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Colorado Academic
STANDARDS

Mathematics

Adopted: December 10, 2009

Overview of Changes
Mathematics Standards

Principles of the Standards Review Process

The Colorado Model Content Standards revision process was informed by these guiding principles:

- Begin with the end in mind; define what prepared graduates need to be successful using 21st century skills in our global economy.
- Align K-12 standards with early childhood expectations and higher education.
- Change is necessary.
- Standards will be deliberately designed for clarity, rigor, and coherence.
- There will be fewer, higher and clearer standards.
- Standards will be actionable.

Notable Changes to the Colorado Model Content Standards in Mathematics

The most evident changes to the Colorado standards are replacing grade-band expectations (K-4, 5-8, and 9-12) with grade-level specific expectations. These are explained here in addition to other changes that are apparent upon comparison between the current mathematics standards and the proposed changes.

1. **Impact of standards articulation by grade level**. The original Colorado Model Content Standards for Mathematics were designed to provide districts with benchmarks of learning at grades 4, 8, and 12. The mathematics standards revision subcommittee was charged with providing more a specific learning trajectory of concepts and skills across grade levels, from early school readiness to postsecondary preparedness. Articulating standards by grade level up to eighth grade in mathematics affords greater specificity (clearer standards) in describing the learning path across levels (higher standards), while focusing on a few key ideas at each grade level (fewer standards).

2. **Articulation of high school standards**. High school standards are not articulated by grade level, but by standard. This is intended to support district decisions about how best to design curriculum and courses – whether through an integrated approach, a traditional course sequence, or alternative approaches such as career and technical education. The high school mathematics standards delineate what all high school students should know and be able to do in order to be well prepared for any postsecondary option. The individual standards are not meant to represent a course or a particular timeframe. All high school students should be able to reach these rigorous standards within four years. Students with advanced capability may accomplish these expectations in a shorter timeframe leaving open options for study of other advanced mathematics.

3. **Integration of P-2 Council's recommendations**. The mathematics subcommittee integrated the *Building Blocks to the Colorado K-12 Content Standards* document into the P-12 mathematics standards, aligning expectations to a great degree. Important mathematics concepts and skills are defined clearly across these foundational years, detailing expectations to a much greater extent for teachers and parents.

4. **Standards are written for mastery**. The proposed revisions to standards define mastery of concepts and skills. Mastery means that a student has facility with a skill or concept in multiple contexts. This is not an indication that instruction at a grade-level expectation begins and only occurs at that grade level. Maintenance of previously mastered concepts and skills and scaffolding future learning are the domain of curriculum and instruction – not standards.

5. **The processes and procedures of school Algebra have been made more explicit**. More specificity about algebraic procedures is apparent in the Patterns, Functions and Algebraic Structures expectations.

   For instance, two high school expectations read:

   - Expressions, equations, and inequalities can be expressed in multiple, equivalent forms.
   - Solutions to equations, inequalities and systems of equations are found using a variety of tools.

   An eighth-grade expectation reads:

   - Properties of algebra, equality, and inequality are used to solve linear equations and inequalities.

6. **Explicit evolution of algebra concepts**. The proposed revisions include a more explicit delineation of algebra concepts across grade levels. Understanding of algebra concepts develops from elementary through middle school with a rigorous study of algebra content in grades 8 and high school.

7. **Emphasis on concepts and skills across grade levels**. The subcommittee deliberately designed the standards to emphasize specific concepts and skills at different grade levels. This allows teachers to focus on fewer concepts at greater depth than in the past.

8. **Integration of technology, most notably at the high school level**. The standards integrate appropriate technology to allow students access to concepts and skills in mathematics in ways that mirror the 21st century workplace.

9. **Greater focus on 3. Data Analysis, Statistics, and Probability across all grade levels**. Information literacy in mathematics involves the ability to manage and make sense of data in more sophisticated ways than in the past. This involves emphasizing 3. Data Analysis, Statistics, and Probability to a greater degree than in the original mathematics standards.

10. **Intentional integration of personal financial literacy (PFL).** Personal financial literacy was integrated preschool through grade twelve in the math standards in order to assure high school graduates are fiscally responsible. House Bill 08-1168 requires standards which includes these skills: goal setting, financial responsibility, income and career; planning, saving and investing, using credit; risk management and insurance.

Below is a quick guide to other changes in the mathematics standards:

Area	Summary of changes	
	Current Standards	**Proposed Revisions**
Number of standards	Colorado has six standards in mathematics	Combine current standards 1 and 6 and standards 4 and 5. There are now four standards
Names of standards	**Standard 1** Number Sense and Number Relationships **Standard 2** Patterns and Algebra **Standard 3** Data and Probability **Standard 4** Geometry **Standard 5** Measurement **Standard 6** Computation	**Standard 1** Number Sense, Properties, and Operations **Standard 2** Patterns, Functions, and Algebraic Structures **Standard 3** Data Analysis, Statistics, and Probability **Standard 4** Shape, Dimension, and Geometric Relationships
Integration of 21st century and postsecondary workforce readiness skills	• Not deliberately addressed in original document.	• A design feature of the revision process. • Intentionally integrated into evidence outcomes.
P-2	• Standards articulated for grade band beginning with kindergarten • Benchmarks articulated by grade band of K-4 with most geared to upper grades.	• Pre-K included. • Grade level expectations articulated for each elementary grade • Clear expectations articulated for grades P-2.
Number of grade level expectations (GLE)	• Average of 27 benchmarks per grade level.	• Average of 10 grade level expectations per grade level (K-8), with 22 for high school.
Integration of Personal Financial Literacy (PFL)	• Not deliberately addressed in original document.	• A design feature of the revision process. • Intentionally integrated into evidence outcomes.

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Mathematics National Expert Reviewer

Dr. Ann Shannon is a mathematics educator with many decades of experience who specializes in standards, assessment, and curriculum. Currently, Shannon works as consultant helping states, districts, and schools to better serve the needs of diverse learners of mathematics.

Dr. Shannon was employed as a research fellow at the Shell Centre for Mathematics Education, University Nottingham, England before moving to the University of California, Berkeley in 1994.

At the University of California, she developed performance assessments for the NSF-funded Balanced Assessment project and the New Standards project. Her 1999 monograph, *Keeping Score*, was published by the National Research Council and drew on her work for Balanced Assessment and New Standards.

Recently Shannon has helped Maine, Georgia, and Rhode Island develop academic standards for learning mathematics.

References

The mathematics subcommittee used a variety of resources representing a broad range of perspectives to inform its work. Those references include:

- Singapore National Curriculum
- Massachusetts Curriculum Framework
- Virginia Standards of Learning
- Finland – National Core Curriculum
- WestEd Colorado Model Content Standards Review
- Achieve *Benchmarks for Elementary, Middle, and High School Mathematics*
- Benchmarks 2061
- College Board *Standards for College Success*
- Guidelines for Assessment and Instruction in Statistics Education (GAISE)
- NCTM Principles and Standards for School Mathematics and Focal Points
- Standards for Success "Understanding University Success"
- Minnesota Academic Standards, Mathematics K-12
- Building Blocks to the Colorado K-12 Content Standards
- National Math Panel Report

Colorado Academic Standards
Mathematics Standards

"Pure mathematics is, in its way, the poetry of logical ideas."
Albert Einstein

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"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."
Dr. Michael Brown, Nobel Prize Laureate

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In the 21st century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how 21st century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The "Continuum of State Standards Definitions" section that follows shows the hierarchical order of the standards components. The "Standards Template" section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

Prepared Graduate Competencies: The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Standard: The topical organization of an academic content area.

High School Expectations: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. *What do students need to know in high school?*

Grade Level Expectations: The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

Evidence Outcomes: The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

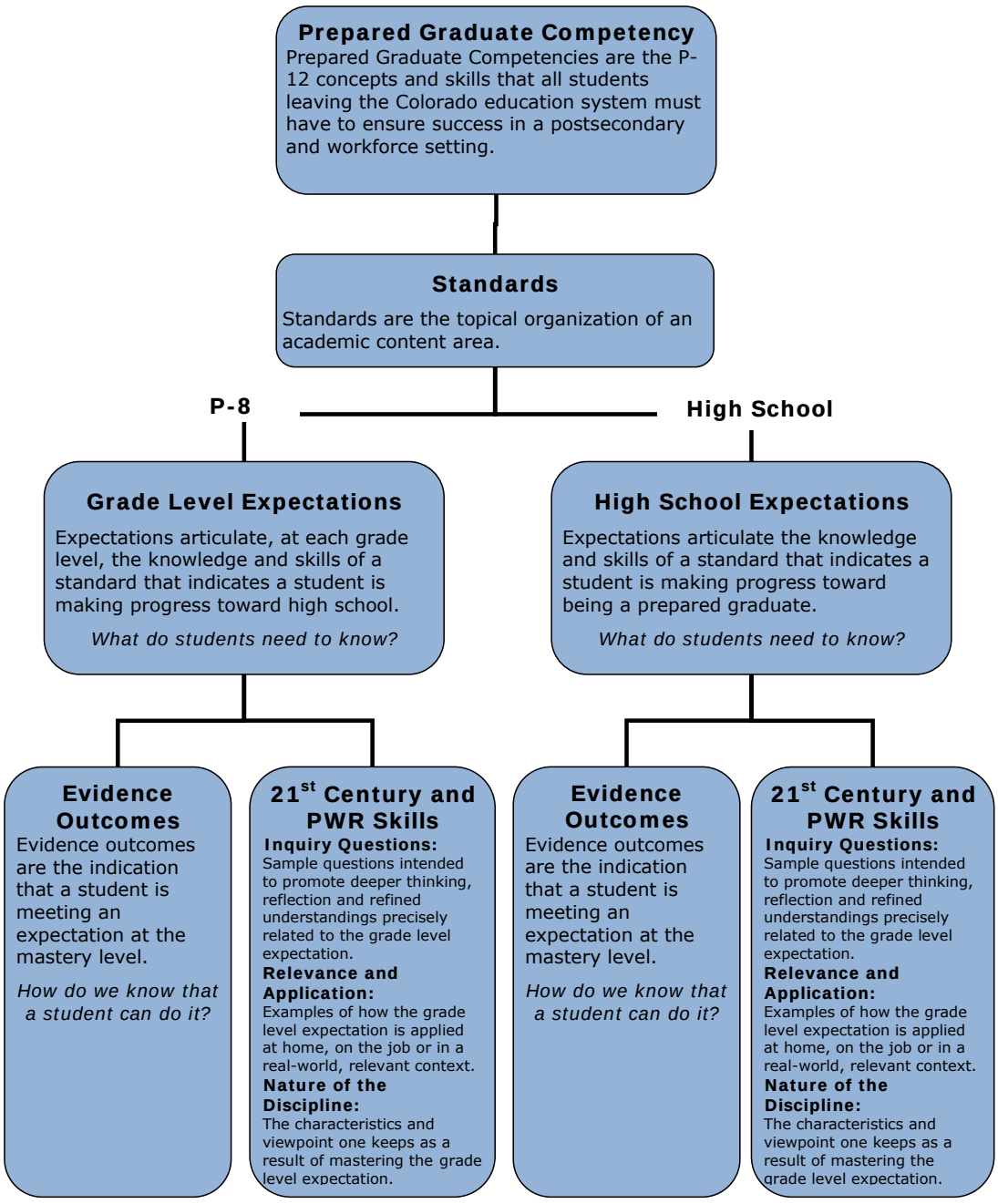
21st Century Skills and Readiness Competencies: Includes the following:

- ***Inquiry Questions:***
  Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

- ***Relevance and Application:***
  Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

- ***Nature of the Discipline:***
  The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Continuum of State Standards Definitions



STANDARDS TEMPLATE

Content Area: NAME OF CONTENT AREA
Standard: The topical organization of an academic content area.

Prepared Graduates:
- The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

High School and Grade Level Expectations

Concepts and skills students master:

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

What do students need to know?

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: Evidence outcomes are the indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*	**Inquiry Questions:** Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
	Relevance and Application: Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
	Nature of the Discipline: The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At the most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Colorado Academic Standards
Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
High School	
1. Number Sense, Properties, and Operations	1. The complex number system includes real numbers and imaginary numbers 2. Formulate, represent, and use algorithms with real numbers flexibly, accurately, and efficiently 3. Systematic counting techniques are used to describe and solve problems
2. Patterns, Functions, and Algebraic Structures	1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables 2. Graphs and tables are used to describe the qualitative behavior of common types of functions 3. Parameters influence the shape of the graphs of functions 4. Expressions , equations, and inequalities can be expressed in multiple, equivalent forms 5. Solutions to equations, inequalities and systems of equations are found using a variety of tools 6. Quantitative relationships in the real world can be modeled and solved using functions
3. Data Analysis, Statistics, and Probability	1. Statistical methods take variability into account, supporting informed decision-making through quantitative studies designed to answer specific questions 2. The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions 3. Visual displays and summary statistics condense the information in data sets into usable knowledge 4. Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic 5. Probability models outcomes for situations in which there is inherent randomness, quantifying the degree of certainty in terms of relative frequency of occurrence
4. Shape, Dimension, and Geometric Relationships	1. Attributes of two- and three-dimensional objects are measurable and can be quantified 2. Objects in the plane and their parts, attributes, and measurements can be analyzed deductively 3. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically 4. Right triangles are central to geometry and its applications

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Eighth Grade	
1. Number Sense, Properties, and Operations	1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line 2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently
2. Patterns, Functions, and Algebraic Structures	1. Linear functions model situations with a constant rate of change and can be represented algebraically, graphically, and using tables 2. Properties of algebra, equality, and inequality are used to solve linear equations and inequalities 3. Graphs and tables can be used to distinguish between linear and nonlinear functions
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Objects in the plane and their parts and attributes can be analyzed 2. Direct and indirect measurements can be used to describe and make comparisons
Seventh Grade	
1. Number Sense, Properties, and Operations	1. In the real number system, rational numbers have a unique location on the number line 2. Formulate, represent, and use algorithms with integers and positive rational numbers flexibly, accurately, and efficiently 3. Proportional reasoning involves comparisons and multiplicative relationships among ratios
2. Patterns, Functions, and Algebraic Structures	1. Relationships involving the constant rate of change are modeled and solved using linear functions
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics with one-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Objects in space and their parts and attributes can be measured and analyzed 2. Proportional reasoning is used to make indirect measurements

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Sixth Grade	
1. Number Sense, Properties, and Operations	1. In the real number system, positive rational numbers are represented in multiple equivalent forms 2. Formulate, represent, and use algorithms with positive rational numbers flexibly, accurately, and efficiently 3. Quantities can be expressed and compared using ratios and rates
2. Patterns, Functions, and Algebraic Structures	1. Patterns can be described using words, tables, and graphs 2. Variables are used to represent unknown quantities
3. Data Analysis, Statistics, and Probability	1. Questions can be answered by collecting and analyzing data and data displays 2. Mathematical models are used to determine probability
4. Shape, Dimension, and Geometric Relationships	1. Polygons can be described, classified, and analyzed by their attributes 2. Standard units provide common language for communicating measurements
Fifth Grade	
1. Number Sense, Properties, and Operations	1. The characteristics of numbers can be used to classify them in various ways 2. In the real number system, commonly used rational numbers have multiple equivalent representations 3. Formulate, represent, and use algorithms to multiply and divide multi-digit whole numbers with flexibility, accuracy, and efficiency
2. Patterns, Functions, and Algebraic Structures	1. Number patterns and relationships can be described using a variety of tools 2. When a relationship exists between two quantities, a change in one results in a change in the other
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics are used to describe and interpret data 2. Mathematical models are used to determine probability, analyze and describe the outcomes of events
4. Shape, Dimension, and Geometric Relationships	1. Geometric figures in the plane and in space are described and analyzed by their attributes 2. Linear measure, area, and volume are fundamentally different and require different units of measure

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Fourth Grade	
1. Number Sense, Properties, and Operations	1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms 2. Formulate, represent, and use algorithms to multiply and divide with flexibility, accuracy, and efficiency 3. Different models and representations can be used to compare fractional parts
2. Patterns, Functions, and Algebraic Structures	1. Number patterns and relationships can be represented by symbols 2. Number properties and relationships can be used to solve problems
3. Data Analysis, Statistics, and Probability	1. Visual displays of classroom data can be used to summarize information across the content areas 2. Mathematical models are used to test predictions about the likelihood of events
4. Shape, Dimension, and Geometric Relationships	1. Geometric figures are described by their attributes and specific location in the plane 2. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time
Third Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships from ones to 10,000 and forms the foundation for efficient algorithms 2. Parts of a whole can be modeled and represented in different ways 3. Formulate, represent, and use algorithms to add and subtract multi-digit whole numbers with flexibility, accuracy, and efficiency 4. Multiplying and dividing are inverse operations modeled in a variety of ways
2. Patterns, Functions, and Algebraic Structures	1. Number patterns are based on operations and relationships 2. Number properties can be used to solve problems
3. Data Analysis, Statistics, and Probability	1. Visual displays of data can be used to answer questions of interest 2. Mathematical models are used to explore and describe fairness
4. Shape, Dimension, and Geometric Relationships	1. Geometric figures are described by their attributes and position in the plane 2. Objects have distinct attributes that can be measured with appropriate tools

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Second Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships from ones to 1,000 and forms the foundation for efficient algorithms 2. Formulate, represent, and use algorithms to add and subtract two-digit whole numbers with flexibility, accuracy, and efficiency 3. Fractions represent parts of a whole object or set
2. Patterns, Functions, and Algebraic Structures	1. Patterns are based on rules 2. Number relationships can be used to develop computation strategies
3. Data Analysis, Statistics, and Probability	1. Visual displays of data can be constructed in a variety of formats 2. Mathematical models are used to describe the likelihood of an outcome or event
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be created and described by quantifiable attributes 2. Some attributes of objects are measurable and can be quantified using different tools
First Grade	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships from ones to 100 and forms the foundation for efficient algorithms 2. Adding and subtracting involve composing and decomposing using a variety of strategies 3. Parts of objects can be shown as fractions
2. Patterns, Functions, and Algebraic Structures	1. Patterns can grow 2. Number relationships can be used to solve problems
3. Data Analysis, Statistics, and Probability	1. Visual displays of data can be created using individual student data
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be created and described by composing and decomposing 2. Measurement is used to compare and order objects and events

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Kindergarten	
1. Number Sense, Properties, and Operations	1. Whole numbers can be used to name, count, represent, and order quantity 2. Adding and subtracting to 10 involves composing and decomposing using a variety of strategies and representations
2. Patterns, Functions, and Algebraic Structures	1. Patterns can repeat 2. Relationships exist between numbers
3. Data Analysis, Statistics, and Probability	1. Visual displays of information can used to answer questions
4. Shape, Dimension, and Geometric Relationships	1. Shapes are described by their characteristics and position 2. Measurement is used to compare and order objects
Preschool	
1. Number Sense, Properties, and Operations	1. Quantities can be represented and counted 2. Counting is a means for solving problems
2. Patterns, Functions, and Algebraic Structures	1. Objects can be sorted based on patterns and relationships
3. Data Analysis, Statistics, and Probability	1. Information and objects can be sorted
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be observed in the world and described in relation to one another 2. Measurement is used to compare objects

21st Century Skills and Readiness Competencies in Mathematics

Mathematics in Colorado's description of 21st century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21st century skills, as follows:

Critical Thinking and Reasoning

Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

Information Literacy

The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

Collaboration

Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

Self-Direction

Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

Invention

Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

Colorado's Description for School Readiness
(Adopted by the State Board of Education, December 2008)
School readiness describes both the preparedness of a child to engage in and benefit from learning experiences, and the ability of a school to meet the needs of all students enrolled in publicly funded preschools or kindergartens. School readiness is enhanced when schools, families, and community service providers work collaboratively to ensure that every child is ready for higher levels of learning in academic content.

Colorado's Description of Postsecondary and Workforce Readiness
(Adopted by the State Board of Education, June 2009)
Postsecondary and workforce readiness describes the knowledge, skills, and behaviors essential for high school graduates to be prepared to enter college and the workforce and to compete in the global economy. The description assumes students have developed consistent intellectual growth throughout their high school career as a result of academic work that is increasingly challenging, engaging, and coherent. Postsecondary education and workforce readiness assumes that students are ready and able to demonstrate the following without the need for remediation: Critical thinking and problem-solving; finding and using information/information technology; creativity and innovation; global and cultural awareness; civic responsibility; work ethic; personal responsibility; communication; and collaboration.

How These Skills and Competencies are Embedded in the Revised Standards
Three themes are used to describe these important skills and competencies and are interwoven throughout the standards: *inquiry questions; relevance and application; and the nature of each discipline.* These competencies should not be thought of stand-alone concepts, but should be integrated throughout the curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without the content to think about, it is equally impossible for students to understand the content of a discipline without grappling with complex questions and the investigation of topics.

Inquiry Questions – Inquiry is a multifaceted process requiring students to think and pursue understanding. Inquiry demands that students (a) engage in an active observation and questioning process; (b) investigate to gather evidence; (c) formulate explanations based on evidence; (d) communicate and justify explanations, and; (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

Relevance and Application – The hallmark of learning a discipline is the ability to apply the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, developing, adapting, and refining solutions for the betterment of society. The application of a discipline, including how technology assists or accelerates the work, enables students to more fully appreciate how the mastery of the grade level expectation matters after formal schooling is complete.

Nature of Discipline – The unique advantage of a discipline is the perspective it gives the mind to see the world and situations differently. The characteristics and viewpoint one keeps as a result of mastering the grade level expectation is the nature of the discipline retained in the mind's eye.

1.Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At the most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: High School

Concepts and skills students master:

1. The complex number system includes real numbers and imaginary numbers

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: (This expectation will be assessed under Standard 2) a. Show that between any two rational numbers there are an infinite number of rational numbers, and that between any two irrational numbers there are also an infinite number of irrational numbers (CO only) b. Express the square root of a negative number using imaginary numbers c. N-CN.2 Use the relation $i^2 = -1$ and the commutative,	• N-CN.1 specifies recognizing complex numbers have the form $a + bi$. (MA.HS.1.1.b) • N-CN.2 added because CO standards do not include arithmetic with complex numbers. • N-CN.3 added because CO standards do not include finding conjugates, moduli, and quotients of complex numbers. • N-CN.4 added because CO standards do not include representing complex numbers on the complex plane.	**Inquiry Questions:** 1. Is there only one meaning of infinite? 2. How many numbers are between any two points on the number line? 3. Are there more complex numbers than real numbers? 4. What is a number system? **Relevance and Application:** 1. The choice of the appropriate measurement tool meets the precision requirements of the measurement task. For example, using a caliper for the manufacture of brake discs or a tape measure for pant size. 2. Complex numbers have applications in fields such as chaos theory and fractals. The familiar image of the Mandelbrot fractal is the Mandelbrot set graphed on the complex plane.

associative, and distributive properties to add, subtract, and multiply complex numbers. d. N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. e. N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. f. N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(1 - \sqrt{3}i)^3 = 8$ *because* $(1 - \sqrt{3}i)$ *has modulus* 2 *and argument* 120°. g. N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	• N-CN.5 added because CO standards do not include representing arithmetic with complex numbers on the complex plane. • N-CN.6 added because CO standards do not include working with distance between points on the complex plane. • N-VM (Vector and Matrix Quantities)—all standards—not included in any CO standard.	**Nature of Mathematics:** 1. Mathematicians build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. 2. Mathematics involves making and testing conjectures, generalizing results, and making connections among ideas, strategies, and solutions.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: High School

Concepts and skills students master:

2. Formulate, represent, and use algorithms with real numbers flexibly, accurately, and efficiently

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: (These evidence outcomes will be assessed in Standard 2, 3, and 4). a. Use appropriate computation methods that encompass estimation and calculation b. Use technology to perform operations (addition, subtraction, multiplication, and division) on numbers written in scientific notation c. Describe factors affecting take-home pay and calculate the impact (PFL) d. Design and use a budget, including income (net take-home pay) and expenses (mortgage, car loans, and living expenses) to demonstrate how living within your means is essential for a secure financial future (PFL)	• 8.EE.4 Content covered in MA.HS.1.2.b. (MA.8.1.2)	**Inquiry Questions:** 1. Can numbers ever be too big or too small to be useful? 2. How much money is enough for retirement? (PFL) 3. Is education worth the cost? (PFL) **Relevance and Application:** 1. The reading, interpreting, and writing of numbers in scientific notation with and without technology is used extensively in the natural sciences such as representing large or small quantities such as speed of light, distance to other planets, distance between stars, the diameter of a cell, and size of a micro–organism. 2. Fluency with computation and estimation allows individuals to analyze aspects of personal finance, such as calculating a monthly budget, estimating the amount left in a checking account, making informed purchase decisions, and computing a probable paycheck given a wage (or salary), tax tables, and other deduction schedules. **Nature of Mathematics:** 1. Using mathematics to solve a problem requires choosing what mathematics to use; making simplifying assumptions, estimates, or approximations; computing; and checking to see whether the solution makes sense.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: High School

Concepts and skills students master:

3. Systematic counting techniques are used to describe and solve problems

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: (This evidence outcome will be assessed in Standard 3) a. Use combinatorics (Fundamental Counting Principle, permutations and combinations) to solve problems in real-world contexts	•	**Inquiry Questions:** 1. Is the lottery really worth playing? 2. How can quantifying certain events lead to better decision-making?
		Relevance and Application: 1. Knowledge of systematic counting techniques helps analyze situations involving multiple outcomes. For example, finding probability in games of chance or the lottery, finding the quantity of possible phone numbers with new area codes, and the number of license plates with combinations of letters and numerals. 2. In an area of biology called phylogenetics, combinatorics are used to study how different groups of organisms evolve.
		Nature of Mathematics: 1. Mathematicians develop strategies to approach complex problems in a systematic way.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Compare and order sets of integers and rational numbers that are expressed as fractions, decimals, or percents b. Given a whole number from 0 - 100, determine whether it is a perfect square or find the two consecutive whole numbers between which its square root lies c. Approximate the location of square roots between two whole numbers on a number line	• 8.NS.1 includes interpreting the decimal expansions of rational and irrational numbers. (MA.8.1.1.c) • 6.NS.5 Content covered in MA.7.1.1.a and MA.8.1.1 (see MA.6.1.1) •	**Inquiry Questions:** 1. Why are real numbers represented by a number line and why are the integers represented by points on the number line? 2. Why is there no real number closest to zero? 3. What is the difference between rational and irrational numbers? **Relevance and Application:** 1. Irrational numbers have applications in geometry such as the length of a diagonal of a one by one square, the height of an equilateral triangle, or the area of a circle. 2. Different representations of real numbers are used in contexts such as measurement (metric and customary units), business (profits, network down time, productivity), and community (voting rates, population density). 3. Technologies such as calculators and computers enable people to order and convert easily among fractions, decimals, and percents. **Nature of Mathematics:** 1. Mathematics provides a precise language to describe objects and events and the relationships among them.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Add, subtract, multiply and divide rational numbers including integers, positive and negative fractions and decimals b. Apply computational methods to solve multi-step application problems involving percents and rational numbers c. Analyze how credit and debt impact personal financial goals (PFL) (CO only)	• 8.EE.1 Content covered in MA.HS.2.4.b. • 8.EE.2 Content covered in MA.HS.2.5.a. • 8.EE.4 Content covered in MA.HS.1.2.b. • 7.NS.3 includes negative rational numbers and specifies real-world and mathematical problems. Content goes beyond MA.8.1.2.a (see MA.7.1.2). •	**Inquiry Questions:** 1. How do operations with rational numbers compare to operations with whole numbers? 2. How do you know if a computational strategy is sensible? 3. Why would estimation be used in problem-solving? **Relevance and Application:** 1. Computational fluency with rational numbers allows individuals to accomplish daily tasks in life and work such as adjusting recipes, comparing the cost of credit from different providers, calculating overtime pay, determining selling prices to make profits, calculating interest, finding percent error, gratuities, or fees. 2. Rational numbers are used extensively in measurement tasks such as home remodeling, clothes alterations, graphic design, and engineering. **Nature of Mathematics:** 1. Mathematicians describe their processes and solutions using careful vocabulary and precise notation.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. In the real number system, rational numbers have a unique location on the number line

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Read, write, locate on number line, compare and order integers and positive rational numbers b. Apply the definition of absolute value with integers, quantifying the distance from zero c. Express large and small numbers using scientific notation	• 6.NS.5 Content covered in MA.7.1.1.a and MA.8.1.1 (see MA.6.1.1) •	**Inquiry Questions:** 1. Why are there negative numbers? 2. How do we compare and contrast numbers? 3. How is scientific notation a useful representation? 4. Are there more rational numbers than integers? **Relevance and Application:** 1. Scientific notation is used to express large and small measurements such as the distance between two planetary bodies, the weight of a microscopic organism, or the size of the US national debt. 2. Communication and collaboration with others is more efficient and accurate using rational numbers. For example, negotiating the price of an automobile, sharing results of a scientific experiment with the public, and planning a party with friends. 3. Negative numbers can be used to represent quantities less than zero or quantities with an associated direction such as debt, elevations below sea level, low temperatures, moving backward in time, or an object slowing down. **Nature of Mathematics:** 1. Mathematicians use their understanding of relationships among numbers and the rules of number systems to create models of a wide variety of situations.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with integers and positive rational numbers flexibly, accurately, and efficiently

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Simplify numeric expressions using the order of operations b. Add, subtract, multiply, and divide integers c. Use mental math and estimation strategies to solve problems involving percents d. Solve problems involving percent of a number, discounts, taxes, simple interest, percent increase, and percent decrease (PFL)	• 7.NS.1 includes adding and subtracting rational numbers and applying the properties of operations. (MA.7.1.2.a) • 7.NS.2 includes multiplying and dividing rational numbers and applying the properties of operations. (MA.7.1.2.a and b) • 7.EE.3 includes problems posed with positive and negative numbers in any form. (MA.7.1.2.d) • 7.NS.3 includes negative rational numbers and specifies real-world and mathematical problems. Content goes beyond MA.8.1.2.a. • 6.EE.1 Content covered in MA.7.1.2.a. (see MA.6.1.1)	**Inquiry Questions:** 1. How do operations with rational numbers compare to operations with integers? 2. How do you know if a computational strategy is sensible? 3. Why does the order of operations exist? 4. What other tasks/processes require the use of a strict order of steps? **Relevance and Application:** 1. The use and understanding algorithms help individuals spend money wisely. For example, compare discounts to determine best buys and compute sales tax. 2. Estimation with rational numbers enables individuals to make decisions quickly and flexibly in daily life such as estimating a total bill at a restaurant, the amount of money left on a gift card, and price markups and markdowns. . 3. People use percentages to represent quantities in real-world situations such as amount and types of taxes paid, increases or decreases in population, and changes in company profits or worker wages). **Nature of Mathematics:** 1. Mathematicians see algorithms as familiar tools in a tool chest. They combine algorithms in different ways and use them flexibly to accomplish various tasks.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

3. Proportional reasoning involves comparisons and multiplicative relationships among ratios

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use ratio relationships to solve for a missing value in a proportion b. Model proportional relationships with bar models, ratio tables, and similar figures c. Explain the difference between a ratio, rate, and unit rate d. Estimate and compute unit cost of consumables (to include unit conversions if necessary) sold in quantity to make purchase decisions based on cost and practicality (PFL)	• 7.RP.2 includes recognizing and representing proportional relationships between quantities. (MA.7.1.3.c) • 7.RP.3 includes solving multi-step ratio and percent problems. (MA.7.1.3.a)	**Inquiry Questions:** 1. What information can be determined from a relative comparison that cannot be determined from an absolute comparison? 2. What comparisons can be made using ratios? 3. How do you know when a proportional relationship exists? 4. How can proportion be used to argue fairness? 5. When is it better to use an absolute comparison? 6. When is it better to use a relative comparison? **Relevance and Application:** 1. The use of ratios, rates, and proportions allows sound decision-making in daily life such as determining best values when shopping, mixing cement or paint, adjusting recipes, calculating car mileage, using speed to determine travel time, or enlarging or shrinking copies. 2. Proportional reasoning is used extensively in the workplace. For example, determine dosages for medicine; develop scale models and drawings; adjusting salaries and benefits; or prepare mixtures in laboratories. 3. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature of Mathematics:** 1. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. In the real number system, positive rational numbers are represented in multiple equivalent forms

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Read, write, compare, convert and order positive rational numbers in a variety of forms including proper and improper fractions, mixed numbers, decimals, and percents b. Express whole numbers as products of prime factors with exponents and use prime factorization to find the greatest common factor and least common multiple of two numbers c. Express the quotient and remainder of a whole number division problem (a/b or a ÷ b) using fractions, terminating decimals, or repeating decimals d. Locate positive fractions and decimals on a number line	• 6.NS.7 includes negative rational numbers and the absolute value of rational numbers. (MA.6.1.1.a) • 6.NS.4 includes using the reverse of the distributive property to factor (e.g., 36 + 8 as 4(9 + 2)). (MA.6.1.1.b) • 6.NS.6 includes negative rational numbers. (MA.6.1.1.d) • 6.NS.5 Content covered in MA.7.1.1.a and MA.8.1.1.a. • 6.EE.1 Content covered in MA.7.1.2.a. • 5.NF.3 Content covered in MA.6.1.1.c and MA.6.1.2.a. (see MA.5.1.2) • 5.MD.2 Content covered in MA.6.1.1.d. (see MA.5.3.1) •	**Inquiry Questions:** 1. Why not look for the least common factor of numbers? 2. Why not look for the greatest common multiple of numbers? 3. Can every even integer greater than two be expressed as the sum of two primes? 4. Why is there no limit to the number of ways a fraction can be represented? **Relevance and Application:** 1. Flexibility with number representations provides a choice of the most efficient form such as using decimals when dealing with money or inputting data in spreadsheets; using fractions in cooking or completing household projects; and using percents with discounts, sales tax or tips. 2. Comprehension of the order of numbers in different forms allows for communication in different situations such as translating between metric and customary measurements, providing weather reports, and pilots talking with control towers. 3. Knowledge of the relative size of fractions and decimals helps to make comparisons such as sports statistics, discounts, costs, profits, interest rates, or gallons of gas at a gas station. 4. The understanding of prime and composite number relationships develops number sense and is the basis of many aspects of mathematics including simplifying division problems and rational algebraic expressions, finding common denominators, and analyzing several unsolved problems in mathematics including the Goldenbach conjecture and the twin prime conjecture. **Nature of Mathematics:** 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with positive rational numbers flexibly, accurately, and efficiently

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Model and compute the addition, subtraction, multiplication and division of positive fractions, decimals, and combinations of fractions and decimals b. Solve multi-step word problems involving fractions, decimals and whole numbers c. Estimate sums, differences, products and quotients of rational numbers using common fractions, common decimals, and whole numbers d. Compare and round positive numbers from thousandths through millions	• 6.NS.1 includes interpreting models of division of fractions by fractions. (MA.6.1.2.b) • 6.NS.8 Content partially addressed in MA.7.2.1.b and covered in MA.8.2.1.e. • 5.NF.3 Content covered in MA.6.1.1.c and MA.6.1.2.a. (see MA.5.1.2) • 5.NBT.7 Content covered in MA.6.1.2.a. (see MA.5.1.2) • 5.NF.1 Content covered in MA.6.1.2.a (see MA.5.1.2) • 5.NF.4 relates to MA.5.1.3.b, with content included in MA.6.1.2.a. • 5.NF.7 Content related to MA.6.1.2.a and b. (see MA.5.1.3) • 5.NF.2 Content covered in	**Inquiry Questions:** 1. Do adding and multiplying always result in an increase? Why? 2. Do subtracting and dividing always results in a decrease? Why? 3. Why might estimation be better than an exact answer? 4. When might an estimate be the only possible answer? 5. How do operations with fractions and decimals compare to operations with whole numbers? 6. Is $0.\overline{9}$ equal to one? 7. How do you know whether a fraction can be represented as a repeating or terminating decimal? **Relevance and Application:** 1. Computational fluency with fractions is necessary for activities in daily life such as cooking, and measuring for household projects and crafts. 2. Estimation with rational numbers enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, how many floor tiles to buy with given dimensions, the amount of carpeting needed for a room, or fencing required for a backyard. 3. Rational numbers are an essential component of mathematics. Understanding fractions, decimals, and percentages is the basis for probability, proportions, measurement, money, algebra, and geometry.

	MA.6.1.2.b. (see MA.5.1.2) • 5.NF.6 Content covered in MA.6.1.2.b. (see MA.5.1.3) • 5.NBT.3b Content covered in MA.6.1.2.d. (see MA.5.1.2) • 5.NBT.4 Content covered in MA.6.1.2.d. (see MA.5.1.2) •	**Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

3. Quantities can be expressed and compared using ratios and rates

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Apply the multiplicative identity to create equivalent fractions and to reduce fractions to simplest form b. Express the comparison of two whole number quantities using differences, part-to-part ratios, and part-to-whole ratios in real contexts, including investing and saving (PFL) c. Compute unit rates in real-world situations involving mixtures, concentrations, and distance-time relationships	• 4.NF.1 relates to MA.4.1.3.c, with content also related to MA.6.1.3.a. • 5.NF.5 Content included in MA.6.1.3.b and c. (see MA.5.1.3) •	**Inquiry Questions:** 1. What is the golden ratio and where does it appear in nature? 2. How are ratios different from fractions? 3. What is the difference between quantity and number? **Relevance and Application:** 1. Knowledge of ratios and rates allows sound decision-making in daily life such as determining best values when shopping, creating mixtures, adjusting recipes, calculating car mileage, using speed to determine travel time, or making saving and investing decisions. 2. Ratios and rates are used to solve important problems in science, business, and politics. For example developing more fuel-efficient vehicles, understanding voter registration and voter turnout in elections, or finding more cost-effective suppliers. 3. Rates and ratios are used in mechanical devices such as bicycle gears, car transmissions, and clocks. **Nature of Mathematics:** 1. Mathematicians develop simple procedures to express complex mathematical concepts.

Content Area: Mathematics

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Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. The characteristics of numbers can be used to classify them in various ways

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Apply concepts of squares, primes, composites, factors, and multiples to solve problems b. Use the identity, associative, commutative, and distributive properties to solve problems c. Describe and use divisibility rules for two, three, four, five, six, nine, and 10 to solve problems	• 4.OA.4 relates to MA.4.1.2.c, with some content addressed in MA.5.1.1a, c. •	**Inquiry Questions:** 1. What characteristics can be used to classify numbers into different groups? 2. Why is there no limit to the number of ways a fraction can be represented? 3. Why do the divisibility rules work? **Relevance and Application:** 1. Comprehension of the relationships between primes, composites, multiples, and factors develop number sense. The relationships are used to simplify computations with large numbers, algebraic expressions, and division problems, and to find common denominators. 2. Exponents are helpful with both large and small quantities. For example using exponents and scientific notation on calculators, understanding units in the metric system, and representing the distance around the equator. **Nature of Mathematics:** 1. Mathematicians take ideas apart and put them back together, sometimes in different ways that lead to new insights.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. In the real number system, commonly used rational numbers have multiple equivalent representations

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Find equivalent forms of commonly used fractions, decimals, and percents using models, drawings, and computational strategies b. Use common fractions and percents to calculate parts of whole numbers in problem situations including comparisons of savings rates at different financial institutions (PFL) c. Model addition, subtraction, and multiplication of fractions, decimals, and percents d. Compose and decompose multi-digit whole numbers and decimals based on	• Patterns in the number of zeroes of the product when multiplying with powers of 10 and patterns of decimal placement in multiplication (MA.5.1.2.c) • 5.NBT.1 specifies recognizing how the value of a digit in one place is ten times the value of the place to the right, etc. (MA.5.1.2.d) • 5.NBT.3b Content covered in MA.6.1.2.d. • 5.NBT.4 Content covered in MA.6.1.2.d. • 5.NBT.7 Content covered in MA.6.1.2.a. • 5.NF.1 Content covered in MA.6.1.2.a • 5.NF.2 Content covered in	**Inquiry Questions:** 1. Why can't the denominator of a fraction be zero? 2. Are there more fractions than whole numbers? 3. Why can a decimal model always be immediately read as a fraction, but a fraction model cannot always be immediately read as a decimal? 4. Is there a smallest fraction? Why? 5. Is there a decimal closest to one? Why? **Relevance and Application:** 1. Fluent conversion between commonly used fractions, decimals, and percents helps to make daily decisions such as determining discounts in stores for comparison shopping, interpreting sports statistics, and comparing savings rates. 2. Situations from daily life can be modeled using operations with fractions, decimals, and percents such as determining the quantity of paint to buy or the number of pizzas to order for a large group. 3. Rational numbers are used to represent data and probability such as getting a certain color of gumball out of a machine, the probability that a batter will hit a home run, or the percent of a mountain covered in forest.

place value e. Represent numbers to 1,000,000 with expanded notation and exponents	MA.6.1.2.b. • 5.NF.3 Content covered in MA.6.1.1.c and MA.6.1.2.a. • 4.NF.3.a, b, c, d Content could be covered in MA.5.1.2.a, b, c. (see MA.4.1.3) • 4.NF.4.a, b, c Content could be covered in MA.5.1.2.a, b, c. (see MA.4.1.3) • 4.NF.5 Content could be covered in MA.5.1.2.c. (see MA.4.1.3) • 4.NBT (footnote 2) allows whole numbers to 1,000,000; covered in MA.5.1.2.e. (see MA.4.1.1.a) •	**Nature of Mathematics:** 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

3. Formulate, represent, and use algorithms to multiply and divide multi-digit whole numbers with flexibility, accuracy, and efficiency

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use flexible methods of computing including standard algorithms to multiply and divide multi-digit numbers by two-digit factors or divisors b. Model multiplication and division using area, linear, and grouping models c. Interpret remainders and select the most useful form of the quotient in division problems d. Select and use appropriate methods to estimate products and quotients or calculate them mentally depending on the context and numbers involved	• 5.OA.1 added because CO standards do not include evaluating numerical expressions. • 5.OA.2 added because CO standards do not include writing and interpreting numerical expressions. • 5.NF.4 specifies types of models to use, including modeling multiplication of fractions. (MA.5.1.3.b) Content included in MA.6.1.2.a. • 5.NF.5 Content included in MA.6.1.3.b and c, • 5.NF.6 Content covered in MA.6.1.2.b. • 5.NF.7 Content related to MA.6.1.2.a and b. • 4.NBT.5 relates to MA.4.1.2.a, with multiplying two 2-digit numbers covered in MA.5.1.3.a.	**Inquiry Questions:** 1. How are multiplication and division related? 2. What makes one strategy or algorithm better than another? 3. What do remainders mean and how are they used? 4. When is the "correct" answer not the most useful answer? **Relevance and Application:** 1. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra. 2. There are many models of multiplication and division such as the area model for tiling a floor, the fundamental counting principle to determine the number of combinations of ice cream toppings and ice cream flavors, and the repeated addition to group people for games.

	• 4.OA.3 relates to MA.4.1.2.d, with interpreting remainders covered in MA.5.1.3.c. •	**Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians develop simple procedures to express complex mathematical concepts.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Read and write numbers from one to 100,000 and explain place value for five-digit numbers b. Compose and decompose multi-digit numbers based on place value c. Read and write numbers to the hundredths place d. Identify the value of any given digit in a number with decimals to the hundredths place e. 4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that	• 4.NBT (footnote 2) allows whole numbers to 1,000,000. (MA.4.1.1.a) Content covered in MA.5.1.2.e. • 4.NBT.2 includes comparing two multi-digit numbers … using >, =, and < symbols. (MA.4.1.1.b) • 4.NF.7 added because CO standards do not specify decimal comparisons. •	**Inquiry Questions:** 1. Why isn't there a "oneths" place in decimal fractions? 2. How might the most commonly used number system be different if humans had only five fingers instead of ten? 3. What is the difference between the base ten counting system and the system used for telling time? 4. How is the place value in the Roman counting system different from the place value system used today? **Relevance and Application:** 1. Decimal place value is the basis of the monetary system and provides information about how much items cost, how much change should be returned, or the amount of savings that has accumulated. 2. Knowledge and use of place value for large numbers provides context for population, distance between cities or landmarks, and attendance at events. 3. Place value is applied to represent a myriad of numbers using only ten symbols.

comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.		**Nature of Mathematics:** 1. Mathematicians use numbers like writers use letters to express ideas.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms to multiply and divide with flexibility, accuracy, and efficiency

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use flexible and efficient methods of computing including standard algorithms to solve three- or four-digit by one-digit multiplication or division problems b. Estimate using strategies such as front end or rounding to justify the reasonableness of solutions to problems c. Demonstrate fluency with multiplication facts and their related division facts 0 to 12 d. Explain why multi-digit multiplication and	• 4.NBT.5 specifies multiplying two 2-digit numbers. (MA.4.1.2.a) Content covered in MA.5.1.3.a. • 4.OA.1 and 4.OA.2 emphasize interpreting multiplication as a comparison (e.g., "times as many") and solving word problems involving multiplicative comparison, and distinguishing it from additive comparison. (MA.4.1.2.c) • 4.OA.4 specifies finding all factor pairs for whole numbers 1 to 100, recognizing multiples and factors, determining multiples of one-digit numbers, and identifying primes and composites. (MA.4.1.2.c) Some content	**Inquiry Questions:** 1. When is an estimate better than an exact answer? 2. How close is close enough in an estimate? 3. How does place value affect the accuracy of an estimate? 4. Is it possible to make multiplication and division of large numbers easy? 5. How does the commutative property help with learning the basic multiplication facts? **Relevance and Application:** 1. Efficient and reasonable estimation allows quick determination of whether numbers make sense such as deciding the amount of food to purchase for a party, determining the cost of dinner at a restaurant, or calculating the number of trees in a park. 2. Comprehension of place value helps to comprehend multiplication, division, and other basic algorithms. 3. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra.

division procedures work based on place value properties and use them to solve problems	addressed in MA.5.1.1a, c. • 4.OA.3 includes solving multi-step word problems using the four operations, including problems in which remainders must be interpreted. (MA.4.1.2.d) Interpreting remainders covered in MA.5.1.3.c. • 4.NBT.5 and 4.NBT.6 specify using equations, rectangular arrays, and/or area models to illustrate and explain multiplication and division. (MA.4.1.2.d) • 3.NBT.1 relates to MA.3.1.1.c, with content also related to MA.4.1.2.b. •	**Nature of Mathematics:** 1. Mathematicians have developed mathematical symbols and procedures to simplify calculations that apply to many situations.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

3. Different models and representations can be used to compare fractional parts

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Solve comparison problems using models of fractions with like and unlike denominators through 10 b. Estimate and justify the reasonableness of solutions to problems involving comparison of fractions c. Demonstrate equivalent fractions, decimals, and percents using drawings and models	• 4.NF (footnote 3) includes fractions with denominators 12 and 100, but excludes 7ths and 9ths. (MA.4.1.3.a) • 4.NF.3 includes mixed numbers. (MA.4.1.3.a, b, c) • 4.NF.2 includes comparing fractions with common numerators, comparing to benchmark fractions, recognizing when comparisons are valid, and using symbols >, +, or <. (MA.4.1.3.a, b) • 4.NF.1 emphasizes explaining equivalences of fractions *a*/*b* and *(n×a)/(n×b)*, attending to how the number and size of parts differ even though the two fractions themselves are the same size. (MA.4.1.3.c) Content related to MA.6.1.3.a. • 4.NF.3a, b, c, d Content could be covered in MA.5.1.2.a, b, c. • 4.NF.4a, b, c Content could be covered in MA.5.1.2.a, b, c. • 4.NF.5 Content could be covered in MA.5.1.2.c. •	**Inquiry Questions:** 1. How can different fractions represent the same quantity? 2. How are fractions used as models? 3. Why are fractions so useful? 4. What would the world be like without fractions? **Relevance and Application:** 1. Fractions and decimals are used any time there is a need to apportion such as sharing food, cooking, making savings plans, creating art projects, timing in music, or portioning supplies. 2. Fractions are used to represent the chance that an event will occur such as randomly selecting a certain color of shirt or the probability of a certain player scoring a soccer goal. 3. Fractions are used to measure quantities between whole units such as number of meters between houses, the height of a student, or the diameter of the moon. **Nature of Mathematics:** 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. The whole number system describes place value relationships from ones to 10,000 and forms the foundation for efficient algorithms

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Read and write numbers from one to 10,000 and explain place value for four-digit numbers b. Generalize the change represented when moving from one place to another place in a number c. Compose and decompose multi-digit numbers based on place value	• 3.NBT.1 specifies rounding whole numbers to the nearest 10 or 100. (MA.3.1.1.c) Content related to MA.4.1.2.b. •	**Inquiry Questions:** 1. How big is 10,000? 2. How do patterns in our place value system assist in comparing whole numbers? 3. How might the most commonly used number system be different if humans had twenty fingers instead of ten? **Relevance and Application:** 1. Knowledge and use of place value for large numbers provides context for distance in outer space, prehistoric timelines, and ants in a colony. 2. The building and taking apart of numbers provide a deep understanding of the base 10 number system. **Nature of Mathematics:** 1. Mathematicians use numbers like writers use letters to express ideas.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Parts of a whole can be modeled and represented in different ways

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use drawings, models, and numerals to represent fractions (halves, thirds, fourths, sixths, eighths) based on a whole shape, number set, or number line b. Estimate and justify the reasonableness of solutions to problems involving representations of fractions c. Describe why equivalent fractions are two ways of modeling the same quantity using a model or drawing	• 3.NF.3.c specifies expressing whole numbers as fractions and recognizing fractions that are equivalent to whole numbers. (MA.3.1.2.IQ.1) • 3.NF.3.d specifies comparing two fractions with the same numerator or denominator by reasoning about their size, recognizing when comparisons are valid, using the symbols >, =, or <. (MA.3.1.2.c, MA.3.1.2.IQ.2, MA.3.1.2.RA.2) •	**Inquiry Questions:** 1. How many ways can a whole number be represented? 2. How can a fraction be represented in different, equivalent forms? 3. How do we show part of something? **Relevance and Application:** 1. Fractions are used to share fairly with friends and family such as sharing a closet with a sibling, and splitting the cost of lunch. 2. Equivalent fractions demonstrate equal quantities even when they are presented differently such as knowing that 1/2 of a box of crayons is the same as 2/4, or that 2/6 of the class is the same as 1/3. **Nature of Mathematics:** 1. Mathematicians use visual models to solve problems.

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Third Grade

Concepts and skills students master:

3. Formulate, represent, and use algorithms to add and subtract multi-digit whole numbers with flexibility, accuracy, and efficiency

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use number sense to estimate and justify the reasonableness of solutions to problems b. Use flexible methods of computing, including student-generated strategies and standard algorithms c. Estimate using strategies such as front-end estimation or landmark numbers	• 3.NBT.2 specifies fluency adding and subtracting within 1000. (MA.3.1.3.b) • 1.OA.8 Content addressed in MA.3.1.3.b. (see MA.1.1.2) •	**Inquiry Questions:** 1. What makes a computational method sensible to use? 2. What makes a good estimate? 3. How would you know a new computation method works? **Relevance and Application:** 1. Algorithms can be used to add and subtract to solve problems such as finding the total number of fish in two tanks, the total number of blocks to school, or the number of cookies remaining after giving three to a friend. 2. Estimation determines the reasonableness of an answer. For example, divide 102 by 5 and get 2, then realize 102 is close to 100 and 100 divided by 5 is 20, so 2 must be too small. 3. Estimation helps to analyze the size of objects such as how many books will fit into a large packing box. **Nature of Mathematics:** 1. Mathematicians have developed mathematical symbols and procedures to simplify calculations that apply to many situations.

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Third Grade

Concepts and skills students master:

4. Multiplying and dividing are inverse operations modeled in a variety of ways

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Demonstrate fluency with multiplication and division facts with single-digit factors b. Describe relationships between related facts and between multiplication and division c. Represent multiplication and division problems with drawings, models, number sentences, and stories d. Model strategies to achieve a personal financial goal using arithmetic operations (PFL)	• 3.NBT.3 specifies multiplying one-digit whole numbers by multiples of 10. (MA.3.1.4.a) • 3.OA.6 specifies understanding division as an unknown-factor problem. (MA.3.1.4.b) • 3.OA.3 specifies solving word problems using multiplication and division within 100. (MA.3.1.4.c) •	**Inquiry Questions:** 1. How are multiplication and division related? 2. How can you use a multiplication or division fact to find a related fact? 3. Why was multiplication invented? Why not just add? 4. Why was division invented? Why not just subtract? **Relevance and Application:** 1. Many situations in daily life can be modeled with multiplication and division such as how many tables to set up for a party, how much food to purchase for the family, or how many teams can be created. 2. Use of multiplication and division helps to make decisions about spending allowance or gifts of money such as how many weeks of saving an allowance of $5 per week to buy a soccer ball that costs $32?. 3. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra. **Nature of Mathematics:** 1. Mathematicians often learn concepts on a smaller scale before applying them to a larger situation.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. The whole number system describes place value relationships from ones to 1,000 and forms the foundation for efficient algorithms

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Read and write numbers to 1,000 and identify place value for three-digit numbers b. Describe relationships between ones, tens, and hundreds c. Explain the value of a digit in a three-digit number d. Order a collection of whole numbers	• 2.NBT.3 specifies reading and writing numbers to 1000 using expanded notation. (MA.2.1.1.a) • 2.NBT.4 specifies comparing two three-digit numbers, using >, <, and = symbols. (MA.2.1.1.d) • 1.NBT.3 Content related to MA.2.1.1.d (see MA.1.1.1) •	**Inquiry Questions:** 1. How big is 1,000? 2. How does the position of a digit in a number affect its value? 3. What is the difference between first and one? **Relevance and Application:** 1. The ability to read and write numbers allows communication about quantities such as the cost of items, number of students in a school, or number of people in a theatre. 2. Place value allows people to represent large quantities. For example, 725 can be thought of as 700 + 20 + 5. **Nature of Mathematics:** 1. Mathematicians use place value to represent many numbers with only ten digits.

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Second Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms to add and subtract two-digit whole numbers with flexibility, accuracy, and efficiency

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Demonstrate fluency with basic addition and subtraction facts to sums of 20 b. Find the value of a collection of coins and choose coins to have a given value c. Create stories and models, including linear and difference, to illustrate addition and subtraction d. Select and use appropriate methods to estimate sums and differences or calculate them mentally depending on the context and numbers involved e. Apply addition and subtraction concepts to financial decision-	• 2.OA.1 specifies using addition within 100 to solve one- and two- step word problems. (MA.2.1.2.a) • 2.OA.2 specifies knowing from memory all sums of two one-digit numbers. (MA.2.1.2.a) • 2.NBT.5 specifies addition and subtraction using strategies based on place value, properties of operations (MA.2.1.2.a), and commutative and associative properties of addition, addressed in MA.3.2.2.a, b, c. • 2.NBT.6 specifies adding up to four two-digit numbers using strategies based on place value and properties of operations (MA.2.1.2.a), content also addressed in MA.3.2.2.a, b, c. • 2MD.8 specifies solving word problems involving dollar bills, quarters, dimes, nickels and	**Inquiry Questions:** 1. What are the ways numbers can be broken apart and put back together? 2. What strategies are used to estimate the answer? 3. What could be a result of not using pennies (taking them out of circulation)? **Relevance and Application:** 1. Addition is used to find the total number of objects such as total number of animals in a zoo, total number of students in first and second grade. 2. Subtraction is used to solve problems such as how many objects are left in a set after taking some away, or how much longer one line is than another. 3. The ability to estimate helps to judge whether answers are reasonable such as results on a calculator, or an answer given by someone else seems feasible. 4. The understanding of the value of a collection of coins helps to determine how many coins are used for a purchase or checking that the amount of change is correct.

making (PFL) (CO only)	pennies. (MA.2.1.2.b and e) •	**Nature of Mathematics:** 1. Mathematicians use visual models to understand addition and subtraction.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Second Grade

Concepts and skills students master:

3. Fractions represent parts of a whole object or set

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Partition basic shapes, using common fractions such as 1/2, 1/3, and 1/4 b. Partition sets using common fractions such as 1/2, 1/3, 1/4	• MA.G2.G.3 specifies partitioning circles and rectangles; describing shares using the words half of, a third of, etc.; describing the whole as two halves, three thirds, etc.; and recognizing that equal shares of identical wholes need not have the same shape. (MA.2.1.3.b) •	**Inquiry Questions:** 1. How are whole numbers and fractions represented? 2. What is the meaning of the numerator and denominator in a fraction? 3. Why are fractions useful? **Relevance and Application:** 1. Fractions help to determine the fairness of situations where sharing is involved such as sharing toys, or money. 2. The understanding of fractions helps to make sense of situations involving parts such as 1/4 hour, 1/2 of a dollar, ¼ note in music, or 1/3 of a family. 3. Fractions help to communicate clearly about partial amounts such as ½ cup of flour, ¼ of an orange, and ½ of the stadium is filled with people.

		Nature of Mathematics: 1. Mathematics involves the understanding that not everything can be represented with whole numbers.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: First Grade

Concepts and skills students master:

1. The whole number system describes place value relationships from ones to 100 and forms the foundation for efficient algorithms

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Count, read, and write numbers to 100 b. Estimate quantities less than 100 c. Represent quantities using tens units and ones units d. Locate numbers up to 100 on a number display e. Compare two sets of objects, including pennies, up to at least 25 using language such as "three more or three fewer" (PFL)	• 1.NBT.1 specifies that students count to 120 starting at any number less than 120 (M.1.1.1.a) • 1.NBT.3 Content related to MA.2.1.1.d • 1.NBT.4 Content related to MA.2.2.2.a and b • 1.NBT.5 Content related to MA.2.2.2.a, b, c • 1.NBT.6 Content related to MA.2.2.2. a, b, c • K.CC.1 specifies counting to 100 by ones and tens, related to MA.K.1.1.a, covered in MA.1.1.1.a. • K.NBT.1 relates to MA.K.2.2.b, with content covered in MA.1.1.1.c. •	**Inquiry Questions:** 1. Can numbers always be related to tens? 2. Why not always count by one? 3. Why was a place value system developed? 4. How does a position of a digit affect its value? 5. How can I tell if I've made a good guess (estimate)? 6. How big is 100? **Relevance and Application:** 1. Estimation allows people to think about how many objects are in a set without counting. 2. Locating numbers on a number line helps to see the relative size of numbers. 3. The comparison of numbers helps to communicate and to make sense of the world. (For example, if someone has two more dollars than another, gets four more points than another, or takes out three fewer forks than needed **Nature of Mathematics:** 1. Mathematics involves visualization and representation of ideas. 2. Numbers are used to count and order both real and imaginary objects.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: First Grade

Concepts and skills students master:

2. Adding and subtracting involve composing and decomposing using a variety of strategies

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use addition when putting sets together and subtraction for breaking sets apart or describing the difference between sets b. Use number relationships such as doubles, one more or one less, and the relationship between composing and decomposing to solve addition and subtraction problems c. Identify coins and find the value of a collection of two coins(PFL) (CO only) d. Demonstrate fluency with basic addition and related subtraction facts through sums to 10	• 1.OA.1 specifies using addition and subtraction within 20 to solve word problems. (MA.1.1.2.a, b, d) • 1.OA.2 specifies solving word problems that call for addition of three whole numbers whose sum is equal to or less than 20. (MA.1.2.d) • 1.OA.8 specifies determining an unknown whole number in addition and subtraction equation relating to three whole numbers and is addressed in MA.3.1.3.b. • K.G.5 Content covered in MA.1.4.1.a. (see MA.K.4.2) • K.G.6 Content covered in MA.1.4.1.a, c. (see MA.K.4.2) • K.G.4 Content covered in MA.1.4.1.b. (see MA.K.4.2) •	**Inquiry Questions:** 1. What is addition and how is it used? 2. What is subtraction and how is it used? 3. How are addition and subtraction related? **Relevance and Application:** 1. Addition and subtraction are used to model real-world situations such as computing saving or spending, finding the number of days until a special day, or determining an amount needed to earn a reward. 2. Fluency with addition and subtraction facts helps to quickly find answers to important questions. **Nature of Mathematics:** 1. Mathematicians use addition and subtraction to take numbers apart and put them back together in order to understand number relationships.

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: First Grade

Concepts and skills students master:

3. Parts of objects can be shown as fractions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Identify unit fractions 1/2, 1/3, and 1/4 as parts of wholes or parts of groups b. Understand fractions as equal shares or parts	•	**Inquiry Questions:** 1. What do fractions tell us? 2. What are some things in the world that have parts?
		Relevance and Application: 1. Fractions help people to understand fairness such as dividing a set of toys into equal parts, or sharing time on a swing. 2. Fractions are used to understand parts found in everyday life such as fraction parts of a family, beats in music, or parts of an hour.
		Nature of Mathematics: 1. Mathematicians create visual representations of problems and ideas that reveal relationship and meaning. 2. The nature of mathematics involves curiosity, integrity, diligence, and fairness.

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Whole numbers can be used to name, count, represent, and order quantity

Evidence Outcomes	CSS Notes	21st Century Skills and Readiness Competencies
Students can: a. Count and represent objects to 20 b. Identify, read, and write corresponding numerals c. Compare sets up to 10 objects and use language to describe more, less, or same d. Compare two sets of objects to at least 25 using language such as "more," "less," or "the same" e. Identify small groups of objects –fewer than five without counting, including zero as "no objects" f. Estimate quantities less than 20	• K.CC.1 specifies counting to 100 by ones and tens. (MA.K.1.1.a) Counting to 100 covered in MA.1.1.1.a. •	**Inquiry Questions:** 1. Why do we count things? 2. Is there a wrong way to count? Why? 3. How do you know when you have more or less? 4. What does it mean to be second and how is it different than two? **Relevance and Application:** 1. Counting is used constantly in everyday life such as counting plates for the dinner table, people on a team, pets in the home, or trees in a yard. 2. Numerals are used to represent quantities. 3. People use numbers to communicate with others such as two more forks for the dinner table, one less sister than my friend, or six more dollars for a new toy. **Nature of Mathematics:** 1. Mathematics involves visualization and representation of ideas. 2. Numbers are used to count and order both real and imaginary objects.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Kindergarten

Concepts and skills students master:

2. Adding and subtracting to 10 involves composing and decomposing using a variety of strategies and representations

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use objects including coins, and drawings to model addition and subtraction problems to 10 (PFL) b. Identify numbers one more or one less than a given number up to 10 c. Determine if more than or less than is needed to change one quantity to another	•	**Inquiry Questions:** 1. What happens when two quantities are combined? 2. What happens when a set of objects is separated into different sets? **Relevance and Application:** 1. People combine quantities to find a total such as number of boys and girls in a classroom or coins for a purchase. 2. People use subtraction to find what is left over such as coins left after a purchase, number of toys left after giving some away. **Nature of Mathematics:** 1. Mathematicians create models of problems that reveal relationships and meaning. 2. Mathematics involves the creative use of imagination.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Preschool

Concepts and skills students master:

1. Quantities can be represented and counted

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Count and represent objects including coins to 10 (PFL) b. Match a quantity with a numeral	**Inquiry Questions:** 1. What do numbers tell us? 2. Is there a biggest number?
	Relevance and Application: 1. Counting helps people to determine how many such as how big a family is, how many pets there are, how much money is in a wallet. 2. People sort things to make sense of sets of things such as sorting pencils, toys, or clothes.
	Nature of Mathematics: 1. Numbers are used to count and order both real and imaginary objects.

Content Area: Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Preschool

Concepts and skills students master:

2. Counting is a means for solving problems

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use vocabulary of same, different, more, and less to express number relationships b. Solve problems answering the question "how many" to 10	**Inquiry Questions:** 1. How does counting help people? 2. What are numbers?
	Relevance and Application: 1. People use comparisons to communicate to others. For example, he has less than I do, I need more, or we have the same amount. 2. People use counting to find how many such as how many petals on a flower, people in the swimming pool, or cats in the window.
	Nature of Mathematics: 1. Mathematicians play with numbers. 2. Mathematicians use a common language to communicate.

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Understand the structure and properties of our number system. At the most basic level numbers are abstract symbols that represent real-world quantities
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: High School

Concepts and skills students master:

1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Determine* when a relation is a function using a table, a graph, or an equation b. Demonstrate the relationship between all forms of linear functions using point-slope, slope-intercept, and standard form of a line c. Represent* linear, quadratic, absolute value, power, exponential, logarithmic, rational, trigonometric (sine and cosine), and step functions in a table, graph, and equation and convert from one representation to another d. Determine the inverse (expressed graphically or in tabular form) of a function from a graph or table e. Categorize sequences as arithmetic, geometric, or neither and develop formulas for the general terms related to arithmetic and geometric	• F-TF.2 includes using the unit circle to represent trigonometric functions. (MA.HS.2.1.c) • F-BF.4b and d include verifying by composition that one function is the inverse of another and producing an invertible function by restricting the domain of a non-invertible function. (MA.HS.2.1.d) • F-BF.5 includes understanding exponential and logarithmic functions as inverses. (MA.HS.2.1.d)	**Inquiry Questions:** 1. Why are relations and functions represented in multiple ways? 2. How can a table, graph, and function notation be used to explain how one function family is different from and/or similar to another? 3. What is an inverse? 4. How is "inverse function" most likely related to addition and subtraction being inverse operations and to multiplication and division being inverse operations? 5. How are patterns and functions similar and different? 6. How could you visualize a function with four variables, such as $x^2 + y^2 + z^2 + w^2 = 1$? 7. Why couldn't people build skyscrapers without using functions? **Relevance and Application:** 1. The exploration of multiple representations of functions develops a deeper understanding of the relationship between the variables in the function. 2. The understanding of the relationship between variables in a function allows people to use functions to model relationships in the real world such as compound interest, population growth and decay, projectile motion, or payment plans. 3. Comprehension of slope, intercepts, and common forms of linear equations allows easy retrieval of information from linear models such as rate of growth or decrease, an initial charge for services, speed of an object, or the beginning balance of an account. 4. Understanding sequences is important preparation for calculus. Sequences can be used to represent functions including e^x, e^{x^2}, $\sin x$, and $\cos x$.

sequences f. F-TF.4 (+) Use the unit circle to explain symmetry and periodicity of trigonometric functions. *Using all tools including graphing technology	• F-TF.6 includes restricting the domain of trigonometric functions to make them invertible. (MA.HS.2.1.d) • F-IF.3 specifies recognizing sequences as functions and specifying domains. Does not restrict to arithmetic and geometric sequences. (MA.HS.2.1.e) • F-BF.2 includes using sequences to model situations. (MA.HS.2.1.e) • F-TF.4 added because the CO standards do not include using the unit circle to explain symmetry and periodicity in trigonometric functions. •	**Nature of Mathematics:** 1. Mathematicians use multiple representations of functions to explore the properties of functions and the properties of families of functions.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: High School

Concepts and skills students master:

2. Graphs and tables are used to describe the qualitative behavior of common types of functions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Evaluate* a function at a given point in its domain given an equation (including function notation), a table, and a graph b. Identify* the domain and range of a function given an equation (including function notation), a table, and a graph c. Identify* intercepts, zeros (or roots), maxima, minima, and intervals of increase and decrease, and asymptotes of a function given an equation (including function notation), a table, and a graph d. Make qualitative statements about the rate of change of a function, based on its graph or table e. A-APR.2 Know and apply the	• F-IF.2 specifies interpreting function notation in terms of a context. (MA.HS.2.2.a) • A-APR.3 specifies identifying zeroes of polynomial functions. (MA.HS.2.2.c) • F-IF.4 specifies using context. (MA.HS.2.2.c) • F-LE.3 includes comparing the rates of changes of two functions. (MA.HS.2.2.d) • A-APR.2 added because CO standards do not include knowing the Remainder Theorem. • A-APR.4 added because CO standards do not include proving polynomial identities. • A-APR.5 added because CO standards do not include the binomial theorem. • F-IF.9 added because CO	**Inquiry Questions:** 1. Why do we classify functions? 2. The ancient Greeks multiplied binomials and found the roots of quadratic equations without algebraic notation. How can this be done? **Relevance and Application:** 1. The understanding of the qualitative behavior of functions allows interpretation of the qualitative behavior of systems modeled by functions such as time-distance, population growth, decay, heat transfer, and temperature of the ocean versus depth. 2. Knowledge of how to interpret rate of change of a function allows investigation of rate of return and time on the value of investments. 3. Comprehension of rate of change of a function is important preparation for the study of calculus. 4. The ability to analyze a function for the intercepts, asymptotes, domain, range, and local and global behavior provides insights into the situations modeled by the function. For example, epidemiologists could compare the rate of flu infection among people who received flu shots to the rate of flu infection among people who did not receive a flu shot to gain insight into the effectiveness of the flu shot.

Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. f. A-APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example,* g. A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. h. F-IF.9 Compare properties of two functions each represented in a different way. For example… *Using all tools including graphing technology	standards do not include comparing properties of two functions. •	**Nature of Mathematics:** 1. Mathematicians represent concepts concretely and use multiple representations to gain insights into relationships among variables.

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: High School

Concepts and skills students master:

3. Parameters influence the shape of the graphs of functions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Apply* transformations (translation, reflection, dilation) to a parent function, f(x) b. Interpret the results of these transformations verbally, graphically, and symbolically *Using all tools including graphing technology	•	**Inquiry Questions:** 1. What are the effects of performing operations on functions? 2. How does order matter when applying transformations of functions? 3. How are transformations of a function similar to and different from transformations of planar figures? 4. How are parameters different from unknowns? **Relevance and Application:** 1. The understanding of transformations of functions helps to comprehend the effects of change to a math model such as change of initial investment. 2. Graphing technologies allow the testing of conjectures about the effect of translations. 3. Translations of elementary functions are used to model real-world data such as initial height in projectile motion, and initial temperature in a cooling problem. **Nature of Mathematics:** 1. Mathematicians experiment with models such as adjusting parameters, recording data, and readjusting parameters to develop increasingly more accurate models.

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: High School

Concepts and skills students master:

4. Expressions , equations, and inequalities can be expressed in multiple, equivalent forms

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Perform and justify steps in generating equivalent expressions by identifying properties used including the commutative, associative, inverse, identity, and distributive properties b. Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions including those involving nth roots c. Solve equations for one variable in terms of the others d. A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of	• N-RN.3 specifies understanding the closure property of rational and real numbers. (MA.HS.2.4.a) • A-SSE.2 specifies using the structure of an expression to rewrite it. (MA.HS.2.4.a, b) • N-RN.1 specifies explaining the meaning of rational exponents based on the properties of integer exponents. (MA.HS.2.4.b) • A-APR.6 includes rewriting rational expressions. (MA.HS.2.4.b) • A-REI.2 specifies showing how extraneous solutions may arise when solving rational and radical equations. (MA.HS.2.4.b) • A-APR.1 added because CO standards do not include knowing properties of the system of polynomial and performing arithmetic on polynomials. • A-APR.7 added because CO	**Inquiry Questions:** 1. How do symbolic transformations affect an equation, inequality, or expression? 2. How is it determined that two algebraic expressions are equivalent? 3. How are order of operations and operational relationships important when solving multivariable equations? 4. How would a number system work without the commutative property? **Relevance and Application:** 1. The ability to recognize and find equivalent forms of an equation allows the transformation of equations into the most useful form such as adjusting the density formula to calculate for volume or mass. 2. The simplification of algebraic expressions and solving equations are tools used to solve problems in science. Scientists represent relationships between variables by developing a formula and using values obtained from experimental measurements and algebraic manipulation to determine values of quantities that are difficult or impossible to measure directly such as acceleration due to gravity, speed of light, and mass of the earth. 3. The manipulation of expressions and solving formulas are techniques used to solve problems in geometry such as finding the area of a circle, determining the volume of a sphere, calculating the surface area of a prism, and applying the Pythagorean Theorem.

addition, subtraction, and multiplication; add, subtract, and multiply polynomials. e. A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	standards do not include knowing properties of the system of rational expressions and performing arithmetic on rational expressions. • 8.EE.1 Content covered in MA.HS.2.4.b. (see MA.8.1.2) •	**Nature of Mathematics:** 1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: High School

Concepts and skills students master:

5. Solutions to equations, inequalities and systems of equations are found using a variety of tools

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Find* solutions to quadratic and cubic equations and linear inequalities by using appropriate algebraic methods such as factoring, completing the square, graphing or using the quadratic formula b. Find* solutions to equations involving power, exponential, rational and radical functions c. Solve* systems of linear equations and inequalities with two variables d. A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. e. N-CN.9 Know the	• N-CN.7 specifies solving quadratic equations with complex roots. (MA.HS.2.5.a) • N-CN.8 specifies extending polynomial identities to the complex numbers. (MA.HS.2.5.a) • F-IF.8b specifies using properties of exponents to interpret exponential functions. (MA.HS.2.5.b) • F-LE.4 specifies translating between exponential and logarithmic notations. (MA.HS.2.5.b) • A-REI.12 specifies graphing systems of linear inequalities. (MA.HS.2.5.c) • A-REI.5 added because CO standards do not include proofs about equivalent systems of equations. • N-CN.9 added because CO standards do not include the Fundamental Thereom of	**Inquiry Questions:** 1. What makes a solution strategy both efficient and effective? 2. How is it determined if multiple solutions to an equation are valid? 3. How does the context of the problem affect the reasonableness of a solution? 4. What are some similarities in solving all types of equations? 5. Why do different types of equations require different types of solution processes? 6. Why can two equations be added together to get another true equation? 7. Can computers solve algebraic problems that people cannot solve? Why? **Relevance and Application:** 1. Linear programming allows representation of the constraints in a real-world situation identification of a feasible region and determination of the maximum or minimum value such as to optimize profit, or to minimize expense. 2. Effective use of graphing technology helps to find solutions to equations or systems of equations. 3. The understanding and use of equations, inequalities, and systems of equations allows situation analysis and decision-making. For example, it helps people choose cell phone plans, calculate credit card interest and payments, and determine health insurance costs.

Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. f. A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example*, … g. A-REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable h. A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). i. A-REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. *Using all tools including graphing technology	Algebra. • A-REI.7 added because CO standards do not include solving systems that include a non-linear quadratic equation • A-REI.8 added because CO standards do not include using matrices and vectors to solve systems of equations. • A-REI.9 added because CO standards do not include finding the inverse of a matrix and using matrices to solve systems of equations. • A-REI.11 added because CO standards do not include solving systems of non-linear functions • 8.EE.2 Content covered in MA.HS.2.5.a. (see MA.8.1.2) •	**Nature of Mathematics:** 1. Mathematics involves visualization. 2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: High School

Concepts and skills students master:

6. Quantitative relationships in the real world can be modeled and solved using functions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Represent, solve*, and interpret problems in various contexts using linear, quadratic, and exponential functions b. Represent, solve*, and interpret problems involving direct and inverse variations and a combination of direct and inverse variation c. Analyze* the impact of interest rates on a personal financial plan (PFL) d. Evaluate* the costs and benefits of credit (PFL) e. Analyze various lending sources, services, and financial institutions (PFL) f. N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in	• F-LE.1a-c emphasize distinguishing between situations that can be modeled using linear and exponential functions. (MA.HS.2.6.a) • F-TF.5 includes using trigonometric functions to model periodic phenomena. (MA.HS.2.6.a) • F-TF.7 includes using trigonometric functions and inverses to solve equations that arise in modeling contexts. (MA.HS.2.6.a) • A-SSE.1a and b specify interpreting parts of expressions (MA.HS.2.6.a, b) • A.CED.1 includes rational functions. (MA.HS.2.6.a, b) • A.CED.3 specifies representing constraints and considering viablility of solutions based on context. (MA.HS.2.6.a) • F-IF.5 specifies relating the domain of a function to the context of the	**Inquiry Questions:** 1. What phenomena can be modeled with particular functions? 2. Which financial applications can be modeled with exponential functions? Linear functions? 3. What elementary function or functions best represent a given scatter plot of two-variable data? 4. How much would today's purchase cost tomorrow? **Relevance and Application:** 1. The knowledge of how functions model real-world phenomena allows exploration and improved understanding of complex systems such as how population growth may affect the environment , how interest rates or inflation affect a personal budget, how stopping distance is related to reaction time and velocity, and how volume and temperature of a gas are related. 2. Biologists use polynomial curves to model the shapes of jaw bone fossils. They analyze the polynomials to find potential evolutionary relationships among the species. 3. Physicists use basic linear and quadratic functions to model the motion of projectiles.

graphs and data displays. g. N-Q.2 Define appropriate quantities for the purpose of descriptive modeling h. N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. i. F-BF.1c Compose functions. For example… *Using all tools including graphing technology	problem.(MA.HS.2.6.a, b) • N-Q.1 added because CO standards do not include dimensional analysis. • N-Q.2 added because CO standards do not include defining quantities for descriptive modeling in general. • N-Q.3 added because CO standards do not include choosing appropriate level of accuracy. • F-BF.1c added because CO standards do not include composing functions. •	**Nature of Mathematics:** 1. Mathematicians use their knowledge of functions to create accurate models of complex systems. 2. Mathematicians use models to better understand systems and make predictions about future systemic behavior.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Linear functions model situations with a constant rate of change and can be represented algebraically, graphically, and using tables

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Convert from one representation of a linear function to another, including situations, tables, equations (slope-intercept form), and graphs b. Use representations of linear functions to analyze situations and solve problems c. Identify the dependent and independent variable in real-world situations d. Identify and interpret the slope(rate of change) and y-intercept in graphs, in tables, and from equations in slope-intercept form e. Model and graph two linear equations in slope-intercept form on the same coordinate plane and interpret the point of intersection as the solution to the system of equations	• 8.EE.6 added because CO standards do not specify using similar triangles to explain the meaning of slope and deriving the general equations $y = mx$ and $y = mx + b$. • 8.EE.8 includes solving real-world and mathematical problems algebraically. (MA.8.2.1.e) • 8.F.3 includes giving examples of functions that are not linear. (MA.8.2.1.d) • 6.NS.8 Content partially addressed in MA.7.2.1.b and covered in MA.8.2.1.e. (see MA.6.2.1) •	**Inquiry Questions:** 1. How can different representations of linear patterns present different perspectives of situations? 2. How can a relationship be analyzed with tables, graphs, and equations? 3. Why are functions represented in multiple ways? 4. Why is one variable dependent upon the other in relationships? **Relevance and Application:** 1. Fluency with different representations of linear patterns allows comparison and contrast of linear situations such as service billing rates for competing companies or simple interest on savings or credit. 2. The ability to represent linear patterns helps to make predictions or find unknown values such as determining how many hours of labor were performed given the total bill or determining the cost of a monthly phone plan based on minutes used. 3. Understanding slope as rate of change allows individuals to develop and use a line of best fit for data that appears to be linearly related. 4. The ability to recognize slope and y-intercept of a linear function facilitates graphing the function or writing an equation that describes the function. 5. Recognition of the significance of the point of intersection for two linear equations helps to solve problems involving two linear rates such as determining when two vehicles traveling at constant speeds will be in the same place, when two calling plans cost the same, or the point when profits begin to exceed costs.

f. 8.EE.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y*= *mx* for a line through the origin and the equation *y*= *mx* + *b* for a line intercepting the vertical axis at *b*.		**Nature of Mathematics:** 1. Mathematicians represent functions in multiple ways to gain insights into the relationships they model.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

2. Properties of algebra, equality, and inequality are used to solve linear equations and inequalities

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use the distributive, associative, and commutative properties to simplify algebraic expressions b. Solve one-variable equations including those involving multiple steps, rational numbers, variables on both sides, and the distributive property c. Solve inequalities in one variable including negative coefficients and graph the solution on a number line d. Represent the distributive property in a variety of ways including numerically, geometrically, and algebraically	• 6.EE.3 Content covered in MA.8.2.2.a. (see MA.6.2.2) • 6.EE.4 Content covered in MA.8.2.2.a. (see MA.6.2.2) • 6.EE.5 Content covered in MA.8.2.2.b and c. (see MA.6.2.2) • 6.EE.8 Content covered in MA.8.2.2.c. (see MA.6.2.2) • 7.EE.4a and b Content covered in MA.8.2.2.b and c. (see MA.7.2.1) •	**Inquiry Questions:** 1. Do algebraic properties work with numbers or just symbols? Why? 2. Why are there different ways to solve equations? 3. How are properties applied in other fields of study? **Relevance and Application:** 1. Procedural fluency with algebraic methods allows use of linear equations and inequalities to solve problems in fields such as banking, engineering, and insurance. For example, it helps to calculate the total value of assets or find the acceleration of an object moving at a linearly increasing speed. 2. Comprehension of the structure of equations allows use spreadsheets effectively to solve problems that matter such as showing how long to pay off debt, or representing data collected from science experiments. **Nature of Mathematics:** 1. Mathematicians can think both forward and backward through a problem. An equation is like the end of a story about what happened to a variable. By reading the story backward, and undoing each step, mathematicians can find the value of the variable.

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

3. Graphs and tables can be used to distinguish between linear and nonlinear functions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Given a table or graph determine if the function is linear b. Explain the properties of linear functions in tables and graphs	• 8.F.5 includes sketching a graph when the qualitative features of a function are described verbally. (MA.8.2.3.a)	**Inquiry Questions:** 1. How can change best be represented mathematically? 2. Why are patterns and relationships represented in multiple ways? 3. What properties of a function make it a linear function? **Relevance and Application:** 1. Recognition that non-linear situations is a clue to non-constant growth over time helps to understand such concepts as compound interest rates, population growth, appreciations, and depreciation. 2. Linear situations allow for describing and analyzing the situation mathematically such as using a line graph to represent the relationships of the circumference of circles based on diameters. **Nature of Mathematics:** 1. Mathematics involves multiple points of view. 2. Mathematicians look at mathematical ideas arithmetically, geometrically, analytically, or through a combination of these approaches.

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Relationships involving the constant rate of change are modeled and solved using linear functions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Given a linear situation (including direct variation), identify variables and write an equation in slope-intercept form b. Given a linear equation (including direct variation), substitute input values to create a table and graph coordinate points in all four quadrants c. 7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*	• 7.EE.2 added because CO standards do not specify re-writing and interpreting equivalent expressions in this way. • 7.EE.4a and b Content covered in MA.8.2.2.b and c. • 6.NS.8 Content partially addressed in MA.7.2.1.b and covered in MA.8.2.1.e. (see MA.6.2.1) •	**Inquiry Questions:** 1. Why do humans recognize patterns? 2. What makes a pattern linear? 3. Why is constant rate of change an important concept to model mathematically? **Relevance and Application:** 1. The modeling of patterns in the world with equations allows decision-making and predictions such as the amount of food needed for different population sizes, the unit rate of an item, and the cost for different quantities. 2. Connection of how tables and graphs are related provides new insights for describing and extending patterns such as a steep graph would relate to large increases on a table. **Nature of Mathematics:** 1. Mathematicians use tables and graphs to represent relationships among variables. They use multiple representations to gain insights into the relationships between variables.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Patterns can be described using words, tables, and graphs

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Extend the pattern and describe the rule for arithmetic and geometric sequences b. Model linear situations using tables and graphs, and convert between these two representations c. Given a linear equation, substitute non-negative input values to create a table and graph coordinate points in the first quadrant	•	**Inquiry Questions:** 1. How can patterns be used to provide insight into data? 2. What are some advantages to modeling a linear situation with a table instead of a graph? A graph instead of a table? 3. What is the value of showing patterns in multiple ways such as tables, graphs, sequences? **Relevance and Application:** 1. The creation of a table to show the relationship between profits from a concert and the number of tickets sold allows prediction of whether or not the concert will be profitable. 2. Graphs of linear situations can be used to compare rates such as the amount of energy used by a regular light bulb versus an energy efficient light bulb. **Nature of Mathematics:** 1. Mathematics can be used to show that things that seem complex can be broken into simple patterns and relationships. 2. Mathematics can be expressed in a variety of formats.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Variables are used to represent unknown quantities

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Describe patterns by using words and variables with mathematical symbols b. Evaluate expressions by substituting whole number values for variables	• 6.EE.6 includes using variables to write expressions for real-world and mathematical problems. (MA.6.2.2.a) • 6.EE.2a, b, c include writing and reading algebraic expressions, using specific terms, using order of operations. (MA.6.2.2.b) • 6.EE.3 Content covered in MA.8.2.2.a. • 6.EE.4 Content covered in MA.8.2.2.a. • 6.EE.5 Content covered in MA.8.2.2.b and c. • 6.EE.8 Content covered in MA.8.2.2.c. •	**Inquiry Questions:** 1. If we didn't have variables, what would we use? 2. What purposes do variable expressions serve? 3. What are some advantages to being able to describe a pattern using variables? **Relevance and Application:** 1. Variables allow communication of big ideas with very few symbols. For example, d = r * t is a simple way of showing the relationship between the distance one travels and the rate of speed and time traveled, and $C = \pi d$ expresses the relationship between circumference and diameter of a circle. 2. Description of patterns with words and symbols enhances communication and collaboration. 3. Variables show what parts of an expression may change compared to those parts that are fixed or constant. For example, the price of an item may be fixed in an expression, but the number of items purchased may change. **Nature of Mathematics:** 1. Mathematics involves the use of elegant, simple language to express complex, abstract ideas.

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Number patterns and relationships can be described using a variety of tools

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Analyze and describe patterns and relationships using words, tables, graphs, symbols, and technology b. Explain, extend, and use patterns and relationships in solving problems, including those involving saving and checking accounts such as understanding that spending more means saving less (PFL)	•	**Inquiry Questions:** 1. How can patterns and relationships be used to describe and explain real-life situations? 2. What makes a pattern difficult to describe? 3. How can patterns be used to make predictions? **Relevance and Application:** 1. The recognition and extension of patterns helps to solve problems and make predictions such as how saving and investing can help someone to reach a financial goal, how weather affects sales, or how a child's height changes over time. 2. The understanding of patterns prepares for work with linear functions. For example, working with the pattern 3, 6, 9, 12, … leads to the linear function y = 3x. **Nature of Mathematics:** 1. Mathematics has always depended on the convenience of tools.

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. When a relationship exists between two quantities, a change in one results in a change in the other

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Express change relationships involving whole numbers with if/then statements, input/output boxes, function tables, and rule statements b. Select, describe, and use symbols to express unknown quantities c. Use patterns to solve problems including those involving saving and checking accounts such as the pattern created when saving $10 a month (PFL)	•	**Inquiry Questions:** 1. Does a function table always have to show a pattern? Why? 2. Do changes in one quantity always result in a change in another? How do you know? **Relevance and Application:** 1. Analysis of situations helps to see the effect of changes such as what happens to saving capability if spending increases, what happens to a phone bill when a family makes more calls, what happens to the balance of a checking account when more checks are written or more deposits are made, or what happens to the number of cookies a family can make when they buy more flour. 2. The use of symbols to express unknown quantities helps to find the unknown quantity such as finding the average speed of a bike ride by using the distance traveled and time spent riding, or finding how old a girl's father was when she was born by using her current age and her father's current age. **Nature of Mathematics:** 1. Mathematicians analyze patterns of change to better understand how the world changes with time.

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Number patterns and relationships can be represented by symbols

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use number relationships to find the missing number in a sequence b. Use a symbol to represent and find an unknown quantity in a problem situation c. Complete input/output tables d. Find the unknown in simple equations	• 4.OA.5 includes identifying features of a pattern not explicit in the rule. (MA.4.2.1.a) •	**Inquiry Questions:** 1. How can we predict the next element in a pattern? 2. Why do we use symbols to represent missing numbers? 3. Why is finding an unknown quantity important? **Relevance and Application:** 1. Use of an input/output table helps to make predictions in everyday contexts such as the number of beads needed to make multiple bracelets or number of inches of expected growth. 2. Symbols help to represent situations from everyday life with simple equations such as finding how much additional money is needed to buy a skateboard, determining the number of players missing from a soccer team, or calculating the number of students absent from school. **Nature of Mathematics:** 1. Mathematics involves pattern seeking. 2. Mathematicians use patterns to simplify calculations.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

2. Number properties and relationships can be used to solve problems

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use and describe number patterns for counting by 2, 5, 9, 10, and 11 from a given starting number b. Communicate the inverse relationship between multiplication and division, and use this relationship to efficiently solve and check problems c. Use the commutative and associative properties of multiplication to solve problems	•	**Inquiry Questions:** 1. Does skip counting always produce a pattern? Why? 2. How do patterns help to skip count from randomly selected places? 3. How do patterns assist in making predictions? 4. How are patterns shown? **Relevance and Application:** 1. Patterns in the everyday world help to find solutions to problems such as noticing that trash collection is decreasing after beginning a recycling program. 2. Number patterns help to make predictions. For example, a calendar is used to organize time around the nature patterns of day and night, and a bus schedule follows a pattern, so it is easy to remember. 3. The understanding of patterns helps to understand linear rates. 4. Fluency with forward and backward skip counting assists in learning multiplication and division. For example, learning the five facts is easy if you can count by fives). 5. Comprehension of associative property helps to more easily solve problems such as in (3 + 4) + 6 = 3 + (4 + 6), adding the 4 and 6 first is easier. **Nature of Mathematics:** 1. Mathematics can be used to show that things that seem complex can be broken into simple patterns and relationships.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Number patterns are based on operations and relationships

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Extend simple arithmetic and geometric sequences b. Count by and analyze patterns in multiples of 2, 3, 5, 9, 10,11,25, 50 and 100 c. Use known multiplication facts to solve unknown multiplication problems	• 3.OA.9 specifies explaining arithmetic patterns using properties of operations. (MA.3.2.1.a, b; MA.3.2.1.IQ.1, 2) • 3.OA.8 specifies solving two-step word problems using the four operations. Related footnote 3 states "… students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations)." (MA.3.2.1.c) • 3.MD.1 involves telling and writing time to the nearest minute and measuring time interval in minutes. (MA.3.2.1.RA.1) Concepts may be addressed in MA.4.4.2e. •	**Inquiry Questions:** 1. How do you know when there is a pattern? 2. What patterns do you notice when you count by 25, 50, and 100? 3. How are patterns useful? **Relevance and Application:** 1. The use of a pattern of elapsed time helps to set up a schedule. For example, classes are each 50 minutes with 5 minutes between each class. 2. The ability to use patterns allows problem-solving. For example, a rancher needs to know how many shoes to buy for his horses, or a grocer needs to know how many cans will fit on a set of shelves. **Nature of Mathematics:** 1. Mathematicians use creativity, invention, and ingenuity to understand and create patterns. 2. The search for patterns can produce rewarding shortcuts and mathematical insights.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Number properties can be used to solve problems

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use the commutative property to solve addition and multiplication problems b. Use the associative property to solve addition problems c. Use the relationship between addition and multiplication to solve problems	• 3.OA.5 specifies applying properties of operations as strategies to multiply and divide, provides examples for commutative, associative and distributive properties, and indicates (footnote 2) formal terms need not be used. (MA.3.2.2.a, b, c; MA.3.2.2.RA.2) • 1.OA.3 specifies using the commutative and associative properties (MA.1.2.2.a and b), addressed in MA.3.2.2.a, b. • 2.NBT.5 specifies adding and subtracting using strategies including commutative and associative properties, addressed in MA.3.2.2.a, b, c. (see MA.2.1.2.a) • 2.NBT.6 specifies adding up to four two-digit numbers using strategies also addressed in MA.3.2.2.a, b, c. (see MA.2.1.2.a) •	**Inquiry Questions:** 1. How do you use the commutative property to help you learn the multiplication facts? 2. What types of problems can you solve using multiplication that you cannot solve using addition? **Relevance and Application:** 1. Fluent use of skip counting helps to solve life problems in your life such as efficiently finding the value of a collection of coins or counting people in a crowd. 2. Knowledge of some multiplication facts allows derivation of unknown facts. For example, knowing 15 x 3 = 45 allows you to find 16 x 3. **Nature of Mathematics:** 1. Mathematics involves invention. 2. Mathematicians tinker with ideas and create new explanations, rules, and algorithms to bring clarity to a situation.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. Patterns are based on rules

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Count objects by groups of 2, 5, and 10 b. Identify a missing number in a sequence, and describe a rule c. Create and extend repeating patterns of 3-5 elements using a variety of materials such as numbers, letters, shapes, and manipulatives (CO only)	• 2.NBT.2 specifies counting within 1000, and skip-counting by 100's (MA.2.2.1.a) • 2.OA.3 specifies writing an equation to express an even number as a sum of two equal addends (MA.2.2.1.a) • 1.NBT.4 Content related to MA.2.2.2.a, b (see MA.1.1.1) • 1.NBT.5 Content related to MA.2.2.2.a, b, c (see MA.1.1.1) • 1.NBT.6 Content related to MA.2.2.2. a, b, c (see MA.1.1.1) •	**Inquiry Questions:** 1. What patterns do you notice when you count by 2, 5, and 10? 2. What patterns are in your life? 3. How does finding patterns help in counting? **Relevance and Application:** 1. The understanding of patterns involving two is useful in real contexts such as knowing which side of the street a house is on, or knowing there would be pennies in a collection of coins worth 43 cents. 2. The grouping of items by 2s, 5s, or 10s makes counting more efficient by organizing objects in groups of ten and then counting by tens. 3. Counting by tens prepares for understanding of the base ten number system. 4. Creation and extension of patterns allows the creation of designs or composition of music. For example, it helps with making decorations, invitations, posters, or the refrain of a song or melody). **Nature of Mathematics:** 1. Mathematicians apply patterns to show new understanding. 2. Counting is the building block to inventing sequencing and pattern-finding.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Second Grade

Concepts and skills students master:

2. Number relationships can be used to develop computation strategies

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use ten-based strategies to solve addition and subtraction facts to 20 b. Demonstrate the structure of numbers as tens and ones in addition and subtraction c. Communicate the inverse relationship between addition and subtraction, and use this relationship to efficiently solve and check problems d. 2.NBT.8 Mentally add 10 to 1000 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. e. 2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal	• 2.NBT.8 added because CO standards do not specify these mental strategies for addition and subtraction. • 2.OA.4 added because CO standards do not describe using addition as a foundation for multiplication in this way. • 2.MD.5 added because CO standards do not describe relating operations to measurement in this way. • 2.MD.6 added because CO standards do not specify relating operations to the number line in this way. •	**Inquiry Questions:** 1. How can finding tens help in addition and subtraction problems? 2. How are addition and subtraction related? 3. How can you use addition to help you find a solution to a subtraction problem? 4. How are operations different from numbers? **Relevance and Application:** 1. The use of tens in computation makes adding and subtracting more efficient. For example, simplify 3 + 5 + 7 + 5 by adding the 3 and 7 to make 10 and adding the 5 and 5 to make 10. 2. The use of the inverse relationship between addition and subtraction can help to know when their solutions are correct. For example, paying for a $2 muffin with a $5 bill and using addition to check that the amount of change is correct. 3. Counting in groups both forward and backward strengthens computational skills in later years.

addends. f. 2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. g. 2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1,2,…, and represent whole-number sums and differences within 100 on a number line diagram.		**Nature of Mathematics:** 1. Mathematics is about creating better strategies.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: First Grade

Concepts and skills students master:

1. Patterns can grow

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Count objects by groups of 2 or 5 b. Extend a repeating pattern based on a rule (CO only)	Note: CCS patterns begin in grade 3 with arithmetic patterns, including addition and multiplication tables, and are explained using properties of operations. •	**Inquiry Questions:** 1. What is a pattern? 2. What patterns are in your life? 3. What is the repeating unit of the pattern? 4. How does finding patterns help in counting? **Relevance and Application:** 1. Rules can be used to describe patterns in everyday life such as the cycle of seasons, rhythm in music, or artistic patterns. 2. The grouping of items by twos or fives makes counting more efficient. For example, when counting a large number of objects, you can put them in groups of five; if you lose count, you can quickly count the groups and find your place. 3. The use of tally marks is an example of counting by fives to make recording and reading data easier. **Nature of Mathematics:** 1. Basic repeating patterns can be endless. 2. Mathematical relationships can be expressed both forward and backward.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: First Grade

Concepts and skills students master:

2. Number relationships can be used to solve problems

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use number relationships such as doubles, or plus or minus one to solve problems b. Use the inverse relationship between adding and subtracting to solve problems	• 1.OA.3 specifies using the commutative and associative properties (MA.1.2.2.a and b), addressed in MA.3.2.2.a, b. •	**Inquiry Questions:** 1. How many ways can a set of objects be broken apart and put together? 2. How does using number relationships, such as doubles or plus or minus one, create a number pattern? **Relevance and Application:** 1. Use of number relationships simplifies calculations. For example, if you have 10 cookies and give one away, it is easy to tell how many are left by finding one less than 10 instead of counting all the cookies again. If you and your friend have the same number of stickers, you can count the number of stickers one of you has, and use doubles to quickly find the total number of stickers you have together. **Nature of Mathematics:** 1. Mathematicians are thinkers. They take ideas apart and put them back together, sometimes in different ways that lead to new insights.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Patterns can repeat

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Duplicate a simple pattern (CO only) b. Extend a repeating two-element pattern using a variety of materials such as numbers, letters, shapes, and manipulatives (CO only)	Note: CCS patterns begin in grade 3 with arithmetic patterns, including addition and multiplication tables, and are explained using properties of operations. •	**Inquiry Questions:** 1. Where are patterns found? 2. How do you know there is a pattern? 3. How could you explain a pattern? **Relevance and Application:** 1. People create patterns using materials, sound, and numbers such as clapping patterns, geometric patterns, and counting patterns. 2. Patterns exist in the world around us such as walkway designs, clothing patterns, and carpeting designs. **Nature of Mathematics:** 1. The nature of mathematics involves creativity, invention, and ingenuity. 2. Mathematics involves pattern seeking. Mathematicians look for patterns and regularity. The search for patterns can produce rewarding shortcuts and mathematical insights.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Kindergarten

Concepts and skills students master:

2. Relationships exist between numbers

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Generalize the counting sequence pattern from counting all to knowing "one more" and "one less" b. Communicate the relationship between composing and decomposing numbers	• K.NBT.1 specifies composing and decomposing numbers from 11 to 19 and understanding that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (MA.K.2.2.b) Content covered in MA.1.1.1.c. •	**Inquiry Questions:** 1. How many ways can a set of objects be broken apart and put together? 2. How does using number relationships, such as doubles or plus or minus one, create a number pattern? **Relevance and Application:** 1. Over the course of one winter a herd of cattle will reduce a stock of hay bales one at a time. 2. When building a fireplace the number of bricks increases, on-by-one, until it is completed. **Nature of Mathematics:** 1. Mathematicians see the relationship of actual objects to numbers.

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Preschool

Concepts and skills students master:

1. Objects can be sorted based on patterns and relationships

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Sort by a single attribute b. Match, place in a series, and group objects according to one attribute	**Inquiry Questions:** 1. Where are patterns found? 2. How do you know there is a pattern? 3. How could you explain a pattern? 4. How many elements does it take to have a pattern?
	Relevance and Application: 1. People create patterns using materials, sound, and numbers such as geometric patterns, clapping patterns, and counting patterns. 2. Patterns exist in the world around us such as walkway designs, clothing patterns, carpeting designs, or refrains in a song. 3. People study how both repeating and growing patterns are generated such as in music, or shapes.
	Nature of Mathematics: 1. Mathematicians sort and organize to create patterns.

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: High School

Concepts and skills students master:

1. Statistical methods take variability into account, supporting informed decision-making through quantitative studies designed to answer specific questions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Formulate appropriate research questions that can be answered with statistical analysis b. Determine appropriate data collection methods to answer a research question c. Explain how data might be analyzed to provide answers to a research question	• 7.SP.1 Content covered in MA.HS.3.1.b and MA.HS.3.2.a. (see MA.7.3.1) • 7.SP.2 Content covered in MA.HS.3.1.c. (see MA.7.3.1) •	**Inquiry Questions:** 1. What types of questions and data collection methods are appropriate for statistical analysis? 2. How do you reduce bias in question design and data collection methods? 3. How can the results of a statistical investigation be used to support an argument? 4. How does the method of sampling affect the design of the study? 5. What happens to sample-to-sample variability when you increase the sample size? 6. When should sampling be used? When is sampling better than using a census? **Relevance and Application:** 1. Comprehension of the appropriate design of studies allows determination of the appropriate data collection methods given research questions from various disciplines such as drug testing, candidates for public office, and student surveys. **Nature of Mathematics:** 1. Mathematics involves making conjectures, gathering data, recording results, and making multiple tests.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking

Grade Level Expectation: High School

Concepts and skills students master:

2. The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Identify the characteristics of a well-designed and well-conducted survey b. Identify the characteristics of a well-designed and well-conducted experiment c. Differentiate between the inferences that can be drawn in experiments versus observational studies	• S-IC.3 includes recognizing the differences between sample surveys, experiments, and observational studies. (MA.HS.3.2.c) • 7.SP.1 Content covered in MA.HS.3.1.b and MA.HS.3.2.a. (see MA.7.3.1) •	**Inquiry Questions:** 1. Does a strong correlation imply causation? 2. How can you tell when a claim that cannot be substantiated? 3. How can the quality of a study be gauged? 4. How does sampling build or erode confidence in the claims made? **Relevance and Application:** 1. The ability to evaluate the design of an experiment or sample survey helps to determine if claims, projections, and data presented in news articles or other publications are valid such as drug advertisements, scientific findings, government studies, or political advertisements. 2. Analysis of the characteristics of different studies allows evaluation and justification of the design of a research study such as using experiments to determine unsafe levels of toxins or using observational studies to suggest the relationship between texting and attention. **Nature of Mathematics:** 1. Mathematicians are open to new ideas that provide deeper understanding of the world around them. They use their knowledge of experimental design to identify false or unsubstantiated claims.

Content Area: Mathematics

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: High School

Concepts and skills students master:

3. Visual displays and summary statistics condense the information in data sets into usable knowledge

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Identify and choose appropriate ways to summarize numerical or categorical data using tables, graphical displays, and numerical summary statistics (describing shape, center and spread) and accounting for outliers when appropriate b. Define and explain how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics (CO only) c. Describe the relationship between two categorical variables using percents d. When the relationship between two numerical variables is reasonably linear, apply* the least-squares criterion for line	• S-ID.1 specifies using dot plots, histograms, and box plots to represent data. (MA.HS.3.3.a) • S-ID.3 includes interpreting differences in shape, center, and spread of data and accounting for effects of outliers. (MA.HS.3.3.a) • S-ID.6a added because, in general, CO standards do not include fitting a function to data. • S-ID.6b added because CO standards do not include informally assessing the fit of a function to data. • 6.SP.5 Content covered in MA.HS.3.3.a. (see MA.6.3.1) •	**Inquiry Questions:** 1. What makes data meaningful or actionable? 2. Why should attention be paid to an unexpected outcome? 3. How can summary statistics or data displays be accurate but misleading? **Relevance and Application:** 1. Facility with data organization, summary, and display allows the sharing of data efficiently and collaborating to answer important questions such as is the climate changing, how do people think about ballot initiatives in the next election, or is there a connection between cancers in a community?

fitting, use Pearson's correlation coefficient as a measure of strength, and interpret the slope and y-intercept in the context of the problem e. S-ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* f. S-ID.6b Informally assess the fit of a function by plotting and analyzing residuals. *Using all tools including graphing technology		**Nature of Mathematics:** 1. Mathematicians create visual and numerical representations of data to reveal relationships and meaning hidden in the raw data.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: High School

Concepts and skills students master:

4. Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Define and explain the meaning of significance (both practical and statistical) b. Explain the role of p-values in determining statistical significance (CO only) c. Determine the margin of error associated with an estimate of a population characteristic d. S-IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	• S-IC.5 added because CO standards do not include estimating a population parameter from data. •	**Inquiry Questions:** 1. Can the practical significance of a given study matter more than statistical significance? Why is it important to know the difference? 2. When can generalizations be made from a study? 3. Why is the margin of error in a study important? 4. How is it known that the results of a study are not simply due to chance? **Relevance and Application:** 1. Inference and prediction skills enable informed decision-making based on data such as whether to stop using a product based on safety concerns, or whether a political poll is pointing to a trend. **Nature of Mathematics:** 1. Mathematicians are skeptical of apparent trends. They use their understanding of randomness to distinguish meaningful trends from random occurrences.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: High School

Concepts and skills students master:

5. Probability models outcomes for situations in which there is inherent randomness, quantifying the degree of certainty in terms of relative frequency of occurrence

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Develop* simulations that demonstrate probability as a long-run relative frequency b. Apply and solve problems using the concepts of independence and conditional probability c. Apply and solve problems using the concept of mutually exclusive properties when combining probabilities d. Evaluate* and interpret probabilities using a normal distribution e. Find* and interpret the expected value and standard deviation of a discrete random variable X f. Analyze* the cost of insurance as a method to offset the risk of a situation (PFL) g. S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of	• S-MD.4 specifies developing a probability distribution and finding the expected value of a random variable using empirically determined probabilities. (MA.HS.3.5.a) • S-CP.4 specifies using two-way frequency tables to decide if events are independent and to approximate conditional probabilities. (MA.HS.3.5.b) • S-CP.5 specifies recognizing and explaining the concept of conditional probability and independence in everyday language and situations. (MA.HS.3.5.b) • S-MD.3 specifies developing a probability distribution and finding the expected value of a random variable using theoretical probabilities. (MA.HS.3.5.e) • S-MD.5b includes evaluating and comparing strategies on the basis of expected values.	**Inquiry Questions:** 1. Can probability be used to model all types of uncertain situations? For example, can the probability that the 50th president of the United States will be female be determined? 2. How and why are simulations used to determine probability when the theoretical probability is unknown? 3. How does probability relate to obtaining insurance? **Relevance and Application:** 1. Comprehension of probability allows informed decision-making, such as whether the cost of insurance is less than the expected cost of illness, when the deductible on car insurance is optimal, whether gambling pays in the long run, or whether an extended warranty justifies the cost. 2. Probability is used in a wide variety of disciplines including physics, biology, engineering, finance, and law. For example, employment discrimination cases often present probability calculations to support a claim.

other events ("or," "and," "not"). h. S-MD.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. i. S-MD.6 Use probabilities to make fair decisions *Using all tools including graphing technology	(MA.HS.3.5.e) • S-CP.1 added because CO standards do not include describing events as subsets of a sample space. • S-MD.1 added because CO standards do not include defining a random variable and graphing its distribution. • S-MD.6 added because CO standards do not include using probabilities to make fair decisions. •	**Nature of Mathematics:** 1. Some work in mathematics is much like a game. Mathematicians choose an interesting set of rules and then play according to those rules to see what can happen. 2. Mathematicians explore randomness and chance through probability.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Given a scatter plot, calculate quadrant count ratio to quantify the magnitude and strength of the association between two variables for numeric data as positive, negative, or no correlation b. Given a scatter plot suggesting a linear relationship, draw a line of fit to make predictions c. Use time series plots (line graphs) to analyze the trend of a set of data over time	• 8.SP.4 Content covered in MA.HS.3.3.a. •	**Inquiry Questions:** 1. How is it known that two variables are related to each other? 2. How is it known that an apparent trend is just a coincidence? 3. How can correct data lead to incorrect conclusions? 4. How do you know when a credible prediction can be made? **Relevance and Application:** 1. The ability to analyze and interpret data helps to distinguish between false and relationships such as developing superstitions from seeing two events happen in close succession versus identifying a credible correlation. 2. Data analysis provides the tools to use data to model relationships, make predictions, and determine the reasonableness and limitations of those predictions. For example, predict whether staying up late affects grades, or the relationships between education and income, between income and energy consumption, or between the unemployment rate and GDP). **Nature of Mathematics:** 1. Mathematicians discover new relationship embedded in information.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Visual displays and summary statistics with one-variable data condense the information in data sets into usable knowledge

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Distinguish between median as middle number and mean as balance point for an ordered set of data b. Use Mean Absolute Deviation (MAD) to analyze the spread of a set of data c. Construct and interpret dot plots, histograms, stem-and-leaf plots, and circle graphs d. Construct and interpret a box plot using the five-number summary and identify the interquartile range (IQR) for a set of data e. Compare sets of data using shape (skewed, normal, uniform), with appropriate measures of central tendency (mean, median, mode), and appropriate measures of spread (range, IQR, MAD) f. Given a frequency table, calculate relative frequencies	• 7.SP.6 includes using frequency tables to approximate the probability of chance events. (MA.7.3.1.f) • 7.SP.1 Basic foundations introduced in grade 3 (MA.3.3.1.c) and grade 6 (MA.6.3.1b, d), but content covered in MA.HS.3.1.b and MA.HS.3.2.a. • 7.SP.2 Content covered in MA.HS.3.1.c. • 6.SP.3 Content covered in MA.7.3.1.a and b. (see MA.6.3.1) • 6.SP.4 Content covered in MA.7.3.1.c and d. (see MA.6.3.1) • 6.SP.2 Content covered in MA.7.3.1.e. (see MA.6.3.1) •	**Inquiry Questions:** 1. Why are there so many ways to describe data? 2. When is one data display better than another? 3. When is one statistical measure better than another? **Relevance and Application:** 1. Comprehension of how to analyze and interpret data allows better understanding of large and complex systems such as analyzing employment data to better understand our economy, or analyzing achievement data to better understand our education system. 2. Different data analysis tools enable the efficient communication of large amounts of information such as listing all the student scores on a state test versus using a box plot to show the distribution of the scores. **Nature of Mathematics:** 1. Mathematicians leverage strategic displays to reveal data.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Questions can be answered by collecting and analyzing data and data displays

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Formulate questions for populations larger than the classroom b. Recognize that a sample may not represent a population accurately c. Recognize bias in surveys d. Utilize appropriate techniques to design a random sample e. Recognize the use of deceptive scales on a graph that make differences look much larger than they are, or the use of pictographs with areas that are proportioned incorrectly (CO only)	• 6. SP.2 Content covered in MA.7.3.1.e. • 6.SP.3 Content covered in MA.7.3.1.a and b. • 6.SP.4 Content covered in MA.7.3.1.c and d. • 6.SP.5 Content covered in MA.HS.3.3.a. •	**Inquiry Questions:** 1. What makes a good question for a survey? 2. When and why would the results of a survey be questioned? 3. How might the sample for a survey affect the results of the survey? 4. What kinds of survey data are used on the Internet? How is reliability determined? 5. Does using numbers to support a claim make the claim more believable? 6. How can you tell if you are being misled? 7. What kinds of problems are caused by misleading statistics? **Relevance and Application:** 1. Comprehension of the survey methodology leads to better decision-making. For example, people use surveys on the Internet to gather information that is of interest or value such as best local restaurants, and most frequently watched videos. 2. The ability to recognize how data can be biased or misrepresented allows critical evaluation of claims and avoids being misled. For example, data can be used to evaluate products that promise effectiveness, or surveys that claim to show strong opinions. **Nature of Mathematics:** 1. Mathematicians are informed consumers of information. They evaluate the quality of data before using it to make decisions.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Mathematical models are used to determine probability

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Determine probabilities through experiments or simulations b. Express the probability of an event using fractions, decimals, and percents c. Make a table, tree diagram or an organized list to determine possible outcomes of two or more compound events d. Predict outcomes of events using experimental and theoretical probabilities	•	**Inquiry Questions:** 1. Why is it important to consider all of the possible outcomes of an event? 2. Is it possible to predict the future? How? 3. What are situations in which probability cannot be used? **Relevance and Application:** 1. The ability to efficiently and accurately count outcomes allows systemic analysis of such situations as trying all possible combinations when you forgot the combination to your lock or deciding to find a different approach when there are too many combinations to try; or counting how many lottery tickets you would have to buy to play every possible combination of numbers. 2. The knowledge of theoretical probability allows the development of winning strategies in games involving chance such as knowing if your hand is likely to be the best hand or is likely to improve in a game of cards. **Nature of Mathematics:** 1. Mathematicians approach problems systematically. When the number of possible outcomes is small, each outcome can be considered individually. When the number of outcomes is large, a mathematician will develop a strategy to consider the most important outcomes such as the most likely outcomes, or the most dangerous outcomes.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Visual displays and summary statistics are used to describe and interpret data

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Formulate a question and hypothesis to design appropriate data collection and display methods b. Select and create appropriate displays of data including double bar graphs, time plots, and line graphs c. Interpret data using the concepts of shape of distribution, range, mode, median and mean d. Draw conclusions, and make convincing arguments based on categorical and numerical data analysis	• 5.MD.2 Content covered in MA.6.1.1.d. •	**Inquiry Questions:** 1. What information would you gather about students in your school? 2. What is the best way to show data? 3. How can you make sense out of the data you collect? 4. If something is true about the students in your school, is it likely to be true about students in other schools? Why or why not? **Relevance and Application:** 1. The collection of information from individual students can help you better understand the whole school or the whole neighborhood for better decision-making. For example, if the principal determines how many students will attend a basketball game, he/she can decide how many bleacher seats to set up. **Nature of Mathematics:** 1. Mathematics helps people collect and use information to make good decisions.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. Mathematical models are used to determine probability, analyze and describe the outcomes of events

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Organize all possible outcomes of events in a list or chart b. Use fractions, decimals, and percents to quantify the likelihood of events c. Explain why a game involving chance devices such as number cubes or spinners is fair or unfair d. Compare individual data to class data collected from chance devices to describe the differences in outcomes based on sample size	• Note: Concepts of chance and probability are covered extensively in grade 7 and high school.	**Inquiry Questions:** 1. Why would you play a game that is unfair? How do you know when you've conducted enough trials to be able make a conclusion? 2. Why are fractions, decimals, and percents good ways to quantify the likelihood of an event? **Relevance and Application:** 1. The ability to quantify uncertainly allows people to understand risk in everyday life such as losing a game, getting injured while skateboarding, or losing money in an investment. 2. The understanding of chance helps to appreciate unlikely events when they occur such as appreciating seeing a rainbow because it is unlikely to happen again soon, or appreciating losing a game of chance 10 times in a row because it is a very unlikely event. **Nature of Mathematics:** 1. Mathematicians explore the concepts of fairness, luck, and likelihood through experimentation. 2. Mathematics helps people make good decisions in uncertain situations.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Visual displays of classroom data can be used to summarize information across the content areas

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Compose questions to generate data related to grade level areas of study b. Collect data from class experiments or multi-classroom surveys c. Create data displays appropriate to data collected d. Describe data using the concept of shape of the distribution	• 4.MD.4 specifies making a line plot to display measurement data in fractions of a unit and using the information to solve problems involving addition and subtraction of fractions. (MA.4.3.1.c, d) • 2.MD.10 content related to MA.2.4.1.b is also addressed in MA.3.3.1.c. •	**Inquiry Questions:** 1. What can you learn by collecting data? 2. What makes a data representation useful? 3. How do you choose the best representation for data? 4. What can the shape of data in a display tell you? **Relevance and Application:** 1. The collection and analysis of data provides understanding of how things work. For example, conducting probability experiments helps to better understand probability, and measuring the temperature every day for a year helps to better understand weather. 2. The selection of an appropriate data display to suit the audience aids with understanding depending on the audience. For example, you may display the results of a penny drive to the whole school differently from how you would display it to the PTO. **Nature of Mathematics:** 1. Mathematics helps people use data to learn about the world.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

2. Mathematical models are used to test predictions about the likelihood of events

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Formulate a question to test a prediction, and conduct an experiment using chance devices, such as coins, spinners, and number cubes, to test predictions b. Represent the outcomes of experiments with fractions, and describe using the concepts of impossible, unlikely, likely, and certain c. Describe the likelihood of real-life situations using the concepts of impossible, unlikely, likely and certain (PFL)	Note: Concepts of chance and probability are covered extensively in grade 7 and high school. •	**Inquiry Questions:** 1. How can you know all of the possible outcomes for an event? 2. How can knowing the likely outcomes in a situation help you make decisions? 3. In what situations is every possible outcome equally likely? 4. In what situations are some possible outcomes not equally likely? 5. Why are fractions a good way to describe the likelihood of an event? **Relevance and Application:** 1. Consideration of likely and unlikely outcomes allows better decision-making. For example, if you are likely to lose a game, you may choose not to play; or since falling and getting injured is a possible outcome when you ride your bike, you may choose to wear a helmet. **Nature of Mathematics:** 1. Mathematicians support anyone who needs advice about the likelihood of an outcome.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Visual displays of data can be used to answer questions of interest

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Compose questions to generate data b. Collect and organize data from simple experiments or surveys in class c. Create picture graphs, bar graphs, dot plots, and frequency tables from a data set d. Describe data using the concepts of mode, clusters and gaps	• 3.MD.4 specifies generating linear measurement data using rulers marked with halves and fourths of an inch, and displaying the data on a line plot with appropriate horizontal scale. (MA.3.3.1.b) • 3.MD.3 specifies drawing a scaled picture graph and a scaled bar graph to represent a data set with several categories, and using the information to solve one- and two-step problems. (MA.3.3.1.c, d) •	**Inquiry Questions:** 1. How do data displays help us to understand information? 2. What can data tell you about your class or your school? **Relevance and Application:** 1. The collection and use of data provides better understanding of people and the world such as knowing what games classmates like to play, how many siblings friends have, or personal progress made in sports. **Nature of Mathematics:** 1. Mathematical data can be represented in both static and animated displays.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Mathematical models are used to explore and describe fairness

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Investigate chance devices such as coins, spinners, and number cubes b. Apply the concepts of impossible, unlikely and likely c. Determine if a chance device is fair or unfair	Note: Concepts of chance and probability are covered extensively in grade 7 and high school. •	**Inquiry Questions:** 1. How can you tell how likely an event is? 2. Why are chance devices such as spinners used in some games but not others? 3. What is fair? 4. How would you know a game is unfair? 5. What does it mean to be lucky or unlucky? **Relevance and Application:** 1. The understanding of chance helps to appreciate unlikely events when they occur such as appreciating seeing a rainbow because it is unlikely to happen again soon, or appreciating losing a game of chance 10 times in a row because it is a very unlikely event. **Nature of Mathematics:** 1. Mathematics helps people understand and deal with uncertainty and unfairness in life.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. Visual displays of data can be constructed in a variety of formats

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Construct picture graphs and bar graphs from a data set b. Read and explain information in picture graphs and bar graphs c. Describe data using concepts of median and range d. 2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	• 2.MD.9 added because CO standards do not specify generating and displaying measurement data on line plots. • 2.MD.10 specifies using information presented in a bar graph to solve simple putting together, taking apart and comparing problems. (MA.2.3.1.b), also addressed in MA.3.3.1.c. •	**Inquiry Questions:** 1. What are the ways data can be displayed? 2. How do data displays help us understand information? 3. What can data tell you about the people you survey? 4. What makes a good survey question? **Relevance and Application:** 1. People use data to answer questions and describe the world such as how many classmates are buying lunch today, how much it rained yesterday, or in which month are there the most birthdays. **Nature of Mathematics:** 1. Mathematics can be displayed as symbols.

Content Area: Mathematics

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: Second Grade

Concepts and skills students master:

2. Mathematical models are used to describe the likelihood of an outcome or event

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Collect data using chance devices, such as spinners and describe outcomes as likely or unlikely b. Apply the concepts of likely or not likely to decisions from daily life (PFL)	Note: Concepts of chance and probability are covered extensively in grade 7 and high school. •	**Inquiry Questions:** 1. How can you tell how likely an event is? 2. How do we communicate the likelihood of an event? 3. What does it mean to be lucky or unlucky? **Relevance and Application:** 1. People use the ideas of "likely" and "unlikely" to understand risks found in everyday life such as the chance of injury while crossing the street, losing your gloves, or the chance of tickets to a show being sold out. **Nature of Mathematics:** 1. Resiliency depends on the ability to understand and deal with uncertainty in life.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: First Grade

Concepts and skills students master:

1. Visual displays of data can be created using individual student data

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Contribute individual data to classroom data display b. Read information from picture graphs, bar graphs, and tally charts c. Describe data by applying the concepts of largest, smallest and most often	•	**Inquiry Questions:** 1. What are the ways data can be displayed? 2. How do data displays help us understand information? 3. What kinds of questions generate data? 4. What questions can be answered by a data representation? **Relevance and Application:** 1. People use graphs and charts to communicate information and learn about a class or community such as the kinds of cars people drive, or favorite ice cream flavors of a class. **Nature of Mathematics:** 1. Mathematicians organize and explain random information.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Visual displays of information can used to answer questions

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Collect classroom data b. Identify and compare own data to group's data c. Describe bar graphs to answer questions such as more or less and simple trends	•	**Inquiry Questions:** 1. What can you learn about yourself by looking at data from your whole class? 2. How could you share data without using a data display? **Relevance and Application:** 1. Data helps people learn about themselves and others. For example, I am the only student in my class who has read a certain book, or most students in my class walk to school. **Nature of Mathematics:** 1. Mathematicians observe and identify objects and phenomena.

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Preschool

Concepts and skills students master:

1. Information and objects can be sorted

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use words to describe attributes of objects b. Sort and group similar objects into simple categories c. Contribute to class display of information (charts, graphs)	**Inquiry Questions:** 1. Why do we put things in a group? 2. What is the same about these objects and what is different? 3. What are the ways to sort objects?
	Relevance and Application: 1. Data displays help organize and understand information such as how many blocks in a set are red, blue, or green. 2. Sorting and grouping allows people to organize their world. For example, we set up time for clean up, and play.
	Nature of Mathematics: 1. Mathematicians organize objects in different ways to learn about the objects and a group of objects.

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: High School

Concepts and skills students master:

1. Attributes of two- and three-dimensional objects are measurable and can be quantified

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Calculate (or estimate when appropriate) the perimeter and area of a two-dimensional irregular shape b. Justify, interpret, and apply the use of formulas for the surface area, and volume of cones, pyramids, and spheres including real-world situations c. Solve for unknown quantities in relationships involving perimeter, area, surface area, and volume d. Apply the effect of dimensional change, utilizing appropriate units and scales in problem-solving situations involving perimeter, area, and volume e. G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. f. G-GPE.2 Derive the equation of a parabola given a focus and directrix. g. G-GPE.3 (+) Derive the	• G-GMD.2 specifies using Cavalieri's principle for volume formulas. (MA.HS.4.1.b) • G-GPE.1, 2, 3 added because CO standards do not include conic sections. • G-GMD.4 added because CO standards do not include identifying cross-sections of three-dimensional objects. • G-MG.2 added because CO standards do not include applying geometry to the concept of density. •	**Inquiry Questions:** 1. How are mathematical objects different from the physical objects they model? 2. What makes a good geometric model of a physical object or situation? 3. How might surface area and volume be used to explain biological differences in animals? 4. How is the area of an irregular shape measured? 5. How can surface area be minimized while maximizing volume? **Relevance and Application:** 1. Understanding areas and volume enables design and building. For example, a container that maximizes volume and minimizes surface area will reduce costs and increase efficiency. Understanding area helps to decorate a room, or create a blueprint for a new building. 2. Geometry is used to create simplified models of complex physical systems. Analyzing the model helps understand the system. For example, modeling Earth as a sphere allows us to calculate measures such as diameter, circumference, and surface area. We can also model the solar system, galaxies, molecules, atoms, and subatomic particles.

equations of ellipses and hyperbolas given foci and directrices. h. G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. i. G-MG.2 Apply concepts of density based on area and volume in modeling situations		**Nature of Mathematics:** 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: High School

Concepts and skills students master:

2. Objects in the plane and their parts, attributes, and measurements can be analyzed deductively

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Classify polygons according to their similarities and differences b. Solve for unknown attributes of geometric shapes based on their congruence, similarity, or symmetry c. Know and apply properties of angles including corresponding, exterior, interior, vertical, complementary, and supplementary angles to solve problems. Justify the results using two-column proofs, paragraph proofs, flow charts, or illustrations d. Develop conjectures and solve problems about geometric figures including definitions and properties (congruence, similarity, and symmetry). Justify these conjectures using two-column proofs, paragraph	• G-CO.1 specifies knowing precise definitions of certain geometric objects.(MA.HA.4.2.d) • G-CO.8 includes explaining how criteria for triangle congruence follow from the definition of congruence in terms of rigid motions.(MA.HS.4.2.d) • G-CO.10 specifies theorems about triangles that students should prove.(MA.HS.4.2.d) • G-CO.11 specifies theorems about parallelograms that students should prove.(MA.HS.4.2.d) • G-C.1 specifies proving all circles are similar.(MA.HS.4.2.d) • G-C.5 specifies to derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define radian measure of the angle as the constant of proportionality: derive the formula for the area of a sector. (MA.HS.4.2.d) • G-C.2 added because CO standards do not include identifying and describing relationships among	**Inquiry Questions:** 1. How would the idea of congruency be used outside of mathematics? 2. What does it mean for two things to be the same? Are there different degrees of "sameness?" 3. What makes a good definition of a shape? 4. Do perfect circles naturally occur in the physical world? **Relevance and Application:** 1. Geometry is used to create simplified models of complex physical systems. Analyzing the model helps to understand the system and is used for such applications as creating a floor plan for a house, or creating a schematic diagram for an electrical system.

proofs, flow charts, or illustrations e. G-C.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	circles and relate objects. •	**Nature of Mathematics:** 1. Geometry involves the generalization of ideas. Geometers seek to understand and describe what is true about all cases related to geometric phenomena.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: High School

Concepts and skills students master:

3. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Make conjectures involving two-dimensional objects represented with Cartesian coordinates. Justify these conjectures using two-column proofs, paragraph proofs, flow charts, and/or illustrations b. Represent transformations (reflection, translation, rotation, and dilation) using Cartesian coordinates c. Develop arguments to establish what remains invariant and what changes after a transformation (reflection, translation, rotation, and dilations). Justify these conjectures using two-column proofs, paragraph proofs, flow charts, and/or illustrations d. Using construction tools, including technology, make conjectures about relationships among properties of shapes in the plane including those formed through transformation. Justify these conjectures using two-column proofs, paragraph proofs, flow charts, and/or illustrations	• G-GPE.5 specifies proving the slope criteria for parallel and perpendicular lines. (MA.HS.4.3.a) • G-CO.2 includes describing transformations as functions. (MA.HS.4.3.b) • G-CO.4 includes developing definitions of transformations in terms of angles, circles, perpendicular lines, parallel lines and line segments. (MA.HS.4.3.b) • G-SRT.1a-b emphasizes work with dilations. (MA.HS.4.3.b, c) • G-SRT.2 specifies similarity transformations. (MA.HS.4.3.c) • G-CO.3 specifies describing the transformations that carry a certain figure onto itself (MA.HS.4.3.c) • G-CO.13 specifies objects that students should be able to construct. (MA.HS.4.3.d) • G-C.3 specifies constructing inscribed and circumscribed circles of a triangle and	**Inquiry Questions:** 1. What advantage, if any, is there to using the Cartesian coordinate system to analyze the properties of shapes? 2. What does it mean for two lines to be parallel? How can you physically verify that two lines are really parallel? 3. What happens to the coordinates of the vertices of shapes when different transformations are applied in the plane? **Relevance and Application:** 1. Comprehension of transformations aids with innovation and creation. For example, dilations are used to enlarge or shrink pictures, rigid motions to make new patterns for clothing or architectural design, and transformation applications for computer graphics and animation.

	proving properties of inscribed quadrilaterals. (MA.HS.4.3.d) • G-C.4 specifies constructing a tangent to a circle. (MA.HS.4.3.d) • 8.G.2 Content covered in MA.HS.4.3.c. (see MA.8.4.1) • 8.G.1 Content covered in MA.HS.4.3.d. (see MA.8.4.1) • 8.G.3 Content covered in MA.HS.4.3.d. (see MA.8.4.1) • 8.G.4 Content covered in MA.HS.4.3.d. (see MA.8.4.1) •	**Nature of Mathematics:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: High School

Concepts and skills students master:

4. Right triangles are central to geometry and its applications

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Apply right triangle trigonometry (sine, cosine, and tangent) to find indirect measures of lengths and angles b. Apply the Pythagorean theorem and its converse to solve real-world problems c. Determine the midpoint of a line segment and the distance between two points in the Cartesian coordinate plane d. F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. e. F-TF.8 Prove the Pythagorean identity $\sin^2\theta+\cos^2\theta=1$ and use it to calculate trigonometric ratios. f. F-TF.9 (+) Prove the addition and subtraction	• F-TF.3 specifies using special right triangles. (MA.HS.4.4.a) • G-SRT.6 specifies understanding how trigonometric ratios stem from relationships between similar triangles. (MA.HS.4.4.a) • G-SRT.7 specifies explaining and using the relationship between sine and cosine of complementary angles. (MA.HS.4.4.a) • G-SRT.10 specifies proving the laws of Sines and Cosines. (MA.HS.4.4.a) • G-SRT.11 specifies understanding and applying the laws of Sines and Cosines. (MA.HS.4.4.a) • G-GPE.6 includes finding the point on a segment that partitions the segment in a given ratio. (MA.HS.4.4.c) • F-TF.1 added because CO standards do not include understanding radians.	**Inquiry Questions:** 1. How can you determine the measure of something that you can't measure physically? 2. How a corner square is made? 3. How are mathematical triangles different from built triangles in the physical world? How are they the same? **Relevance and Application:** 1. Knowledge of right triangle trigonometry allows modeling and application of angle and distance relationships such as surveying land boundaries, shadow problems, angles in a truss, and the design of structures.

formulas for sine, cosine, and tangent and use them to solve problems.	• F-TF.8 added because CO standards do not include proving the Pythagorean identity. • F-TF.9 added because CO standards do not include proving the trigonometric formulas. •	**Nature of Mathematics:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Objects in the plane and their parts and attributes can be analyzed

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Classify quadrilaterals and apply angle and side properties, including the sum of the interior angles b. Apply properties of complementary, supplementary, and vertical angle relationships c. Apply properties of parallel lines including corresponding angles and alternate interior angles	• 8.G.1 Content covered in MA.HS.4.3.d. • 8.G.2 Content covered in MA.HS.4.3.c. • 8.G.3 Content covered in MA.HS.4.3.d. • 8.G.4 Content covered in MA.HS.4.3.d. • 5.G.4 Content beyond related CO standards MA.6.4.1.c and MA.8.4.1.a. (see MA.5.4.1.c) • 7.G.5 Content covered in MA.8.4.1.b and c. (see MA.7.4.1) •	**Inquiry Questions:** 1. How can geometric relationships among lines and angles be generalized, described, and quantified? 2. What relationships exist between quadrilaterals? 3. How do line relationships affect angle relationships? 4. Why do we classify shapes? **Relevance and Application:** 1. The understanding of basic geometric relationships helps to use geometry to construct useful models of physical situations such as blueprints for construction, or maps for geography. 2. Development of classification systems based on properties of objects aids in organizing and understanding large groups of objects such as the use of binomial nomenclature to classify living organisms on Earth. **Nature of Mathematics:** 1. Geometric objects are abstracted and simplified versions of physical objects. 2. Geometers describe what is true about all cases by studying the most basic and essential aspects of objects and relationships between objects.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

2. Direct and indirect measurements can be used to describe and make comparisons

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use properties of similar triangles to find unknown lengths b. Use the Pythagorean Theorem to find unknown lengths in right triangles c. Use proportional reasoning to estimate distance, weight, and capacity d. Use proportional reasoning to convert among measures including dimensional analysis e. 8.G.6 Explain a proof of the Pythagorean Theorem and its converse.	• 8.G.7 includes finding distance between two points on a coordinate plane. (MA.8.4.2.b) • 8.G.8 includes finding lengths in 3-D figures. (MA.8.4.2.b) • 8.G.6 added because CO standards do not include explaining a proof of the Pythagorean Theorem. •	**Inquiry Questions:** 1. Why might indirect measurement be considered estimation? 2. When is indirect measurement more appropriate than direct measurement? 3. Why does the Pythagorean Theorem only apply to right triangles? **Relevance and Application:** 1. The understanding of indirect measurement strategies allows measurement of features in the immediate environment such as playground structures, flagpoles, and buildings. 2. Knowledge of how to use right triangles and the Pythagorean Theorem enables design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads. **Nature of Mathematics:** 1. Mathematicians study proportional insight in order to convert and compare.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Objects in space and their parts and attributes can be measured and analyzed

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Develop and apply formulas and procedures for the surface area and volume of right cylinders and right prisms b. Develop and apply formulas and procedures for area of regular polygons, circumference and area of circles, and area of composite figures c. Identify and construct two-dimensional nets of prisms and cylinders d. 7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. e. 7.G.3 Describe the two-dimensional figures that	• 7.G.1 includes solving problems of scale involving geometric figures. (MA.7.4.1.a) • 7.G.2 added because CO standards do not include drawing geometric shapes, with emphasis on triangles. • 7.G.3 added because CO standards do not include describing cross-sections of 3-D geometric figures. • 7.G.5 Content covered in MA.8.4.1.b and c. • 5.MD.5b Content covered in MA.7.4.1.a. (see MA.5.4.2) • 6.G.2 specifies volumes of solids with fractional edge lengths, which may be included in MA.7.4.1.a. (see MA.6.4.1) • 6.G.4 Content covered in MA.7.4.1c. (see MA.6.4.1) •	**Inquiry Questions:** 1. Can two shapes have the same volume but different surface areas? Why? 2. Can two shapes have the same surface area but different volumes? Why? 3. How are surface area and volume like and unlike each other? 4. What do surface area and volume tell about an object? 5. How are one-, two-, and three-dimensional units of measure related? **Relevance and Application:** 1. The ability to find volume and surface helps to answer important questions such as how to minimize waste by redesigning packaging, or understanding how the shape of a room affects its energy use. 2. The application of area measurement of different shapes aids with everyday tasks such as buying carpeting, determining watershed by a center pivot irrigation system, finding the number of gallons of paint needed to paint a room, decomposing a floor plan, or designing landscapes.

result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.		**Nature of Mathematics:** 1. Mathematicians realize that measurement always involves a certain degree of error. 2. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Proportional reasoning is used to make indirect measurements

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Describe the relationship between the circumference and diameter of a circle b. Read and interpret scales on maps c. Use proportions to convert from one set of units to another within customary and metric systems using standard units of measure for length, weight, capacity and time	• 7.G.4 includes deriving the relationship between the circumference and area of a circle. (MA.7.4.2.c) • 5.MD.1 Content covered in MA.7.4.2.c. (see MA.5.4.2) •	**Inquiry Questions:** 1. What's so special about pi? 2. Why did people define standard units of measure? **Relevance and Application:** 1. The ability to convert units allows collaboration around the world. For example, monetary exchange rates allow countries that use different currency to trade goods and collaborate on business ventures; measurement conversion allows countries using different measurement systems to trade goods. 2. When people understand the prefixes in the metric system, all measures in the system can be easily found. For example, knowing that 'kilo' means one-thousand allows an individual to know that a kilogram is 1,000 grams, and a kilowatt is 1,000 watts. **Nature of Mathematics:** 1. Mathematicians recognize commonalities among situations that appear to be quite different. For example, proportions are relevant for solving problems in such varied fields as art, construction, finance, and manufacturing.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Polygons can be described, classified, and analyzed by their attributes

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Develop and apply formulas and procedures for finding area of triangles, parallelograms, and trapezoids b. Describe properties of polygons up to ten sides using accurate vocabulary and notation c. Classify triangles and apply angle and side properties, including the sum of the interior angles d. Use accurate geometric notation to describe angles, lines, and segments e. 6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same	• 6.G.1 includes solving real-world and mathematical problems. (MA.6.4.1.a) • 6.G.2 specifies volumes of solids with fractional edge lengths, which may be included in MA.7.4.1.a. • 6.G.3 added because CO standards do not include drawing polygons in the coordinate plane. • 6.G.4 Content covered in MA.7.4.1.c. • 5.G.4 Content beyond related CO standards MA.6.4.1.c and MA.8.4.1.a. (see MA.5.4.1.c) •	**Inquiry Questions:** 1. What does area tell you about a figure? 2. What properties affect the area of figures? 3. Why do we classify shapes? **Relevance and Application:** 1. Knowledge of how to find the areas of different shapes helps do projects in the home and community. For example how to use the correct amount of fertilizer in a garden, buy the correct amount of paint, or buy the right amount of material for a construction project. 2. Development of a precise vocabulary to describe physical objects leads to clear communication of ideas such as the difference between a circle and a disc, or a triangular prism and a rectangular prism.

first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.		**Nature of Mathematics:** 1. Mathematicians recognize commonalities among objects that appear quite different. For example, a basketball, a planet, the Sun, and an atom can all be modeled as spheres.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Standard units provide common language for communicating measurements

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Connect metric prefixes to place value b. Measure to the nearest sixteenth of an inch c. Select and use appropriate units to accurately measure length, weight, capacity and time in problem-solving situations d. Use a protractor to measure angles to the nearest degree	• 3.MD.2 Content covered in MA.6.4.2.c. (see MA.3.4.2) • 4.MD.6 Content covered in MA.6.4.2.d. (see MA.4.4.1) •	**Inquiry Questions:** 1. How does the metric system relate to our number system? 2. Why doesn't everyone in the world use the same measurement system? Should they? 3. When does precision in measurement matter and why? 4. What are you measuring when you measure an angle? Why doesn't the measure of an angle change as you move further out along the rays of the angle? **Relevance and Application:** 1. The metric system is the system of measurement used in science and many countries. Therefore people must understand the metric system to participate in scientific endeavors and collaborate with people in other countries. 2. Understanding when accuracy is needed and when an estimate will suffice enables efficient use of time and energy. For example, the accurate measurement of expensive materials is essential, but the amount of salt in a recipe may not be. 3. Recognition that the degree of precision in measurements depends on the measurement tool helps to choose the right tool for the job. For example, if accuracy is required, it is best to invest in the right tool. **Nature of Mathematics:** 1. Mathematicians recognize that different situations require different degrees of accuracy. 2. Mathematicians recognize commonalities among objects that appear to be quite different. For example, height is a property of buildings, animals, plants, and ocean waves.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Geometric figures in the plane and in space are described and analyzed by their attributes

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Relate two-dimensional shapes to three-dimensional shapes using faces, edges, and vertices b. Predict and describe the results of transformations: translations, reflections, rotations c. Classify and compare angles d. Apply concepts of parallel, perpendicular, congruence and line symmetry e. 5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	• 5.G.3 added because CO standards do not include understanding attributes of categories of shapes. • 5.G.4 specifies classifying all two-dimensional figures. (MA.5.4.1.c) Content beyond related CO standards MA.6.4.1.c and MA.8.4.1.a. •	**Inquiry Questions:** 1. How do geometric relationships help us solve problems? 2. Is a square still a square if it's tilted on its side? 3. How are three-dimensional shapes different from two-dimensional shapes? 4. What would life be like in a two-dimensional world? 5. Why is it helpful to classify things like angles or shapes? **Relevance and Application:** 1. The understanding and use of spatial relationships helps to predict the result of motions such as how articles can be laid out in a newspaper, what a room will look like if the furniture is rearranged, or knowing whether a door can still be opened if a refrigerator is repositioned. 2. The application of spatial relationships of parallel and perpendicular lines aid in creation and building. For example, hanging a picture to be level, building windows that are square, or sewing a straight seam. **Nature of Mathematics:** 1. Geometry is a system that can be used to model the world around us or to model imaginary worlds.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. Linear measure, area, and volume are fundamentally different and require different units of measure

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Accurately measure length to the nearest 1/8 inch or millimeter b. Determine the perimeter of polygons and area of rectangles c. Distinguish between appropriate units for area and linear measures d. Model volume using cubic units e. Use, apply, and select appropriate scales on number lines, graphs, and maps f. 5.MD.5c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping	• 5.MD.3a, b specify recognizing volume as an attribute of solid objects and understanding the definition of volume. (MA.5.4.2.d) • 5.MD.5a specifies finding the volume of right rectangular prisms with whole number side lengths by packing it with cubes. (MA.5.4.2.d) • 5.MD.5c added because CO standards do not include recognizing volume as additive and finding volume of figures composed of other figures. • 5.MD.1 Content covered in MA.7.4.2.c. • 5.MD.5b Content covered in MA.7.4.1.a. • 3.G.2 relate to MA.3.1.2.a, with content addressed in MA.4.2.2.a and MA.5.4.2.b. • 3.MD.7a, b, c, d relate to MA.3.1.4c, with content	**Inquiry Questions:** 1. What kinds of questions can be answered by measuring? 2. What are the ways to describe the size of an object or shape? 3. How does what we measure influence how we measure? 4. What would the world be like without a common system of measurement? **Relevance and Application:** 1. The use of measurement tools allows people to gather data, organize, and share data to collaborate with others such as sharing results from science experiments, or showing the growth rates of different types of seeds. 2. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods.

parts, applying this technique to solve real world problems.	addressed in MA.4.4.2.a and MA.5.4.2.b. • 4.MD.3 relates to MA.4.4.2.a, b, with content covered in MA.5.4.2.a, c. • 4.MD.2 relates to MA.4.4.2.c, d, e, with some content covered in MA.5.4.2.e. •	**Nature of Mathematics:** 1. Mathematicians use tools and techniques to accurately determine measurement. 2. People use measurement systems to specify attributes of objects with enough precision to allow collaboration in production and trade.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Geometric figures are described by their attributes and specific location in the plane

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Identify parallel, perpendicular, and intersecting line segments in the plane and within geometric shapes b. Create geometric designs using transformations :reflections, translations, and rotations c. Compare geometric figures according to the attributes of congruence, symmetry, and angle size d. Name and locate points specified by ordered number pairs on a coordinate grid	• 4.G.2 includes classifying figures based on parallel or perpendicular lines and angle size; and identifying right triangles. (MA.4.4.1.a, c) • 4.MD.5a and b emphasize understanding concepts of angle measurement with reference to a circle, considering the fraction of the circular arc, and relating "one-degree angles" to determine the measure of an angle that turns through *n* one-degree angles. (MA.4.4.1.c) • 4.MD.7 includes recognizing angle measure as additive, and solving addition and subtraction problems to find unknown angles on a diagram. (MA.4.4.1.c) • 4.MD.6 Content covered in MA.6.4.2.d. • Note: Concepts of transformations are covered extensively in grade 8 and high school content.	**Inquiry Questions:** 1. What are the ways to compare and classify geometric figures? 2. Why do we classify shapes? 3. How does using a coordinate grid help in the study of shapes and their relationships? 4. Do lines exist in the world around us? 5. What shapes fit together well to make designs? **Relevance and Application:** 1. The coordinate grid is a basic example of a system for mapping relative locations of objects. It provides a basis for understanding latitude and longitude, GPS coordinates, and all kinds of geographic maps. 2. Symmetry is used to analyze features of complex systems and to create works of art. For example, symmetry is found in living organisms, the art of MC Escher, and the design of tile patterns, and wallpaper.

	• 3.G.1 relates to MA.3.4.1.a, with content covered in MA.4.4.1.c. •	**Nature of Mathematics:** 1. Geometry's attributes give the mind the right tools to consider the world around us.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

2. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Model area using square units b. Distinguish between area and perimeter c. Convert using unit equivalencies within the standard measurement system (yards to feet and feet to inches, pounds to ounces, gallons to quarts) d. Convert using unit equivalencies within the metric measuring system (meters to centimeters, kilometers to meters, and liters to milliliters) e. Estimate and measure elapsed time to the nearest quarter hour f. Select an appropriate tool and unit for measuring length, weight, and capacity	• 4.MD.3 specifies applying the area and perimeter formulas for rectangles. (MA.4.4.2.a, b) Content covered in MA.5.4.2.a, c. • 4.MD.1 includes kilograms and grams. (MA.4.4.2.d) • 4.MD.2 specifies solving word problems involving a variety of measurements, including time and money, and involving fractions and decimals, using number line diagrams featuring a measurement scale. (MA.4.4.2.c, d, e) Some content covered in MA.5.4.2.e. • 2.G.2 Content related to MA.2.4.1.a is covered in MA.4.4.2.a. • 3.MD.5 relates to MA.3.4.1.a, with concepts covered in MA.4.4.2.a. • 3.MD.6 relates to MA.3.4.2.IQ.2, with content	**Inquiry Questions:** 1. How do you decide when close is close enough? 2. How can you describe the size of geometric figures? 3. Why do we need standard units of measure? 4. Why do we measure time? **Relevance and Application:** 1. Accurate use of measurement tools allows people to create and design projects around the home or in the community such as flower beds for a garden, fencing for the yard, wallpaper for a room, or a frame for a picture. 2. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods.

	covered in MA.4.4.2.a. • 3.MD.7a, b, c, d relate to MA.3.1.4c, with content addressed in MA.4.4.2.a and MA.5.4.2.b. • 3.G.2 relates to MA.3.1.2.a, with content addressed in MA.4.2.2.a and MA.5.4.2.b. • 3.MD.8 relates to MA.3.4.2.b, and content may be covered in MA.4.4.2.a, b. • 3.MD.1 relates to MA.3.2.1.RA.1, and may be addressed in MA.4.4.2.e. •	**Nature of Mathematics:** 1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Geometric figures are described by their attributes and position in the plane

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Construct and describe two-dimensional shapes by attributes and properties such as sides, angles, and symmetry b. Recognize and demonstrate transformations – reflections, translations, and rotations – of basic shapes or designs c. Use geometric properties of points and line segments to describe figures	• 3.MD.5 specifies recognizing area as an attribute of plane figures and understanding concepts of area measurement. (MA.3.4.1.a) Concepts covered in MA.4.4.2.a) • 3.G.1 specifies understanding that shapes in different categories may share attributes which can define a larger category, and recognizing examples and non-examples of special quadrilaterals. (MA.3.4.1.a; MA.3.4.1.IQ.3) Content covered in MA.4.4.1.c; MA.4.4.1.IQ.1) • Note: Concepts of transformations are covered extensively in grade 8 and high school content. •	**Inquiry Questions:** 1. What words in geometry are also used in daily life? 2. Why are there so many different words used in geometry? 3. How can you describe geometric figures? **Relevance and Application:** 1. Recognition of geometric shapes and transformations allows people to describe and change their surroundings such as creating a work of art using geometric shapes, or design a pattern to decorate. 2. Points, lines, and planes are the concepts that form the basis of Euclidian geometry. 3. Visualizing the motion of objects in space is a basic skill that allows people to imagine new ways of designing and organizing objects in space such as re-arranging a room to be more comfortable, designing an interface to be easier to use, or imagining new designs for buildings, sculptures, or furniture. **Nature of Mathematics:** 1. Geometry expresses dimension.

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Objects have distinct attributes that can be measured with appropriate tools

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Use standard units to measure to the nearest 1/2 or whole inch or centimeter b. Estimate and measure distance and perimeter	• 3.MD.2 includes measuring and estimating liquid volumes in grams, kilograms, and liters, and solving related one-step word problems. (MA.3.4.2.IQ.2) Content covered in MA.6.4.2.c. • 3.MD.6 specifies measuring areas using unit squares, including improvised units. (MA.3.4.2.IQ.2) Content covered in MA.4.4.2.a. • 3.MD.7a, b, c, d relate to area multiplication and addition, including developing the area relationship with sides of rectangles, using tiling to show area of a rectangle with sides *a* and *b+c* illustrating the distributive property, and showing area as additive by decomposing figures into non-overlapping rectangles (related to area models of multiplication in MA.3.1.4.c). Content addressed in MA.4.4.2.a and MA.5.4.2.b. • 3.MD.8 involves solving problems with perimeters of	**Inquiry Questions:** 1. How important is precise measurement? How close is close enough? 2. Why are there different units to measure different things? 3. Does everyone in the world agree on the length of an inch? **Relevance and Application:** 1. Measurement helps people make sense of the world and make daily decisions. For example, Is it cold enough to wear a jacket? When is it recess? What is the weight of a fair share of candies? 2. People use estimates of distance and perimeter to describe places and things in the community. For example, About how far is it to Denver or to the store? About how far is it around the lake? 3. People use systems of measurement to trade with other people. For example, my pumpkin weighs two pounds and yours only weighs one pound. Calculate if it is fair to trade one of my pumpkins for two of your pumpkins.

	polygons, including finding missing side lengths and finding rectangles with the same perimeter and different areas or with the same area and different perimeters. (MA.3.4.2.b) Content may be covered in MA.4.4.2.a, b. • 3.G.2 specifies partitioning shapes into equal parts and expressing the area of each part as a unit fraction of the whole. (related to MA.3.1.2.a) Content addressed in MA.4.2.2.a and MA.5.4.2.b. •	**Nature of Mathematics:** 1. Mathematicians use tools and techniques to accurately determine measurement. 2. Mathematicians use measurable attributes to describe countless objects.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. Shapes can be created and described by quantifiable attributes

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Recognize, describe, and create geometric figures according to given quantifiable attributes such as number of sides and size b. Identify symmetry in two-dimensional figures c. Use quantifiable attributes to describe and estimate size of objects	• 2.G.1 specifies drawing shapes having specific attributes, such as a given number of angles. (MA.2.4.1.a) • 2.G.2 specifies partitioning a rectangle into rows and columns of same-size squares and counting to find the total number of them. (MA.2.4.1.a) Content covered in MA.4.4.2.a. •	**Inquiry Questions:** 1. How can we describe geometric figures? 2. What makes this shape have symmetry? 3. What can you use to describe the size of this object? 4. Is it easier to draw shapes or describe them with words? **Relevance and Application:** 1. Representation of ideas through drawing is an important form of communication. Some ideas are easier to communicate through pictures than through words such as the idea of a circle, or an idea for the design of a couch. 2. Symmetry is used analyze features of complex systems and to create works of art. For example, symmetry is found in living organisms, the art of MC Escher, and the design of tile patterns, and wallpaper. **Nature of Mathematics:** 1. Geometry provides a system to describe, organize, and represent the world around us.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Second Grade

Concepts and skills students master:

2. Some attributes of objects are measurable and can be quantified using different tools

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Identify the measurable attribute and appropriate unit of measure for an object b. Use common objects as non-standard units c. Use standard linear measuring tools to measure to the nearest whole unit d. Identify common units of time, weight, and temperature and their appropriate use	• 2.MD.2 specifies measuring an object's length using two different units of measure, and describing how the measurements relate to the units. (MA.2.4.2.c) • 2.MD.3 specifies estimating lengths in inches, feet, centimeters, and meters. (MA.2.4.2.c, d) • 2.MD.7 specifies telling and writing time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (MA.2.4.2.d) •	**Inquiry Questions:** 1. What are the different things we can measure? 2. How do we decide which tool to use to measure something? 3. If you think something is heavy, will everyone else agree? 4. If you find the length of a book is eight inches, will everyone else agree? 5. What would happen if everyone created and used their own rulers? **Relevance and Application:** 1. Measurement is used to understand and describe the world including sports, construction, and explaining the environment. **Nature of Mathematics:** 1. Mathematicians use measurable attributes to describe countless objects with only a few words.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: First Grade

Concepts and skills students master:

1. Shapes can be created and described by composing and decomposing

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Recognize, describe, and make shapes according to given relationships, attributes, or properties b. Sort geometric figures and describe how they are alike and different c. Combine and take apart shapes to create new shapes and describe results	• 1.G.2 specifies composing three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape (MA.1.4.1.c)	**Inquiry Questions:** 1. How can we describe geometric figures? 2. How are these shapes alike and how are they different? 3. Can you make this shape with other shapes? 4. How can we organize all the objects in the room into groups? 5. What shapes can be combined to create a square? 6. What shapes can be combined to create a circle? **Relevance and Application:** 1. Many objects in the world can be described using geometric shapes and relationships such as architecture, objects in your home, and things in the natural world. Geometry gives us the language to describe these objects. 2. Representation of ideas through drawing is an important form of communication. Some ideas are easier to communicate through pictures than through words such as the idea of a circle, or an idea for the design of a couch. **Nature of Mathematics:** 1. Geometers use shapes to represent the similarity and difference of objects.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: First Grade

Concepts and skills students master:

2. Measurement is used to compare and order objects and events

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Measure the length of common objects using nonstandard units such as created units, popsicle sticks, or paper clips b. Compare and order objects by length and weight c. Distinguish units of time (day, night, morning, afternoon, hours) and connect them to common events d. Compare and order units of time	• 1.MD.3 specifies telling and writing time in hours and half-hours using analog and digital clocks. (MA.1.4.2.c) •	**Inquiry Questions:** 1. How can you tell when one thing is bigger than another? 2. Why do we measure objects and time? 3. How are length and time different? How are they the same? **Relevance and Application:** 1. Time measurement is a means to organize and structure each day and our lives, and to describe tempo in music. 2. Measurement helps to understand and describe the world such as comparing heights of friends, or describing how heavy something is, or how much something holds. **Nature of Mathematics:** 1. With only a few words, mathematicians use measurable attributes to describe countless objects.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Shapes are described by their characteristics and position

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Recognize and informally describe two dimensional shapes with varying orientation, sizes, and shapes b. Use relational vocabulary, such as above, below and next to, to describe spatial relationships	•	**Inquiry Questions:** 1. How are shapes like each other? How are they different? 2. What are ways to describe where an object is? 3. Where do you see shapes in the environment? 4. What are all the things you can think of that are round? What is the same about these things? **Relevance and Application:** 1. Shapes help people describe the world. For example, a box is a cube, the Sun looks like a circle, and the side of a dresser looks like a rectangle. 2. People communicate where things are by their location in space using words like next to, below, or between. **Nature of Mathematics:** 1. Geometry helps discriminate one characteristic from another. 2. Geometry clarifies relationships between and among different objects.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Kindergarten

Concepts and skills students master:

2. Measurement is used to compare and order objects

Evidence Outcomes	CCS Notes	21st Century Skills and Readiness Competencies
Students can: a. Recognize and compare attributes of length, height, weight, capacity of objects b. Use estimates of measurements from everyday experiences c. Order several objects by length, height, weight, capacity, or price (PFL) d. K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.	• K.G.4 Content covered in MA.1.4.1.b. • K.G.5 Content covered in MA.1.4.1.a. • K.G.6 Content covered in MA.1.4.1.a, c. • KMD.3 added because CO standards do not include classifying objects and sorting categories in this way. •	**Inquiry Questions:** 1. How can you tell when one thing is bigger than another? 2. How is height different from length? 3. How is weight different from capacity? **Relevance and Application:** 1. Measurement helps to understand and describe the world such as in cooking, playing, or pretending. 2. People compare objects to communicate and collaborate with others. For example we describe items like the long ski, the heavy book, the expensive toy. **Nature of Mathematics:** 1. A system of measurement provides a common language that everyone can use to communicate about objects.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Preschool

Concepts and skills students master:

1. Shapes can be observed in the world and described in relation to one another

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Match, sort, group and name basic shapes found in the natural environment b. Follow directions to arrange, order, or position objects	**Inquiry Questions:** 1. How do we describe where something is? 2. Where do you see shapes around you? 3. How can we arrange these shapes?
	Relevance and Application: 1. Shapes and position help students describe and understand the environment such as in cleaning up, or organizing and arranging their space. 2. Comprehension of order and position helps students learn to follow directions. 3. Technology games can be used to arrange and position objects.
	Nature of Mathematics: 1. Geometry affords the predisposition to explore and experiment.

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Preschool

Concepts and skills students master:

2. Measurement is used to compare objects

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe the order of common events b. Group objects according to their size using standard and non-standard forms (height, weight, length, or color brightness) of measurement c. Sort coins by physical attributes such as color or size (PFL)	**Inquiry Questions:** 1. How do we know how big something is? 2. How do we describe when things happened?
	Applying Mathematics in Society and Using Technology: 1. Understanding the order of events allows people to tell a story or communicate about the events of the day. 2. Measurements helps people communicate about the world. For example, we describe items like big and small cars, short and long lines, or heavy and light boxes.
	Nature of Mathematics: 1. Mathematics involves pattern seeking. Mathematicians look for patterns and regularity. The search for patterns can produce rewarding shortcuts and mathematical insights.

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