# Colorado Academic Standards: Mathematics Benchmarking Report Summary



### Introduction

The Colorado Department of Education is committed to providing rigorous academic standards of the highest quality. In preparation for the standards review and revise process, the CDE requested that Center on Standards and Assessment Implementation (CSAI) staff design and conduct an evaluation of the current Colorado Academic Standards (CAS) in relation to selected external referent standards, in order to determine how and to what degree Colorado's current standards compare with other state standards, with national standards, and with international standards; to inform the CDE regarding alignment or gaps between the CAS and these external referents; and to provide guidance to Standards Review Committees in their work.

#### Methodology (pp. 5-10)\*

<u>Alignment Review Criteria</u>: The general criteria were depth and breadth, coherence, and rigor; specific criteria are described in greater detail in the following bulleted list. These criteria were used to ensure that the information in the findings would be appropriate for Colorado's context and, thus, maximally useful to the state. The composition for the criteria were designed as holistic questions of sufficiency and appropriateness that were applied by analysts as they conducted their review.

- **Depth and Breadth**: Do the CAS GLE and EO statements describe sufficient and appropriate depth and breadth of content within each standard, with respect to comparable statements from the CCSS? If not, what content is missing? Are the GLE and EO statements free from extraneous content? If not, what content is extraneous?
- **Coherence**: Are the CAS GLE and EO statements for each standard sequenced appropriately, with respect to comparable statements from the CCSS? Do the GLE and EO statements for each standard begin and end at appropriate points in the content?
- **Rigor**: Do the CAS GLE and EO statements describe content and skill expectations of a reasonable and appropriate level for the given grade, with respect to comparable statements from the selected external referents? Do the CAS GLE and EO statements communicate an appropriate level of rigor?

**External Referent Standards.** The CDE selected the external referent standards to which the CAS would be compared in this step. Included in the selection criteria was whether the standards were from states or countries identified as having strong overall academic performance and quality of their standards. To enable maximal usefulness in guiding standards revisions, sets of standards were sought that would be relevant in many content areas. Additionally, it was thought that by including a review of each set of external referent standards for multiple content areas, the comparison would benefit from identification of cross-content elements or guiding philosophies that might not be apparent in any one content area. To this end, they selected the following sets of standards from the following entities:

- Virginia's Standards of Learning (SOL), and
- Singapore's Mathematics Syllabi

<u>Criteria Used for Comparative Analyses</u>. The external referent comparison was intended to serve as a holistic review of the similarities and differences between the CAS, in their current forms, and each external referent, as compared to similar analyses completed prior to the last revisions of the CAS. Specifically, comparisons were documented in terms of two criteria: organization/structure and content. CSAI staff's considerations for judging each are defined as follows:



- Organization/Structure: Considerations related to standards organization and structure included similarities and differences in grade articulation (standards articulated by individual grade, grade span, course, etc.; crossgrade strands versus no repetition of content), hierarchy of standards (number of levels in standards—e.g., strand, standard, benchmark, indicator), number of standards (numbers of strands, standards, and indicators), design/format, and ways in which intended knowledge and skills are communicated.
- Content: CSAI staff considerations related to standards content included similarities and differences in scope and sequence (depth and breadth of content described in the standards), grade spans (sequencing and distribution of content within and across the grade spans), and wording (specificity of language; focus on action verbs, knowledge, etc.) of the standards.

# **Findings**

## **Overall Findings**

CSAI determined there was strong alignment between the CAS and CCSS mathematics standards. For overall depth and breadth, 85% of Colorado math standards are fully aligned with CCSS math. Eighth grade was found to have the least alignment, with some EO statements describing only parts of CCSS standards. For coherence, 99% of Colorado math standards are strongly aligned to CCSS standards. For rigor, 93% of Colorado math standards are fully aligned to CCSS math, with rigor most misaligned at third and fourth grade. A grade-by-grade summary of all full, partial, and no alignments can be found in Table 1. The EO-by-EO summary can be found in the full CSAI report.

CSAI found that in organization and structure, the math standards were similar to those found in Virginia and Singapore for Grades K-8, but different at high school. Virginia organizes high school standards by course, and Singapore has five secondary (beginning after Primary 6) syllabi with students taking 4-6 years of secondary mathematics, with preuniversity syllabi offered at the highest level. Colorado organizes the CAS around four content strands, while Virginia uses six and Singapore uses three. The differences can be explained by matching two strands in one state or country with a single, broader strand in another.

Colorado's math standards focus on the concepts and skills students are to master at each grade. The Virginia Standards of Learning identify content that is essential at each grade level, and statements about student understandings are in a separate document, the Curriculum Framework. Colorado and Virginia have comparable content strands, but content within the strands at each grade level is not similar between the states. Virginia places greater emphasis on patterns in elementary and introduces many concepts earlier than found in the CAS. Statistics and probability standards in Colorado are, in general, more rigorous than in Virginia, and Colorado has standards for Personal Financial Literacy. At the high school level, Virginia's standards describe seven courses while Colorado does not subdivide high school content into courses or grades. For additional differences between the CAS and Virginia's standards, see Pages 23-24 in the CSAI report.

Each Singapore syllabi includes information about curriculum, instruction, assessment of student learning, and student affect, none of which is in Colorado's standards. Each syllabus also presents standards along with learning experiences that should be part of the enacted curriculum. At the secondary level, Singapore includes standards specifically written to position content into real-world contexts, something that is usually left for teachers to infer in Colorado. Colorado and Singapore uses similar strand descriptions (algebra, geometry and measurement, etc.) but Singapore identifies "Mathematical Processes" as its own strand. The greatest differences between the Colorado and Singapore standards are related to content focus and placement. Prime numbers are addressed about three years earlier in Colorado than in Singapore, but stem-and-leaf plots are included in Singapore's Secondary 1 syllabus but are not named in Colorado's standards.

Table 1. CSAI alignment of CAS mathematics standards and the Common Core State Standards for Mathematics.



|               |                          |         | Depth and Breadth |           |    |    |   | Coherence |         |           |   |    |   | Rigor   |         |           |    |    |   |
|---------------|--------------------------|---------|-------------------|-----------|----|----|---|-----------|---------|-----------|---|----|---|---------|---------|-----------|----|----|---|
| Grade         | # of CO<br>Standar<br>ds | Fully   |                   | Partially |    | No |   | Fully     |         | Partially |   | No |   | Fully   |         | Partially |    | No |   |
|               |                          | #       | %                 | #         | %  | #  | % | #         | %       | #         | % | #  | % | #       | %       | #         | %  | #  | % |
| Kindergarten  | 22                       | 22      | 10<br>0           | 0         | 0  | 0  | 0 | 22        | 10<br>0 | 0         | 0 | 0  | 0 | 21      | 95      | 1         | 5  | 0  | 0 |
| First Grade   | 22                       | 19      | 86                | 3         | 14 | 0  | 0 | 22        | 10<br>0 | 0         | 0 | 0  | 0 | 21      | 95      | 1         | 5  | 0  | 0 |
| Second Grade  | 29                       | 23      | 79                | 6         | 21 | 0  | 0 | 29        | 10<br>0 | 0         | 0 | 0  | 0 | 29      | 10<br>0 | 0         | 0  | 0  | 0 |
| Third Grade   | 25                       | 24      | 96                | 1         | 4  | 0  | 0 | 25        | 10<br>0 | 0         | 0 | 0  | 0 | 20      | 80      | 5         | 20 | 0  | 0 |
| Fourth Grade  | 30                       | 26      | 87                | 4         | 13 | 0  | 0 | 30        | 10<br>0 | 0         | 0 | 0  | 0 | 23      | 77      | 7         | 23 | 0  | 0 |
| Fifth Grade   | 35                       | 27      | 77                | 8         | 23 | 0  | 0 | 35        | 10<br>0 | 0         | 0 | 0  | 0 | 32      | 91      | 3         | 9  | 0  | 0 |
| Sixth Grade   | 34                       | 24      | 71                | 10        | 29 | 0  | 0 | 34        | 10<br>0 | 0         | 0 | 0  | 0 | 29      | 85      | 5         | 15 | 0  | 0 |
| Seventh Grade | 27                       | 22      | 81                | 5         | 19 | 0  | 0 | 27        | 10<br>0 | 0         | 0 | 0  | 0 | 26      | 96      | 1         | 4  | 0  | 0 |
| Eighth Grade  | 37                       | 21      | 57                | 15        | 41 | 1  | 3 | 36        | 97      | 0         | 0 | 1  | 3 | 36      | 97      | 0         | 0  | 1  | 3 |
| High School   | 115                      | 11<br>0 | 96                | 5         | 4  | 0  | 0 | 11<br>4   | 99      | 1         | 1 | 0  | 0 | 11<br>2 | 97      | 3         | 3  | 0  | 0 |
| Total         | 376                      | 31<br>8 | 85                | 57        | 15 | 1  | 0 | 37<br>4   | 99      | 1         | 0 | 1  | 0 | 34<br>9 | 93      | 26        | 7  | 1  | 0 |