

**Instructional Unit Authors**

Burlington School District

Trevor Aeschliman

Marianne Hinkhouse

Stacy Oldham

Elsie Pimentel

Nichole Swanson

Mitzi Swiatkowski

Mary Pat Weingardt

**Based on a curriculum overview Sample authored by**

Aspen School District

Sarah Beesley

Byers School District

Terrell Price

Montrose School District

Teresa Brown

*This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.*

**Colorado’s District Sample Curriculum Project**

date Posted: march 31, 2014

Mathematics

7th Grade

Colorado Teacher-Authored Instructional Unit Sample

**Unit Title: Data Daze**

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| **Content Area** | Mathematics | | | **Grade Level** | 7th Grade | | |
| **Course Name/Course Code** |  | | | | | | |
| **Standard** | **Grade Level Expectations (GLE)** | | | | | | **GLE Code** |
| 1. Number Sense, Properties, and Operations | 1. Proportional reasoning involves comparisons and multiplicative relationships among ratios | | | | | | MA10-GR.7-S.1-GLE.1 |
| 1. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently | | | | | | MA10-GR.7-S.1-GLE.2 |
| 1. Patterns, Functions, and Algebraic Structures | 1. Properties of arithmetic can be used to generate equivalent expressions | | | | | | MA10-GR.7-S.2-GLE.1 |
| 1. Equations and expressions model quantitative relationships and phenomena | | | | | | MA10-GR.7-S.2-GLE.2 |
| 1. Data Analysis, Statistics, and Probability | 1. Statistics can be used to gain information about populations by examining samples | | | | | | MA10-GR.7-S.3-GLE.1 |
| 1. Mathematical models are used to determine probability | | | | | | MA10-GR.7-S.3-GLE.2 |
| 1. Shape, Dimension, and Geometric Relationships | 1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof | | | | | | MA10-GR.7-S.4-GLE.1 |
| 1. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure | | | | | | MA10-GR.7-S.4-GLE.2 |
| **Colorado 21st Century Skills**    **Critical Thinking and Reasoning:** *Thinking Deeply, Thinking Differently*  **Information Literacy:** *Untangling the Web*  **Collaboration:** *Working Together, Learning Together*  **Self-Direction:** *Own Your Learning*  **Invention:** *Creating Solutions* | | **Mathematical Practices:**   1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning. | | | | | |
| **Unit Titles** | | | **Length of Unit/Contact Hours** | | | **Unit Number/Sequence** | |
| Data Daze | | | 3 weeks | | | 6 | |

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| **Unit Title** | Data Daze | | | **Length of Unit** | 3 weeks |
| **Focusing Lens(es)** | Random  Inference | **Standards and Grade Level Expectations Addressed in this Unit** | MA10-GR.7-S.3-GLE.1 | | |
| **Inquiry Questions (Engaging- Debatable):** | * How can you declare a winner in an election before counting all the ballots? (MA10-GR.7-S.3-GLE.1-IQ.3) * When playing music, does the shuffle function play songs randomly? * How might the sample for a survey affect the results of the survey? (MA10-GR.7-S.3-GLE.1-IQ.1) | | | | |
| **Unit Strands** | Statistics and Probability | | | | |
| **Concepts** | Representative, samples, statistics, generalizations, random, population, parameters, inferences, mean absolute deviations, multiplicative, ratio, additive, comparison, unit size, deviations, visual displays, measures of center, measures of variability, numerical data distributions, random sampling, representative samples, estimates, predictions | | | | |

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| **Generalizations**  **My students will Understand that…** | | **Guiding Questions**  **Factual Conceptual** | |
| Representative samples create statistics from which mathematicians make valid generalizations about a population. (MA10-GR.7-S.3-GLE.1-EO.a.i) | | What is a statistic?  What is a sample?  What is a population?  What are examples of samples that would not be a valid representation of a population? | Why do statistics based on a representative sample support generalizations about a population?  Why do we investigate samples rather entire populations? |
| Random and relatively large samples generate sample statistics that estimate population parameters and support valid inferences. (MA10-GR.7-S.3-GLE.1-EO.a.ii, a.iii, a.iv) | | How can you choose a representative sample of a population?  How do you distinguish between random and bias samples? (MA10-GR.7-S.3-GLE.1-IQ.2)  How do statistics from multiple samples generated from the same population compare to each other? | Why is a random sample a method of generating a representative sample?  Why do random samples better estimate or predict information about a population than non-random samples? |
| Comparing mean absolute deviations requires a multiplicative (ratio) rather than additive comparison because mathematicians compare the unit size of the deviations to each other. (MA10-GR.7-S.3-GLE.1-EO.b.i) | | What is mean absolute deviation?  How do you compare the mean absolute deviations of two data sets? | Why are comparisons of mean absolute deviations stated as a ratio? |
| Visual displays, measures of center, and measures of variability for two numerical data distributions from random samples facilitate informal comparative inferences about two populations. (MA10-GR.7-S.3-GLE.1-EO.b.i, b.ii) | | What are measures of center?  What are measures of variability?  Which visual displays help mathematicians informally assess the degree of overlap between two numerical data distributions? | How do you determine what measure best represents the data?  Why are both measures of center and variability important when comparing two numerical data distributions? |
| **Key Knowledge and Skills:**  **My students will…** | *What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.* | | |
| * Understand statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population (MA10-GR.7-S.3-GLE.1.a.i) * Understand random sampling tends to produce representative samples and support valid inferences (MA10-GR.7-S.3-GLE.1-EO.a.ii) * Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. (MA10-GR.7-S.3-GLE.1-EO.a.i, a.ii) * Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. (MA10-GR.7-S.3-GLE.1-EO.a.iii, a.iv) * Assess the degree of visual overlap Informally of two numerical data distributions with similar variables, measuring the difference between the centers by expressing it as a multiple of a measure of variability. (MA10-GR.7-S.3-GLE.1-EO.b.i) * Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. (MA10-GR.7-S.3-GLE.1-EO.b.ii) | | | |

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| **Critical Language:** includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.  EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *“Mark Twain exposes the hypocrisy of slavery through the use of satire.”* | | |
| **A student in \_\_\_\_\_\_\_\_\_\_\_\_\_\_ can demonstrate the ability to apply and comprehend critical language through the following statement(s):** | | *When comparing two numerical data distributions, I begin by making a visual comparison using a box plot and then compare the means and mean absolute deviations.*  *A random sample of the senior class will better predict the most popular location for prom than only surveying the calculus classes.* |
| **Academic Vocabulary:** | Generate, informally, representative, generalizations, inferences, comparison, estimate, predict | |
| **Technical Vocabulary:** | Stem and leaf plot, box plot, dot plot, sample, statistics, random, population, mean absolute deviations, multiplicative, ratio, additive, unit size, measures of center, mean, median, measures of variability, inter-quartile range, numerical data distributions, random sampling, representative samples, multiple samples, simulated samples, | |

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| **Unit Description:** | Data Daze takes students on a journey into random sampling. Students learn to make inferences about a population based on sample data. Students are also introduced to the idea of variation through the mean absolute deviation. This is the first unit in which students explore the connections between samples and populations. These ideas will be extended at the high school level and linked to the concept of standard deviation. |
| **Unit Generalizations** | |
| **Key Generalization:** | Visual displays, measures of center, and measures of variability for two numerical data distributions from random samples facilitate informal comparative inferences about two populations |
| **Supporting Generalizations:** | Representative samples create statistics from which mathematicians make valid generalizations about a population |
| Random and relatively large samples generate sample statistics that estimate population parameters and support valid inferences |
| Comparing mean absolute deviations requires a multiplicative (ratio) rather than additive comparison because mathematicians compare the unit size of the deviations to each other |

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| **Performance Assessment:** *The capstone/summative assessment for this unit.* | |
| **Claims:**  (Key generalization(s) to be mastered and demonstrated through the capstone assessment.) | Visual displays, measures of center, and measures of variability for two numerical data distributions from random samples facilitate informal comparative inferences about two populations. |
| **Stimulus Material:**  (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization) | You are a product researcher and a cell phone company has hired you to study the cell phone habits of middle school students in your community (e.g., calls, texts, Tweets, Instagram, Vine). The company wants to study how boys use their cell phones in comparison to girls. You will need to create an unbiased sampling method to determine if the two populations have different device needs. Based on your findings you will help the company decide if it is a good business decision to design different cell phone plans for the two populations. For example, does one population text more than the other, or use the phone features and apps more often, or in different ways? |
| **Product/Evidence:**  (Expected product from students) | Students will create a report for the cell phone company based on a survey question they designed related to cell phone usage. The survey data must be numerical (e.g., frequency of tweets, number of apps used). The presentation should include:   * strategy used to create a representative sample of middle school students in the community * visual displays of the sample data for girls versus boys * means, ranges, and mean absolute deviations for each sample   Students should also make recommendations to the cell phone company about marketing and designing cell phone plans based on inferences resulting from their sampling data. |
| **Differentiation:**  (Multiple modes for student expression) | Students can present their findings through video, Prezi, animation, or PowerPoint.  Students can work in groups to create their survey question and collect data.  Computer graphing programs can be used to create the visual displays for students:  <http://nces.ed.gov/nceskids/createagraph/default.aspx>  <http://illuminations.nctm.org/ActivityDetail.aspx?ID=220>  <http://illuminations.nctm.org/ActivityDetail.aspx?ID=204> |

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| **Texts for independent reading or for class read aloud to support the content** | |
| **Informational/Non-Fiction** | **Fiction** |
| *How Many Ants in an Anthill* by Kate Boehm Jerome (Lexile level 710)  *If the World Were a Village: A Book about the World's People* by David Smith (Lexile level 840)  *This Child, Every Child: A Book about the World’s Children* by David Smith (Lexile level 1020) | *Conned Again Watson! Cautionary Tales of Logic, Math, and Probability* by Colin Bruce (Lexile level 950)  *Mind Games* by Jeanne Grunwell (Lexile level 760) |

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| **Ongoing Discipline-Specific Learning Experiences** | | | | |
| 1. | Description: | Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others  [Mathematical Practice 3] | Teacher Resources: | <http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm> (lesson plans contains exemplars that could be replicated for students to critique the reasoning of others)  <http://map.mathshell.org/materials/index.php> (samples and examples of student work to critique the validity of others)  [www.exemplars.com/resources/rubrics/assessment-rubrics](http://www.exemplars.com/resources/rubrics/assessment-rubrics) (standards-based math rubric for the students to assess other’s work) |
| Student Resources: | N/A |
| Skills: | Construct and communicate a complete and concise response, justify a conclusion using correct vocabulary, interpret and critique the validity of other’s conclusions and reasoning, and identify errors and present correct solutions  [Mathematical Practice 4] | Assessment: | Students analyze and defend their solutions for each major learning experience. Careful attention should be paid to precise use of vocabulary and symbols. Periodically throughout the unit, students can also be provided with flawed solutions and asked to identify, describe, and correct the flaw. |
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| 2. | Description: | Think/work like a mathematician – Engaging in the practice of modeling the solution to real world problems | Teacher Resources: | <https://www.sites.google.com/a/cmpso.org/caccss-resources/k-8-modeling-task-force/k-8-modeling-resources> (examples of modeling problems and resources for teachers on teaching and scoring them)  <http://www.insidemathematics.org/index.php/standard-4> (video examples of students modeling with mathematics)  <http://learnzillion.com/lessons/1722-solve-multistep-word-problems-using-model-drawing> (video about modeling) |
| Student Resources: | N/A |
| Skills: | Model real world problems mapping relationships with appropriate models, analyze relationships to draw conclusions, interpret results in relation to context, justify and defend the model, and reflect on whether results make sense | Assessment: | Modeling Problems  Students can use statistical models to represent and analyze relationships between two real world samples to make inferences about populations in relation to the context of the problem. |
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| 3. | Description: | Mathematicians fluently add, subtract, multiply, and divide rational numbers | Teacher Resources: | N/A |
| Student Resources: | <http://www.mangahigh.com/en_us/games> (math games for building fluency with rational numbers)  <http://coolmath-games.com/> (math games for building fluency with rational numbers)  <http://hotmath.com/games.html>(math games for building fluency with rational numbers) |
| Skills: | Add, subtract, multiply and divide rational numbers, the culmination of fluency skills extended to negative numbers | Assessment: | Fluency Problems  Students can build fluency through consistent practice with all four operations on rational numbers. |
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| **Prior Knowledge and Experiences** |
| Student familiarity with the mean, median, and graphical representations (e.g., bar graph, histogram, box plots) provides a strong foundation for this unit. The first two learning experiences in this unit review these concepts. Teachers should adjust the length of the experiences based on students’ prior exposure to these concepts. This is the final unit in seventh grade. A major concept within this grade level is proportional reasoning, which is extended to include statistical concepts in this unit. |

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| **Learning Experience # 1** | | |
| The teacher may provide a population for which counting would be either impossible or inefficient to determine its size (e.g., a tree population) so that students can try their hand at creating a method for estimating the size of a population. | | |
| **Teacher Notes:** | This learning experience is intended as a pre-assessment of students’ understanding of populations and samples. Teachers may observe students strategies and listen to their conversations to determine their prior knowledge. Estimating a population size will be revisited later in the unit when students have learned more sophisticated strategies. | |
| **Generalization Connection(s):** | Representative samples create statistics from which mathematicians make valid generalizations about a population | |
| **Teacher Resources:** | <http://map.mathshell.org/materials/tasks.php?taskid=386&subpage=expert> (lesson about using sampling to estimate a population) | |
| **Student Resources:** | N/A | |
| **Assessment:** | Students mastering the concept and skills of this lesson should be able to answer questions such as:  Why is counting an inefficient and sometimes impossible strategy when determining the size of a population?  How confident are you in your estimate of the size of the population?  Beyond counting every object in the population, what other strategies could you try to improve your estimate?  What are some examples of populations, which might be hard to count and yet an estimate of their size is important?  What would you do differently if asked to estimate again? | |
| **Differentiation:**  (Multiple means for students to access content and multiple modes for student to express understanding.) | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| The teacher may modify the population to make it smaller or less complex | Students can estimate the size of a less complex population |
| **Extensions for depth and complexity:** | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| N/A | N/A |
| **Key Knowledge and Skills:** | * Understand statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population * Understand random sampling tends to produce representative samples and support valid inferences | |
| **Critical Language:** | Estimate, explain, population, inefficient, efficient | |

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| **Learning Experience # 2** | | |
| The teacher may show data in a variety of graphical representations so that students can discuss the advantages of different types of graphical representations to communicate data. | | |
| **Teacher Notes:** | This learning experience is also a pre-assessment activity to help teachers determine what prior knowledge and skills students have with analyzing a variety of graphical representations. | |
| **Generalization Connection(s):** | Representative samples create statistics from which mathematicians make valid generalizations about a population  Visual displays, measures of center, and measures of variability for two numerical data distributions from random samples facilitate informal comparative inferences about two populations | |
| **Teacher Resources:** | <http://www.mdk12.org/instruction/curriculum/mathematics/standard4/grade7.html> (school Improvement in Maryland provides examples of graphs in their sample assessments)  <http://www.docstoc.com/docs/61672468/Graphical-Representation-of-Data-GRAPHICAL> (discussion on the advantages of different types of graphical representations) | |
| **Student Resources:** | N/A | |
| **Assessment:** | Students mastering the concept and skills of this lesson should be able to answer questions such as:  How can line plots, bar graphs and box and whisker plots help you to describe data?  What are the similarities and differences between line plots, bar graphs and box and whisker plots? | |
| **Differentiation:**  (Multiple means for students to access content and multiple modes for student to express understanding.) | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| <http://au.ixl.com/math/year-6> (extra practice analyzing graphs in the section data and graphs) | Students can answer questions about graphs to support fluency with each of the graphical models |
| **Extensions for depth and complexity:** | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| <http://www.docstoc.com/docs/61672468/Graphical-Representation-of-Data-GRAPHICAL> (discussion on the advantages of different types of graphical representations)  <http://www.eisd.net/cms/lib04/TX01001208/Centricity/Domain/599/DoubleBubbleMap.pdf> (thinking map for comparing and contrasting) | Students can complete a graphic organizer comparing two different types of graphs |
| **Key Knowledge and Skills:** | * Understand statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population * Understand random sampling tends to produce representative samples and support valid inferences | |
| **Critical Language:** | Line plots, bar graphs, box and whisker plots, analyze | |

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| **Learning Experience # 3** | | |
| The teacher may provide students with the opportunity to measure items in order to create a data set (e.g., foot size, hand span) so that students can explore the meaning of the mean, median and range.  *Enactive*: Students can use cubes to determine the median by lining up the set of data from least to greatest. Students can use cubes to find the mean by leveling off the set of data. Students can use cubes to find the range of a set of data by seeing the difference between the largest and smallest data point.  *Iconic*: Students can create a line plot and determine the mean, median and range on the line plot.  *Symbolic*: Students can find the mean, median and range of a given set of data in a table. | | |
| **Generalization Connection(s):** | Representative samples create statistics from which mathematicians make valid generalizations about a population | |
| **Teacher Resources:** | [www.teachersnotebook.com/product/mhanson01/smartboard-interactive-mean-median-mode-and-range-practice](http://www.teachersnotebook.com/product/mhanson01/smartboard-interactive-mean-median-mode-and-range-practice) (practice with mean, median and range)  [www.bbc.co.uk/bitesize/ks2/maths/data/mode\_median\_mean\_range/play](http://www.bbc.co.uk/bitesize/ks2/maths/data/mode_median_mean_range/play) (video about the mean, median, and range) | |
| **Student Resources:** | N/A | |
| **Assessment:** | Students mastering the concept and skills of this lesson should be able to answer questions such as:  How can you find the mean, median and range of a given data set?  When might a median be better for calculating your math grade?  Why are the median and the mean both called measures of center? | |
| **Differentiation:**  (Multiple means for students to access content and multiple modes for student to express understanding.) | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| The teacher may provide students with smaller sets of data | Students can determine the mean, median, and range using manipulatives for smaller sets of data |
| **Extensions for depth and complexity:** | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| <http://www.youtube.com/watch?v=-LtFWjtTBSw> (video creating a graphic novel using an online template) | Students can create a short graphic novel titled the outliers that illustrates how outliers affect the mean and the median |
| **Key Knowledge and Skills:** | * Understand statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population * Understand random sampling tends to produce representative samples and support valid inferences | |
| **Critical Language:** | Mean, median, range, algorithm, calculate, data set, and line plot | |

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| **Learning Experience # 4** | | |
| The teacher may describe a population and give examples of different types of sampling methods so that students can identify what makes a sample either biased or unbiased.  *Enactive*: Students can explain and discuss their understanding of bias in samples as they look at the examples of the sampling methods.  *Iconic*: Students can label the different types of sampling to demonstrate their understanding of which factors contribute to sample bias.  *Symbolic*: Students can choose a population, create a sampling method, and justify what makes their sampling method representative of the population. | | |
| **Teacher Notes:** | Students tend to struggle with the concept of bias or fairness in a sample, often confusing this concept with their own concept of fair. For example, the students may declare a sample to be biased if they are not included in the sample or if the results disagree with their opinions. | |
| **Generalization Connection(s):** | Random and relatively large samples generate sample statistics that estimate population parameters and support valid inferences | |
| **Teacher Resources:** | <http://www.rsm.rcs.k12.tn.us/teachers/Robertsont/6.5.3%20Biased%20Samples%20Lesson,%20Practice,%20and%20Quiz.pdf> (lesson on what makes a sample biased)  <http://eric.ed.gov/?id=EJ600186> “How Do Students Think About Statistical Sampling Before Instruction?” by Victoria Jacobs  <http://www.utexas.edu/academic/ctl/assessment/iar/teaching/gather/method/survey-Sampling.php?task=research> (describes various types of sampling) | |
| **Student Resources:** | <http://learnzillion.com/lessons/1844-identify-a-random-sample> (video lesson on identifying a random sample) | |
| **Assessment:** | Students mastering the concept and skills of this lesson should be able to answer questions such as:  What makes a sample unbiased?  What should you look for to determine whether a sample is biased?  Why might it be useful to have more than one sample?  What are some examples of samples that would not be a valid representation of a population?  How do you distinguish between a random and biased sample?  How can you choose a representative sample of a population?  Why do random samples better estimate or predict information about a population than non-random samples? | |
| **Differentiation:**  (Multiple means for students to access content and multiple modes for student to express understanding.) | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| The teacher may provide a completed model comparing biased and unbiased samples  The teacher may provide explanations of sampling methods that are biased and unbiased | Students can recognize the differences between a biased and an unbiased sample by separating examples of samples into groups based on the types of sampling methods provided by the teacher |
| **Extensions for depth and complexity:** | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| <http://www.abs.gov.au/websitedbs/a3121120.nsf/home/statistical+language+-+census+and+sample> (pros and cons of census versus sample)  <http://www.scienceclarified.com/dispute/Vol-2/Should-statistical-sampling-be-used-in-the-United-States-Census.html#b> (article on pros and cons of census versus sample) | Students can create a summary of the pros and cons of conducting a census as opposed to taking a random sample of a population |
| **Key Knowledge and Skills:** | * Understand statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population * Understand random sampling tends to produce representative samples and support valid inferences * Use data from a random sample to draw inferences about a population with an unknown characteristic of interest * Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions | |
| **Critical Language:** | Biased, unbiased, sample, population, representative, accurate, random | |

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| **Learning Experience # 5** | | |
| The teacher may model several sampling methods for determining a population size so that students can explore the efficiency of sampling methods in comparison to counting an entire population.  *Enactive*: Students can determine the size of populations (e.g. bag of marbles, jellybeans) by using sampling methods.  *Iconic*: Students can create ratio tables to examine the proportionality between samples and populations.  *Symbolic*: Students can create a visual representation of the data collected from numerous samples, and create inferences based on the visual representations of the samples. | | |
| **Teacher Notes:** | Students often struggle with inferences made about a population based on a sample. The teacher may want to lead a discussion about the concept of statistical measures as estimates of information that would be very difficult to obtain exactly. | |
| **Generalization Connection(s):** | Random and relatively large samples generate sample statistics that estimate population parameters and support valid inferences | |
| **Teacher Resources:** | <http://eric.ed.gov/?id=EJ580572> “Capture and Recapture: Your Students’ Interest in Statistics” by June Morita  <http://cms.cerritos.edu/uploads/MathGateways/Ratios/Mario%20Title%20V%20Experiment%20-%20combined%20file.pdf> (teaching notes for the Capture and Recapture lesson)  <http://learnzillion.com/lessons/1848-make-inferences-about-a-population-by-analyzing-random-samples> (video on making inferences by analyzing random samples)  <http://www.illustrativemathematics.org/illustrations/974> (examples task about sampling)  <http://www.pbslearningmedia.org/resource/midlit11.math.splsamp/population-sampling-fish/> (video about population sampling) | |
| **Student Resources:** | <http://www.hoodamath.com/tutorials/7thgrade/Using_Measures_of_Center_to_Draw_Inferences_About_Two_Populations_Example_1.html> (video example of drawing inferences about populations) | |
| **Assessment:** | Students mastering the concept and skills of this lesson should be able to answer questions such as:  When does a sample sufficiently represent the population?  How can a random sample of a larger population be used to draw inferences?  How do statistics (e.g., mean, median, range) from multiple samples generated from the same population compare to each other? | |
| **Differentiation:**  (Multiple means for students to access content and multiple modes for student to express understanding.) | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| The teacher may provide students with a method for sampling and pair students at similar levels of understanding | Students can determine the size of a population using a sampling method provided by the teacher |
| **Extensions for depth and complexity:** | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| <http://wildlife.state.co.us/Pages/Home.aspx> (Colorado Division of Wildlife) | Students can write a report about sampling methods used to determine the size of a population of animals used by the Department of Natural Resources |
| **Key Knowledge and Skills:** | * Understand statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population * Understand random sampling tends to produce representative samples and support valid inferences * Use data from a random sample to draw inferences about a population with an unknown characteristic of interest * Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions | |
| **Critical Language:** | Variability, distribution, inference, sample, population | |

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| **Learning Experience # 6** | | |
| The teacher may revisit the first learning experience so that students can critique the strengths and weaknesses of their initial methods for sampling and find a more accurate and efficient method. (e.g., a tree population). | | |
| **Teacher Notes:** | This learning experience revisits the first learning experience but students can now use more sophisticated methods by applying their understanding of line plots, samples, means, medians, and ranges. | |
| **Generalization Connection(s):** | Random and relatively large samples generate sample statistics that estimate population parameters and support valid inferences | |
| **Teacher Resources:** | <http://map.mathshell.org/materials/tasks.php?taskid=386&subpage=expert> (counting trees formative assessment lesson) | |
| **Student Resources:** | <http://www.bbc.co.uk/bitesize/ks2/maths/data/mode_median_mean_range/play/> (game to practice mean, median, and range) | |
| **Assessment:** | Students mastering the concept and skills of this lesson should be able to answer questions such as:  Why would you use a mean or median to estimate the population of the trees?  What size sample is sufficient to estimate the population?  Do the estimates from your sample accurately represent the population? Why or why not?  What is a sample?  What is a population?  Why do statistics based on a representative sample support generalizations about a population?  Why do we investigate samples rather than entire populations? | |
| **Differentiation:**  (Multiple means for students to access content and multiple modes for student to express understanding.) | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| The teacher may provide students with possible solution strategies for sampling <http://map.mathshell.org/materials/tasks.php?taskid=386&subpage=expert> (counting trees formative assessment lesson with possible solution strategies) | Students can analyze possible solution strategies in order to create a more accurate and efficient strategy |
| **Extensions for depth and complexity:** | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| N/A | N/A |
| **Key Knowledge and Skills:** | * Assess the degree of visual overlap Informally of two numerical data distributions with similar variables, measuring the difference between the centers by expressing it as a multiple of a measure of variability | |
| **Critical Language:** | Estimate, diagram, method, accurate, explain | |

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| **Learning Experience # 7** | | |
| The teacher may give students a mean for a context (e.g., number of people per home) so that students can create several possible data distributions for the given mean to explore the concept of variation (e.g., Mean Absolute Deviation).  *Enactive*: Students can create a variety of line plots with sticky notes for the given mean.  *Iconic*: Students can rank the line plots from those with the most to the least variation and describe their reasoning for the ranking.  *Symbolic*: Students can use MAD (Mean Absolute Deviation) to determine the variation from the mean. Students can then interpret the meaning of the MAD in a provided context. | | |
| **Generalization Connection(s):** | Comparing mean absolute deviations requires a multiplicative (ratio) rather than additive comparison because mathematicians compare the unit size of the deviations to each other  Visual displays, measures of center, and measures of variability for two numerical data distributions from random samples facilitate informal comparative inferences about two populations | |
| **Teacher Resources:** | <http://www.learner.org/courses/learningmath/data/pdfs/session5/mads_1.pdf> (article by Gary Kader describing this learning experience in more detail)  <http://www.illustrativemathematics.org/7.SP.B> (tasks related to this concept, offensive Linemen or college athletes)  <http://learnzillion.com/lessons/1462-make-comparative-inferences-using-the-mad> (video about the MAD) | |
| **Student Resources:** | [www.ixl.com/math/algebra-1/mean-absolute-deviation](http://www.ixl.com/math/algebra-1/mean-absolute-deviation) (practice calculating the MAD)  <http://tulyn.com/absolute_value.htm> (review of absolute value) | |
| **Assessment:** | Students mastering the concept and skills of this lesson should be able to answer questions such as:  What is the mean absolute deviation?  How is variability of data sets measured?  Why do we measure the variability of a data set?  What would happen if we didn’t use the absolute value when calculating the MAD?  What can you tell about a data that has a small versus a large MAD? | |
| **Differentiation:**  (Multiple means for students to access content and multiple modes for student to express understanding.) | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| <http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&ved=0CC8QFjAB&url=http%3A%2F%2Fmahaffeymath.wikispaces.com%2Ffile%2Fview%2FMean%2BAbsolute%2BDeviation%2Bgraphic%2Borganizer.docx&ei=_bRyUo_wEc_eyQHw9YHwBg&usg=AFQjCNGihLNDIvCk3AYssA4i6iGhVZGAVg&bvm=bv.55819444,d.aWc> (graphic organizer for MAD) | Students can calculate the MAD using a graphic organizer |
| **Extensions for depth and complexity:** | **Access** (Resources and/or Process) | **Expression** (Products and/or Performance) |
| <http://sahilmohnani.wordpress.com/2013/06/02/absolute-mean-deviation/> (discussion on the MAD versus standard deviation)  <http://www.eisd.net/cms/lib04/TX01001208/Centricity/Domain/599/DoubleBubbleMap.pdf> (thinking map for comparing and contrasting) | Students can create a reflection addressing the similarities and differences between standard deviation and mean absolute deviation |
| **Key Knowledge and Skills:** | * Understand statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population * Understand random sampling tends to produce representative samples and support valid inferences * Use data from a random sample to draw inferences about a population with an unknown characteristic of interest * Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions * Assess the degree of visual overlap Informally of two numerical data distributions with similar variables, measuring the difference between the centers by expressing it as a multiple of a measure of variability * Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations | |
| **Critical Language:** | Absolute value, deviation, variance, distribution, mean absolute deviation (MAD) | |