr>
<th>Standard</th>
<th>Grade Level Expectations (GLE)</th>
<th>GLE Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A. Number Sense, Properties, and Operations</td>
<td>1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line</td>
<td>MA10-GR.8-S.1-GLE.1</td>
</tr>
<tr>
<td>2. A. Patterns, Functions, and Algebraic Structures</td>
<td>1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically</td>
<td>MA10-GR.8-S.2-GLE.1</td>
</tr>
<tr>
<td>2. A. Properties of algebra and equality are used to solve linear equations and systems of equations</td>
<td>MA10-GR.8-S.2-GLE.2</td>
<td></td>
</tr>
<tr>
<td>3. A. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions</td>
<td>MA10-GR.8-S.2-GLE.3</td>
<td></td>
</tr>
<tr>
<td>1. B. Data Analysis, Statistics, and Probability</td>
<td>1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge</td>
<td>MA10-GR.8-S.3-GLE.1</td>
</tr>
<tr>
<td>2. B. Transformations of objects can be used to define the concepts of congruence and similarity</td>
<td>MA10-GR.8-S.4-GLE.1</td>
<td></td>
</tr>
<tr>
<td>3. B. Direct and indirect measurement can be used to describe and make comparisons</td>
<td>MA10-GR.8-S.4-GLE.2</td>
<td></td>
</tr>
</tbody>
</table>

**Colorado 21st Century Skills**

**Critical Thinking and Reasoning:** Thinking Deeply, Thinking Differently

**Information Literacy:** Untangling the Web

**Collaboration:** Working Together, Learning Together

**Self-Direction:** Own Your Learning

**Invention:** Creating Solutions

**Mathematical Practices:**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th>Unit Titles</th>
<th>Length of Unit/Contact Hours</th>
<th>Unit Number/Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey Says...</td>
<td>3 weeks</td>
<td>7</td>
</tr>
</tbody>
</table>
### Generalizations

**My students will Understand that...**

<table>
<thead>
<tr>
<th>Concepts</th>
<th>Factual</th>
<th>Guiding Questions</th>
<th>Conceptual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random samples from a population allow statisticians to make inferences about population parameters. (MA10-GR.HS.S.3-GLE.2.a)</td>
<td>How can we reduce the margin of error in a population prediction? How can I use mean and standard deviation of a data set to draw a normal distribution? What happens to sample-to-sample variability when you increase the sample size? (MA10-GR.HS.S.3-GLE.2-IQ.2)</td>
<td>Why is the normal distribution commonly used to model a population and when is this not appropriate? How can the results of a statistical investigation be used to support an argument? (MA10-GR.HS.S.3-GLE.2-IQ1) Why is the margin of error in a study important? (MA10-GR.HS.S.3-GLE.2-IQ.5) How is it known that the results of a study not simply due to chance? (MA10-GR.HS.S.3-GLE.2-IQ.6)</td>
<td></td>
</tr>
<tr>
<td>Validity in sampling, surveys, experiments, observational studies and the interpretation of statistical results depends on randomization. (MA10-GR.HS.S.3-GLE.2-EO.b)</td>
<td>In what ways can a survey be biased? How does randomization factor into the design of an experiment?</td>
<td>Why is randomization an important component of sampling?</td>
<td></td>
</tr>
<tr>
<td>Simulation provides a means to indirectly collect data. (MA10-GR.HS.S.3-GLE.2-EO.b)</td>
<td>How do you design a simulation to model the collection of data that isn’t easily obtainable?</td>
<td>How has the use of technology enhanced our ability to study difficult to measure phenomena?</td>
<td></td>
</tr>
</tbody>
</table>
Colorado Teacher-Authorized Sample Instructional Unit

<table>
<thead>
<tr>
<th>Key Knowledge and Skills:</th>
<th>What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.</th>
</tr>
</thead>
<tbody>
<tr>
<td>My students will...</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Understand statistics as a process for making inferences about population parameters based on a random sample from that population. (MA10-GR.HS-S.3-GLE.2-EO.a.i)</td>
</tr>
<tr>
<td></td>
<td>• Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. (MA10-GR.HS-S.3-GLE.2-EO.a.ii)</td>
</tr>
<tr>
<td></td>
<td>• Recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each. (MA10-GR.HS-S.3-GLE.2-EO.b.i))</td>
</tr>
<tr>
<td></td>
<td>• Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. (MA10-GR.HS-S.3-GLE.2-EO.b.ii, iii)</td>
</tr>
<tr>
<td></td>
<td>• Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (MA10-GR.HS-S.3-GLE.2-EO.b.iv)</td>
</tr>
<tr>
<td></td>
<td>• Evaluate reports based on data. (MA10-GR.HS-S.3-GLE.2-EO.b.vi)</td>
</tr>
<tr>
<td></td>
<td>• Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and recognize that there are data sets for which such a procedure is not appropriate; use calculators, spreadsheets, and tables to estimate areas under the normal curve. (MA10-GR.HS-S.3-GLE.1-EO.a.iv, v)</td>
</tr>
</tbody>
</table>

**Critical Language:** includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.

**EXAMPLE:** A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: “Mark Twain exposes the hypocrisy of slavery through the use of satire.”

<table>
<thead>
<tr>
<th>A student in ____________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):</th>
<th>Statistics is a process for making inferences about population parameters based on a random sample of a population.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Academic Vocabulary:</strong></td>
<td>Inferences, surveys, experiments, observational studies, data, conclusions, interpret, evaluate, recognize, compare, model, explain, spreadsheets, estimate</td>
</tr>
<tr>
<td><strong>Technical Vocabulary:</strong></td>
<td>Statistic, statistics, sampling, mean, standard deviation, data sets, normal distribution, normal curve, margin of error, parameters, random sample, population, validity, sampling, statistical results, randomization, simulation, indirect, data</td>
</tr>
</tbody>
</table>
**Unit Description:** This unit focuses on inferring conclusions about a population from samples. Students explore normal distributions and the concept of quantifying "normal". The concept of normal distributions provides the foundation for making inferences about a population from sample means and standard deviations. The unit then looks at the different ways to collect data about a population. Finally, students investigate simulation the use of simulations to test the validity of statistical conclusions and hypotheses. Throughout this unit students will explore how statistics are used to make policy decisions and engage in the process of creating policies that impact their life.

**Considerations:** Throughout this unit students will be analyze data from the Health Kids Colorado Survey (HKCS) to produce a report, video, or presentation to use in their community to help prevent harmful behaviors among teenagers. This work is done in a partnership with Lake County Build a Generation (LCBAG).

<table>
<thead>
<tr>
<th><strong>Unit Generalizations</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Key Generalization:</strong> Random samples from a population allow statisticians to make inferences about population parameters</td>
</tr>
<tr>
<td><strong>Supporting Generalizations:</strong> Validity in sampling, surveys, experiments, observational studies and the interpretation of statistical results depends on randomization</td>
</tr>
<tr>
<td>Simulation provides a means to indirectly collect data</td>
</tr>
</tbody>
</table>

**Performance Assessment:** *The capstone/summative assessment for this unit.*

**Claims:** (Key generalization(s) to be mastered and demonstrated through the capstone assessment.)

**Stimulus Material:** (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)

Random samples from a population allow statisticians to make inferences about population parameters.

Lake County “Build a Generation” wants to hire you as statistician to examine student data from Lake County High School in one or more of the following areas: alcohol, tobacco, or drug use, health and safety, violence and crime, and education. Data in Colorado can be obtained in each community from the Healthy Kids Colorado Survey (HKCS) ([http://collaboration.omni.org/sites/hkc/Pages/default.aspx](http://collaboration.omni.org/sites/hkc/Pages/default.aspx)). As part of the report they would like you to compare Lake County High School data to state and national data for each area. The audience for the project is a coalition of community leaders including such as: County Health Department, Human Services, City Council, and County Commissioners. The products from your investigation will be used to help craft public policy and spur discussion of changing trends in the community. Information on an how Lake County School District partners with Lake County Build a Generation to do this project can be found at [http://lcbag.org/initiatives/student-programs](http://lcbag.org/initiatives/student-programs).

**Product/Evidence:** (Expected product from students)

Students will produce a report, video, or podcast that explores the data from the school. The final product should interpret and compare their local data to Colorado or National Data sets in order to understand what the data means and tell their own story. Students are asked to examine trends over time, to compare sample means from Lake County versus other data sets, to look for associations (i.e., more boys use tobacco), and to follow a particular group of students through time (i.e., when did most students in this grade first try alcohol?).

**Differentiation:** (Multiple modes for student expression)

Students can express their interpretations verbally through a video, presentation, infographics. Students can work in collaborative groups on the project building on one another’s strengths. Students can extend the analysis to examine correlations (e.g., do students that drink alcohol also smoke marijuana?).
# Ongoing Discipline-Specific Learning Experiences

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Teacher Resources</th>
<th>Student Resources</th>
<th>Skills</th>
<th>Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others</strong>&lt;br&gt;[Mathematical Practice 3]</td>
<td><a href="http://www.insidemathematics.org/index.php/standard-3">http://www.insidemathematics.org/index.php/standard-3</a>&lt;br&gt;<a href="http://quizlet.com/22134361/cpm-index-cards-of-teaching-strategies-flash-cards/">http://quizlet.com/22134361/cpm-index-cards-of-teaching-strategies-flash-cards/</a> (teaching strategies to get students talking)</td>
<td>N/A</td>
<td>Provide justification for arguments through a series of logical steps while using correct mathematical vocabulary. Analyze and critique the arguments of other students</td>
<td>Students justify their statistical inferences. Students can also analyze the reasoning of a fellow student. Students will need to be precise with their language such as the use samples versus populations.</td>
</tr>
<tr>
<td></td>
<td><strong>Think/work like a mathematician – Engaging in the practice of modeling the solution to real world problems</strong>&lt;br&gt;[Mathematical Practice 4]</td>
<td><a href="http://www.corestandards.org/Math/Content/HSM">http://www.corestandards.org/Math/Content/HSM</a>&lt;br&gt;<a href="http://blog.mrmeyer.com/?p=16301">http://blog.mrmeyer.com/?p=16301</a> (an entry about an approach to modeling)&lt;br&gt;<a href="http://threeacts.mrmeyer.com">http://threeacts.mrmeyer.com</a> (examples of 3-act problems that are good for modeling)</td>
<td>N/A</td>
<td>Model real world problems mapping relationships with appropriate models of functions, analyze relationships to draw conclusions, interpret results in relation to context, justify and defend the model, and reflect on whether results make sense</td>
<td>Modeling Problems Students use statistics to analyze relationships between samples and populations and create models.</td>
</tr>
</tbody>
</table>

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# Texts for independent reading or for class read aloud to support the content

<table>
<thead>
<tr>
<th>Informational/Non-Fiction</th>
<th>Fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The Signal and the Noise</strong> by Nate Silver (Lexile level 1260)&lt;br&gt;www.fivethirtyeight.com (online newspaper focused on articles using statistics)</td>
<td></td>
</tr>
</tbody>
</table>
### Description:
Mathematicians are fluent with seeing structure in expressions and using this structure to rewrite expressions.

### Teacher Resources:
- [http://ccssmath.org/?page_id=2091](http://ccssmath.org/?page_id=2091) (links to student resources for practicing fluency with rewriting expressions)

### Student Resources:

### Skills:
Rewrite expressions to reveal something about the behavior of the corresponding function

### Assessment:
Students build fluency with rewriting expressions to reveal something about the behavior of the corresponding function through consistent practice with the symbolic and graphical forms of functions.

### Prior Knowledge and Experiences
Student familiarity with measure of center and spread and graphical representations of one-variable data. It is also helpful if students have some familiarity with two-way frequency tables and probability theory. Students will continue to develop their understanding of the difference between correlation and causation in this unit, in addition to the need for randomization when creating samples.

### Learning Experience #1
The teacher may provide a variety of one-variable data sets (see teacher resources) so that students can begin to explore the concept of normal distributions.

**Enactive:** Students can sort a variety of graphical representations (e.g., histograms, line plots) of normal and not normal data (e.g., ACT scores versus attendance at Broncos games) to determine the properties of data that can be fit to a normal curve (e.g., centered at the mean, symmetrical, bell shaped).

**Iconic:** Students can graph data for ACT scores to study how data is distributed under a normal curve (e.g., how many students scored between 16 and 24?)

**Symbolic:** Students can calculate the percentage of students that scored one, two and three standard deviations from the mean ACT score.

### Teacher Notes:
The data set for the ACT is readily available (see teacher resources) and will make sense to many juniors, but there are other sets of data that could be used in its place. This learning experience is an appropriate place to introduce the 68-95-99 rule, but z-scores will be covered in learning experience #2. It is important to return to the data sets that were not normal from the enactive part of the learning experience during the symbolic part and provide students opportunities to explain why the 68-95-99 does not make sense with these populations this will help to avoid the common misconception that all populations are normal.

### Generalization Connection(s):
Random samples from a population allow statisticians to make inferences about population parameters.

### Teacher Resources:
- [http://www.mathematicsvisionproject.org/secondary-mathematics-iii.html](http://www.mathematicsvisionproject.org/secondary-mathematics-iii.html) (the first three lessons of module 8 shows examples of normal and not-normal data and data on the ACT distribution)
- [http://phet.colorado.edu/en/simulation/plinko-probability](http://phet.colorado.edu/en/simulation/plinko-probability) (online simulation of a plinko board that simulates a normal distribution)
**Colorado Teacher-Authorized Sample Instructional Unit**

<table>
<thead>
<tr>
<th>Student Resources:</th>
<th><a href="https://www.khanacademy.org/math/probability/statistics-inferential/normal_distribution/v/introduction-to-the-normal-distribution">https://www.khanacademy.org/math/probability/statistics-inferential/normal_distribution/v/introduction-to-the-normal-distribution</a> (Khan Academy video introduction to the normal distribution)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assessment:</strong></td>
<td>Students mastering the concept and skills of this lesson should be able to answer questions such as: Why is the normal distribution commonly used to model a population and when is it not appropriate?</td>
</tr>
<tr>
<td><strong>Differentiation:</strong></td>
<td>(Multiple means for students to access content and multiple modes for student to express understanding.)</td>
</tr>
<tr>
<td><strong>Access (Resources and/or Process)</strong></td>
<td><a href="http://www.nctm.org/resources/content.aspx?id=32702">http://www.nctm.org/resources/content.aspx?id=32702</a> (core math tools suite provides software to do multiple types of simulation on a computer)</td>
</tr>
<tr>
<td><strong>Expression (Products and/or Performance)</strong></td>
<td>Students can generate a uniform and normal data distribution by rolling one number cube versus two number cubes (or use a computer simulation) and explain why only the sum of the two number cubes creates a normal distribution</td>
</tr>
<tr>
<td><strong>Extensions for depth and complexity:</strong></td>
<td><strong>Access (Resources and/or Process)</strong></td>
</tr>
<tr>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>
| **Key Knowledge and Skills:** | • Understand statistics as a process for making inferences about population parameters based on a random sample from that population  
• Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and recognize that there are data sets for which such a procedure is not appropriate; use calculators, spreadsheets, and tables to estimate areas under the normal curve |
| **Critical Language:** | Histogram, center, standard deviation, normal curve, mean, percentage |

**Learning Experience # 2**

The teacher may provide related data sets with a variety of means and standard deviations (e.g., state test scores) so that student can begin to understand the need for standardization of statistics (e.g., z-scores) in order to make valid inferences.

**Enactive:** Students can study the data of state test scores from a variety of classes and give their first impression on which teacher helped their students improve the most compared to the state average (i.e., Who is the best teacher?).

Example: Ms. Math Teacher
- Mean Score from 2013: 140
- Colorado Mean Score 2013: 155
- Colorado Standard Deviation 2013: 10
- Mean Score from 2014: 150
- Colorado Mean Score 2014: 162
- Colorado Standard Deviation 2014: 12

Example: Mr. English Teacher
- Mean Score from 2013: 140
- Colorado Mean Score 2013: 142
- Colorado Standard Deviation 2013: 8
- Mean Score from 2014: 150
- Colorado Mean Score 2014: 143
- Colorado Standard Deviation 2014: 15

**Iconic:** Students can graph the normal curves and plot the test scores for each year on the curves (i.e., Ms. Math Teacher will have two normal curves one for 2013 and one for 2014) and discuss how the graph provides a visual of which class is “further to the right” on the normal curve.

**Symbolic:** Students can calculate the z-score for each class of students and compare the improvement of the students for each teacher. Students can provide determine and justify which teachers helped their students improve the most compared to the state average.
## Teacher Notes:
When creating or choosing data it may be helpful to have at least one set of data where the mean score of the group of students has gone down and the Colorado mean score also went down. Students will likely assume that the students did not improve but they actually did. Students will likely bring up the many issues with determining who is the best teacher using this method such as students moving, a change in tests or even the characteristics of teachers not measured by test scores, it is valuable for students to discuss these ideas as they become critical consumers of statistics.

## Generalization Connection(s):
Random samples from a population allow statisticians to make inferences about population parameters.

## Teacher Resources:
- [http://www.regentsprep.org/Regents/math/algtrig/ATS2/NormalLesson.htm](http://www.regentsprep.org/Regents/math/algtrig/ATS2/NormalLesson.htm) (introduction to the concept and vocabulary of a normal curve that includes calculating z-scores)
- [https://www.illustrativemathematics.org/illustrations/216](https://www.illustrativemathematics.org/illustrations/216) (sample problem on z-scores)
- [https://www.illustrativemathematics.org/illustrations/1020](https://www.illustrativemathematics.org/illustrations/1020) (sample problem on z-scores and normal distribution)

## Student Resources:

## Assessment:
Students mastering the concept and skills of this lesson should be able to answer questions such as:
- Why do we use z-scores?
- How do z-scores provide a way to compare samples from different populations?
- How do you quantify how extraordinary a performance was (e.g., test score, sporting event)?
- How are z-scores calculated?
- What is the relationship between z-scores and the normal distribution?
- How does the 68-95-99 rule relate to z-scores?

## Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)

### Access (Resources and/or Process)

### Expression (Products and/or Performance)
Students can use a graphic organizer for the normal curve to calculate z-scores and explain the meaning of the scores in relation to the normal curve and the context of the problem.

## Extensions for depth and complexity:

### Access (Resources and/or Process)

### Expression (Products and/or Performance)
Students can analyze the value added model for evaluating teacher effectiveness to produce a presentation for the school board on how the model measures student growth using normal curves. Students can create a method for evaluating teacher effectiveness and explain why their model is fair and balanced.
Key Knowledge and Skills:
- Understand statistics as a process for making inferences about population parameters based on a random sample from that population
- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and recognize that there are data sets for which such a procedure is not appropriate; use calculators, spreadsheets, and tables to estimate areas under the normal curve

Critical Language:
- Histogram, center, standard deviation, normal curve, mean, percentage

Learning Experience # 3

The teacher may provide research questions (e.g., What do American eat every day?, Will the ballot measure pass or fail in the next election?, Does having a television in your bedroom cause difficulties with sleeping?), so that students can develop an understanding and begin to analyze the three main ways of answering research questions: surveys, experiments and observational studies.

Enactive: Students can create a method for collecting data to answer one of several research questions (see above) and, as a class, analyze the similarities and differences between the methods they created with a particular focus on randomization.

Iconic: Students can sort a variety of research questions in categories and write descriptions for each of their categories.

Symbolic: Students can discuss the categories they created in the iconic stage and compare them to the definitions of sample surveys, experiments and observational studies. Students can then create a graphic organizer (e.g., Frayer model) for each of the three main types of research.

Teacher Notes:
A key part of the discussion about this learning experience is randomization. When students are asked, “How can you be sure that your survey or experiment is not biased in any way?”, they may give answers such as, “Ask a random group of people from town . . . “. However, the teacher should consider pushing the students to describe exactly how they might ask a random group. Can you wait at the grocery store? Can you look in the phone book? Can you walk around downtown? This question is extremely difficult and helps students to understand the dangers when proper randomization is not present.

Generalization Connection(s):
Validity in sampling, surveys, experiments, observational studies and the interpretation of statistical results depends on randomization

Teacher Resources:
- [https://www.illustrativemathematics.org/illustrations/1029](https://www.illustrativemathematics.org/illustrations/1029) (example problem differentiating between experiments and observational studies, including a question about randomization)
- [https://www.illustrativemathematics.org/illustrations/122](https://www.illustrativemathematics.org/illustrations/122) (example problem of determining the best way to determine how 1000 students at a school feel)
- [http://www.mathematicsvisionproject.org/secondary-mathematics-iii.html](http://www.mathematicsvisionproject.org/secondary-mathematics-iii.html) (module 8, pages 42 to 46 of the teacher edition, explores the difference between explore sample surveys, experiments, and observational studies)
### Student Resources:

### Assessment:
Students mastering the concept and skills of this lesson should be able to answer questions such as:
- How does randomization factor into the design of an experiment?
- Why is randomization an important component of sampling?
- What is the best way to collect data to answer opinion questions?
- What type of research allows you to determine cause-and-effect relationships?
- How do you make sure your data was collected randomly?
- Why might you choose an observational study rather than an experiment?

<table>
<thead>
<tr>
<th>Access (Resources and/or Process)</th>
<th>Expression (Products and/or Performance)</th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="https://learnzillion.com/lessonsets/475-distinguish-between-surveys-experiments-and-observational-studies-relate-randomization-to-each">https://learnzillion.com/lessonsets/475-distinguish-between-surveys-experiments-and-observational-studies-relate-randomization-to-each</a> (videos about each type of study)</td>
<td>Students can complete their graphic organizer about each type of research methodology by watching videos about them</td>
</tr>
</tbody>
</table>

### Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)

### Extensions for depth and complexity:

<table>
<thead>
<tr>
<th>Access (Resources and/or Process)</th>
<th>Expression (Products and/or Performance)</th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="http://www.people-press.org/methodology/questionnaire-design/question-wording/">http://www.people-press.org/methodology/questionnaire-design/question-wording/</a> (advice about creating high quality survey questions)</td>
<td>Students can produce two different surveys on a topic, one with a very biased set of questions and that limits the biases in the questions</td>
</tr>
<tr>
<td><a href="https://www.surveymonkey.com/blog/en/blog/2013/07/24/a-little-bias-is-a-big-problem/">https://www.surveymonkey.com/blog/en/blog/2013/07/24/a-little-bias-is-a-big-problem/</a> (discussion about the impact of even a little bias in surveys)</td>
<td>Students can create a prezi that describes the impact of the wording of questions on survey results</td>
</tr>
</tbody>
</table>

### Key Knowledge and Skills:
- Recognize the purposes of and differences among sample surveys, experiments, and observational studies and explain how randomization relates to each
- Understand statistics as a process for making inferences about population parameters based on a random sample from that population

### Critical Language:
- Sample survey, experiment, observational studies, randomization

### Learning Experience # 4
The teacher may provide data about sampling scenarios (see possible examples in teacher notes) so that students can understand how simulation can be used to test the likelihood of a sample (e.g., Is this sample an aberration or not?).

**Enactive:** Students can look at data from a sampling scenario and make a hypothesis about the likelihood a sample. Students can develop a simulation to test the likelihood of a sample and run the simulation to create a large number of simulated samples (e.g., 100 trials).

**Iconic:** Students can create a graphical representation (e.g., histogram) of the simulated samples to see a visual representation of the likelihood a sample.

**Symbolic:** Students can analyze the simulation data to make a conclusion about whether their hypothesis was plausible or not based on the distribution of the simulated trials.
## Teacher Notes:

Students often benefit doing an initial simulation by hand, such as flipping a coin or rolling a die. However, computer simulations greatly speed the ability to collect large sets of data. This learning experience provides a foundation for later statistics by connecting sampling theory and probability. Specifically, the teacher may help students to understand how probability relates to sampling a population. There is a chance that the sample mean is far from the population mean, but it is unlikely if the sample size is large enough. Students may notice that their simulations of samples will create a bell curve, which is basis for sampling theory. In addition, students are developing an early understanding of null hypothesis testing. Illustrative mathematics illustrations 125, 244, and 1099 (see links in teacher resources) are all great example of how this learning experience can bring together the three ideas of sampling a population, simulation, and the normal curve.

Example 1: If you randomly sample 100 people from your town and 60% say they will be voting for a tax increase, is it possible that less than 50% of your town of 3,000 people will vote for the tax increase? Is it probable?

Example 2: If a sports team has a 50% chance of winning any football game, what is the probability that the team goes 0-9? What if they have a 25% chance of winning any game? A 16% chance?

Example 3: For example, using actual data from the Healthy Kids Colorado Survey, let us suppose that 30% of Lake County 12th graders have drank alcohol in the last 30 days, and 20% of Colorado 12th graders have drank alcohol in the last 30 days. Is there a statistical difference between these two values? How do you know?

## Generalization Connection(s):

Simulation provides a means to indirectly collect data

## Teacher Resources:

- [http://www.nctm.org/resources/content.aspx?id=32702](http://www.nctm.org/resources/content.aspx?id=32702) (core math tools suite provides software to do multiple types of simulation on a computer)
- [https://www.illustrativemathematics.org/illustrations/1411](https://www.illustrativemathematics.org/illustrations/1411) (problem on the law of large numbers—how larger samples are more accurate)
- [https://www.illustrativemathematics.org/illustrations/125](https://www.illustrativemathematics.org/illustrations/125) (problem explores the probability that a sample of student opinion in a large high school represents the opinion of the student population.)
- [https://www.illustrativemathematics.org/illustrations/244](https://www.illustrativemathematics.org/illustrations/244) (problem on determining if 7 out of 8 correct answers on a true/false test could be random or not, including simulating the scenario with a coin flip)
- [https://www.illustrativemathematics.org/illustrations/1099](https://www.illustrativemathematics.org/illustrations/1099) (same problem on determining if 7 out of 8 correct answers on a true/false test could be random or not, but the simulation has been done for the student and plotted on a dot plot)

## Student Resources:

[http://stattrek.com/experiments/simulation.aspx](http://stattrek.com/experiments/simulation.aspx) (Explanation of simulation and includes a random number generator)

## Assessment:

Students mastering the concept and skills of this lesson should be able to answer questions such as:

- When is simulation appropriate?
- How do you design a simulation so that it will tell you about the event you care about?
- Is it possible to get 9 out of 10 on a true/false test just by guessing? Is it probable?
- How is it known that the results of a study are not simply due to chance?
- How are samples, probability, and the normal curve related?
Colorado Teacher Authored Sample Instructional Unit

<table>
<thead>
<tr>
<th>Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)</th>
<th>Access (Resources and/or Process)</th>
<th>Expression (Products and/or Performance)</th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="https://www.illustrativemathematics.org/illustrations/1099">https://www.illustrativemathematics.org/illustrations/1099</a> (problem asks students to determine if 7 out of 8 correct answers on a true/false test could be random or not and the simulation has been done for the student and plotted on a dot plot)</td>
<td>Student can analyze the results of simulation (scaffold) to determine if a true/false test is random or not</td>
<td>Students can work in groups to create and run simulations on the computer to test their hypotheses</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Extensions for depth and complexity:</th>
<th>Access (Resources and/or Process)</th>
<th>Expression (Products and/or Performance)</th>
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<tr>
<td><a href="http://espn.go.com/nba/hollinger/playoffodds">http://espn.go.com/nba/hollinger/playoffodds</a> (article about determining playoff odds for basketball) <a href="https://www.khanacademy.org/math/probability/statistics-inferential/sampling_distribution/v/central-limit-theorem">https://www.khanacademy.org/math/probability/statistics-inferential/sampling_distribution/v/central-limit-theorem</a> (video about the central limit theorem) <a href="https://www.khanacademy.org/math/probability/statistics-inferential/hypothesis-testing/v/hypothesis-testing-and-p-values">https://www.khanacademy.org/math/probability/statistics-inferential/hypothesis-testing/v/hypothesis-testing-and-p-values</a> (video about hypothesis testing)</td>
<td>Students can produce a report on how the ESPN playoff projections are based simulations and make suggestions for how it could be improved</td>
<td>Students can analyze the scenarios from class using the central limit theorem and null hypothesis testing</td>
</tr>
</tbody>
</table>

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<tr>
<th>Key Knowledge and Skills:</th>
<th>Critical Language:</th>
</tr>
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<tbody>
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<td>• Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation</td>
<td>Simulation, probable, possible, validity, trials, samples, probability, plausible, implausible</td>
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</tbody>
</table>

Learning Experience # 5

The teacher may provide scenarios or reports, which use statistics to make interpretations about a population (see link in teacher resources for examples) so that students can critique interpretations of statistics and begin to understand the dangers of using simple statistics in real-world contexts.

Teacher Notes: This learning experience is a great precursor to the performance assessment for the unit, where students are asked to create their own report that based on statistical interpretations.

Generalization Connection(s): Random samples from a population allow statisticians to make inferences about population parameters

Teacher Resources: [http://map.mathshell.org/materials/lessons.php?taskid=217&subpage=problem](http://map.mathshell.org/materials/lessons.php?taskid=217&subpage=problem) (lesson about determining the biases in reports, the key idea from Algebra 1 of correlation versus causation is also revisited)

Student Resources: N/A
### Assessment:
Students mastering the concept and skills of this lesson should be able to answer questions such as:
- How do you identify bias in statistical reports?
- What are ways surveys can be biased?
- How can the results of a statistical investigation be used to support an argument?
- Does the way a question is asked influence how it might be answered?
- How might the scales on a graph affect the interpretation of them?
- Why is the data collection process important to understand when interpreting data?

### Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)

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<td><a href="http://map.mathshell.org/materials/lessons.php?taskid=217&amp;subpage=problem">http://map.mathshell.org/materials/lessons.php?taskid=217&amp;subpage=problem</a> (Shell Centre lesson includes ideas for helping students to access the material in this learning experience)</td>
<td>N/A</td>
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</table>

### Extensions for depth and complexity:

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<tbody>
<tr>
<td>N/A</td>
<td>Students may produce a report that is based on factual data that has a bias to it</td>
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</table>

### Key Knowledge and Skills:
- Evaluate reports based on data

### Critical Language:
- Bias, correlation, causation, sampling, surveys