Title: The Beliefs And Attitudes Of Special Educators: Mathematics, Mathematics Teaching, And Mathematics Learning

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Abstract/Summary:

The purpose of the study was to explore the beliefs and attitudes of special education teachers have about the discipline of mathematics, teaching mathematics, and learning mathematics. The study utilized a mixed method design that was conducted in two phases. Forty-eight in-service special education teachers participated in Phase One of the study, which consisted of quantitative data collection through surveys related to mathematics anxiety level and alignment of beliefs with the National Council of Teachers of Mathematics (NCTM) *Standards*. A sub-sample of seven teachers was purposefully selected to participate in Phase Two of the study, which consisted of a qualitative data collection through a semi-structured interview. Quantitative results indicated that the study sample had relatively low levels of mathematics anxiety and a relatively high degree of alignment with reform-based mathematics beliefs promoted by the NCTM. Qualitative results expanded upon the quantitative results of Phase One of the study and indicated that the beliefs of the sub-sample participants could be categorized according to beliefs common to general education mathematics teachers.

Subject/Keywords: Mathematics teaching, special education, National Council of Teachres of Mathematics Standards, mathematics anxiety

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THE BELIEFS AND ATTITUDES OF SPECIAL EDUCATORS: MATHEMATICS, MATHEMATICS TEACHING, AND MATHEMATICS LEARNING

by

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The Beliefs and Attitudes of Special Educators:
Mathematics, Mathematics Teaching, and Mathematics Learning

Thesis directed by Professor Deanna Sands

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The form and content of this abstract are approved. I recommend its publication.

Approved: Deanna Sands

DEDICATION

I dedicate this work to my husband, Mark Colsman, whose support, encouragement, and love allowed me to persevere through this long marathon.

I also dedicate this work to my father, Bill Bunney, and grandmother, Angela Bunney. Dad and Grandma, your love and faith in me are a constant source of strength and are the foundation for all of my successes.

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INTRODUCTION

Statement of the General Problem

Achievement in mathematics is essential for entry into postsecondary education (Conley, 2005) and is highly related to future earning potential (Rose & Betts, 2004). In the 21st century, a high level of mathematics and science knowledge is needed even for jobs that do not require a college education (Evan, Gray, & Olchefske, 2006). However, students with specific learning disabilities (SLD) have characteristics that make learning mathematics difficult (Miller & Mercer, 1997) and are at much greater risk for academic failure in mathematics than their grade level peers (Cortiella, 2011). The underperformance of students with SLD in mathematics has the potential to influence the ability of students with SLD to access desirable postsecondary options, resulting in fewer economic opportunities and diminished quality of life. Despite having gaps in achievement, students with SLD have the intellectual ability to succeed in school (Gresham & Vellutino, 2010).

Tracing the path of mathematics achievement for students with SLD reveals a disturbing downward trend as these students progress through the U.S. education system. National mathematics achievement data (National Center for Education Statistics, 2009) show that only 19% of 4th grade students with SLD are proficient in mathematics. By 8th grade, these data are even worse with only 9% of students with SLD scoring as proficient. High school students with SLD have been shown to perform at levels equivalent to third graders without disabilities in computational fluency and significantly low on other measures of mathematics proficiency (Calhoon, Emerson, Flores, & Houchins, 2007). This evidence suggests that the mathematics achievement of students with SLD is not sufficient for entry into universities. Data from Colorado's ACT program (Colorado Department of Education, 2011) reveal that students with learning disabilities do not attain minimum mathematics scores necessary for college admission. It is therefore not surprising that students with disabilities participate in postsecondary education at rates significantly lower than their peers (Wagner, Newman, Cameto, Garza, & Levine, 2005). Even more alarming than the low participation of students with SLD in postsecondary education are data on high school completion for these students. Recent data from the U. S. Department of Education indicate the dropout ratio for 16- through 24-year olds with disabilities is twice that of their peers, 15.5% compared with 7.8% (Chapman, Laird, Ifill, & Kewal-Ramani, 2011).

The Context of the Problem

Despite their capacity to learn, students with SLD face challenges learning mathematics. Thus, the quality of the mathematics instruction that students with SLD receive is critical to their success. Students with SLD receive special education services, which in simple terms, frame the *what*, *where*, and *how* of mathematics instruction for students with disabilities (Zigmond, Kloo, & Volonino, 2009). *What* refers to the curriculum students with disabilities are taught, *where* is the classroom in which students with disabilities are served, and *how* refers to the instructional techniques used for students with disabilities. Implicit in the components identified by Zigmond, Kloo, and Volonino (2009) is *who,* the special educator who is responsible for making decisions about content, instructional delivery models, and instructional techniques.

Special educators have played a prominent role in the education of students with disabilities since the enactment of the Education for the Handicapped Act (EHA) of 1970, which was designed to create support for students with disabilities through the creation of preparation programs for special educators (Katsiyannis, Yell, & Bradley, 2001). Changes to special education legislation over the past three decades have impacted the *what*, *where*, and *how* of services for students with disabilities, with each change also impacting the role of special educators in the education of students with disabilities. Today special educators find their role in mathematics instruction for students with disabilities either as the sole provider of instruction in a separate pullout mathematics class or as delivering support to students placed in a general education class. Regardless of *where* services are provided, the responsibility of special educators to support students with learning disabilities and spur adequate growth in mathematics is critical.

In an effort to improve mathematics achievement for all students, general education researchers have paid extensive attention to improving mathematics instruction. A major reform effort in mathematics was initiated in the 1990s when the National Council of Teachers of Mathematics (NCTM) published the first-ever national standards titled the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989). The National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics*, or the NCTM *Standards* as they became known, triggered a wave of reform in mathematics education (Battista, 1994) that was focused on "increasing conceptual learning, problem solving, and effective mathematical communication for all students" (Simon, 2008, p. 17) and challenged the notions about mathematics, and the teaching and learning thereof, among educators, mathematicians, policy makers, and the public at large (Battista, 1994; Schoenfeld, 2004). The NCTM *Standards* have served as the centerpiece for changes in curriculum and instruction since their publication (Woodward, 2004), yet special educators have tended to have little knowledge or understanding of the *Standards* (Maccini & Gagnon, 2002, 2006). When understanding of the NCTM *Standards* exists, special educators have been wary or critical of the NCTM *Standards* and their applicability to students with learning disabilities (Miller & Hudson, 2007).

As mathematics educators and professional developers sought to support teachers in shifting their instructional practice to align with the reform-based mathematics standards promoted by the NCTM, a line of research emerged that focused on the role of beliefs and attitudes in the mathematics instructional practice of general education teachers. Beliefs research is based on the premise that teacher attitude and beliefs must be addressed if the mathematics instruction experienced by students is to be changed or improved. Thompson (1984) summarized the role of teacher beliefs as a lever for affecting change in mathematics instruction:

> If teachers' characteristic patterns of behavior are indeed a function of their views, beliefs, and preferences about the subject matter and its teaching, then any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by the teachers and how these are related to their instructional practice. (p. 135)

Understanding teacher beliefs and attitudes about mathematics is important for two reasons in improving instruction and outcomes for all students. First, the beliefs teachers hold about the nature of the discipline of mathematics has been hypothesized to affect how teachers portray the discipline through teaching and the assumptions they hold about learning (Ernest, 1989b). For example, according to Ernest's hypothesis, teachers who believe mathematics to be a dynamic discipline centered on solving problems approach teaching differently than a teacher who believes mathematics to be a body of procedures to be mastered and formulas to be memorized. Secondly, teacher attitudes about mathematics have the potential to impact student attitudes and subsequent achievement in mathematics. Teachers' positive attitude toward mathematics has been shown to correlate to student achievement in mathematics (Schofield, 1981). Conversely, Geist (2010) contended that "many teachers who have math anxiety themselves inadvertently pass it on to their students" (p. 29).

Miller and Hudson (2007) noted that "for students with learning disabilities (LD), mathematics is one of the most challenging aspects of the school curriculum" (p. 47). Not only is mathematics academically challenging for students with SLD, mathematics is often associated with negative emotions. Stodolsky (1985) suggested that many students and adults perceive mathematics as difficult, becoming anxious about mathematical activities and disliking the subject. Further, avoidance of mathematics is seen as socially acceptable, "The idea that you are or are not good at math is readily accepted among adults, whereas such distinctions are not made in other fields such as reading, English, or social studies" (Stodolsky, 1985, p. 131).

On this basis, the mathematics beliefs and attitudes of special educators have the potential to positively or negatively influence students with SLD. In order to positively affect the experiences and achievement of students with SLD, an understanding of the beliefs and attitudes of special educators may prove critical. But despite the potential importance of understanding the beliefs and attitudes of special education teachers, and the important role that beliefs and attitudes may play in mathematics education of at risk students, little research has been conducted in this area. Beliefs research in mathematics has focused almost exclusively on general education teachers, with the vast majority of studies exploring the beliefs and attitudes of prospective elementary teachers. A gap in the literature exists related to the mathematics beliefs and attitudes of special educators, who work with the population of students whose achievement is most in need of attention, students with SLD. This study is therefore intended to add to the literature in an area that has largely been ignored.

Purpose and Significance of the Study

The goal of this study was to understand the complex phenomena of special education teachers' beliefs and attitudes about mathematics and the teaching and learning of mathematics. The research question for the study was: "What is the nature of the beliefs and attitudes held by special educators about the discipline of mathematics and the teaching and learning of mathematics?" The research question was explored through the four domain questions: (a) what are the attitudes of special educators about mathematics, (b) what are the beliefs of special educators about the discipline of mathematics, (c) what are the beliefs of special educators about teaching mathematics, and (d) what are the beliefs of special educators about learning mathematics?

Given the scarcity of research related to the mathematics beliefs and attitudes of special educators, the study has the potential to shed light on the unique support special educators may need in order to provide effective mathematics instruction to students. Development of positive student attitudes towards mathematics requires supportive classroom climate (Haladyna, Shaughnessy, & Shaughnessy, 1983). Ensuring a supportive classroom climate is a concern if the teacher charged with providing instructional assistance to students is mathematically anxious. Finally, although a student's view of mathematics is influenced through a number of factors,

> A major influence is undoubtedly the pupil's own experience of learning mathematics…Perhaps most notable are the nature of the class activities the pupil engages in, the choice of methods of solution which is permitted and the teacher's communicated attitude to errors and to mathematical truth. (Ernest, 1989a, p. 558)

There is an urgent need to keep students engaged in mathematics courses in order to impact their mathematics achievement (Bozick, Ingels, & Owings, 2008). A factor in that engagement relates to the quality of mathematics instruction students receive, which in turn is influenced by the beliefs and attitudes of their teachers.

Design

The goal of this study was to understand the complex phenomenon of beliefs and attitudes of special education teachers. The research objective was exploration, that is, to "generate information about unknown aspects of a phenomenon" (Teddlie & Tashakkori, 2009, p. 25). Exploratory research is distinguished from explanatory research which seeks to test hypotheses and theories (Johnson & Christensen, 2008). The exploratory nature of the research questions that guided the present study is consistent with mixed method study design. Understanding the phenomena of beliefs and attitudes is inherently complex and subjective, and as such, requires an inquiry methodology that explores multiple sources and types of data, both quantitative and qualitative. Quantitative data allows measurement of the phenomena while qualitative data allows for investigation into the meaning of the data (Newman, Ridenour, Newman, & DeMarco, 2003). Thus, the rationale for employing mixed research methods in the study was significance enhancement, which permits the researcher to expand the interpretation of findings from qualitative and quantitative strands of a study and thereby enhance, compare, and clarify across methods (Collins, Onwuegbuzie, & Sutton, 2006).

The study was conducted in two phases, one quantitative and one qualitative, both designed to answer the primary research question plus four domain-related questions. Phase One was designed to collect data on the attitudes and beliefs of special educators about mathematics using survey instruments. Out of the full study sample from Phase One a smaller sub-sample was identified for Phase Two qualitative data collection through a semi-structured interview. Data from Phase Two participants' interviews enhanced the quantitative data from the first phase of the study in order to more fully understand the attitudes and beliefs of special educators about mathematics, teaching mathematics, and learning mathematics.

The next section explicates the conceptual framework of the study, discussing the hypothesized impact of teacher mathematics beliefs and attitudes on mathematics instruction.

Conceptual Framework

Delving into the beliefs teachers hold about the discipline of mathematics and beliefs about how mathematics is learned and should be taught has been seen as a way to improve mathematics instruction (Pajares, 1992; Thompson, 1992). As Perry, Ngai-Ying, and Howard (2006) noted, "All teachers of mathematics hold beliefs about mathematics learning and teaching ….These beliefs influence and guide teachers in their decision making and implementation of teaching strategies" (p. 436). Peterson, Fennema, Carpenter, and Loef (1989) stated the case even stronger, "Teachers' beliefs, knowledge, judgments, thoughts, and decisions have a profound effect on the way they teach as well as on students' learning in their classrooms" (p. 2). The goal of beliefs research is to effect positive change in instructional practices of mathematics teachers (Beswick, 2006; Lerman, 1990; McLeod, 1999; Pajares, 1992).

The Relationship Between Teachers' Beliefs and Instructional Practice

The relationship between teacher beliefs and instructional practice in mathematics frequently begins with a question about the nature of mathematics itself. It would seem logical that the instructional decisions made by a teacher who believes mathematics to be about accurate and efficient execution of procedures would look different from those of a teacher who conceives of mathematics as the practice of solving meaningful, real-world problems. In elucidating the connection between a teacher's conceptions of mathematics and instructional practice, Ernest (1989) developed a conceptual framework (Figure I.1), which suggested that a teacher's instructional practice begins with a personal philosophy of mathematics and what it means to do mathematics, which in turn influences their conceptions about teaching and learning mathematics. Ernest distinguished between espoused beliefs and enacted beliefs, suggesting that espoused beliefs are filtered through the constraints and opportunities afforded by the social context and realities of teaching, which translate them into enacted beliefs and classroom practices.

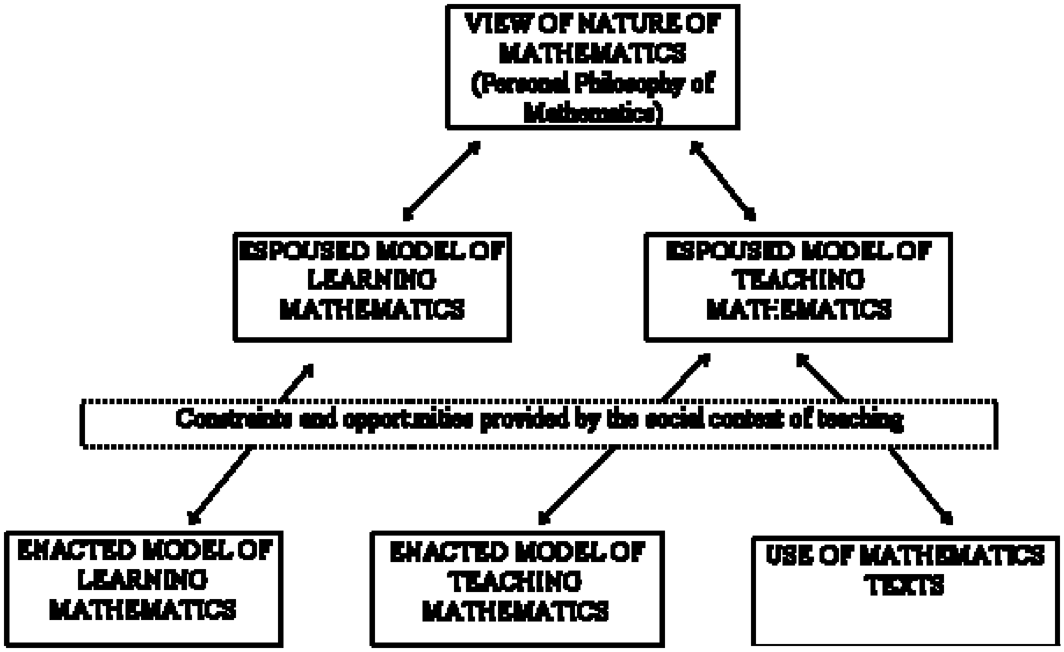

Figure I.1. Conceptual Framework Relating the Beliefs of Mathematics Teachers to Teaching Practices (Ernest, 1989).

Two-way arrows within the framework illustrate how teaching experience has the potential to change one's beliefs. For example, a teacher may believe that the nature of mathematics to be an accumulation of rules and procedures. The belief implies a model of teaching mathematics that emphasizes demonstrating procedures to students and a model of learning mathematics as practicing procedures. In the reality of the classroom context, however, the teacher may find that some students do not understand why procedures work or cannot relate procedures to real-world problems. The disequilibrium created by the teachers' espoused views and the reality of teaching may cause the teacher to reconsider approaches to teaching and broaden his or her view of mathematics to be more than procedures. The teacher may determine that students should be able to apply their mathematical knowledge in real-world contexts leading to an altered view of the nature of mathematics.

In addition to beliefs, teachers bring their attitudes about the subject area into their teaching. In mathematics, attitudes or emotions can be quite strong, to the point of an intensely negative response known as mathematics anxiety (MA). People who suffer from MA report emotions ranging from discomfort to panic when faced with mathematics tasks (Aiken & Dreger, 1961; Ashcraft, 2002; Ho et al., 2000). MA results in mathematics avoidance (Ashcraft & Krause, 2007; Ashcraft & Ridley, 2005; Hembree, 1990) and is related to lower achievement in mathematics (Zakaria & Nordin, 2008). Given the attribution sufferers of MA give to their classroom experiences learning mathematics, the effect of teachers who have MA on their students has been of interest to researchers (Brady & Bowd, 2005).

Ernest's framework suggests how beliefs a teacher holds about the nature of mathematics relates to beliefs about mathematics is learned and should be taught; however, it does not include teacher attitude. Thus, the dimension of attitude is an addition to the conceptual framework for the study.

Mathematics Anxiety as a Mediating Factor in Instructional Practice

Dislike and fear of mathematics is well documented (Beilock, 2008; Geist, 2010; Ho, et al., 2000; Meece, Wigfield, & Eccles, 1990; Shannon & Allen, 1998; Tobias, 1991). For instance, the Math Anxiety Bill of Rights by Davis cited in Tobias (1991) includes as one of its tenets, "I have the right to dislike math". When dislike of the discipline is intense, the emotion causes "self-interference" that inhibits one's performance in mathematics (Shannon & Allen, 1998). Negative emotions and fear of mathematics is of concern to mathematics education because "When negative math and science perceptions are formed, the student's potential to achieve favorably in these subjects is compromised" (Ghee & Khoury, p. 353).

The study of MA emerged in the 1950s originally related to test anxiety (Hembree, 1990). A meta-analysis conducted by Hembree (1990) provided foundational information on the nature and experience of MA. Among the findings were a relationship between MA and both diminished performance in mathematics and the avoidance of mathematics. Hembree (1990) also found that prospective elementary educators experience MA to a much greater degree than other college majors. The potential of teachers' MA to impact student learning has prompted a number of studies learn more about the prevalence and impact of MA in elementary teachers (Austin, Wadlington, & Bitner, 1992; Beilock, Gunderson, Ramirez, & Levine, 2010; Bursal & Paznokas, 2006; Malinsky, Ross, Pannells, & McJunkin, 2006; Swars, Daane, & Giesen, 2006; Wood, 1988). Interestingly, similar studies with special education teachers have not been conducted.

Teachers' positive attitude toward mathematics has been shown to correlate to student achievement in mathematics (Schofield, 1981). Conversely, Geist (2010) contended that "many teachers who have math anxiety themselves inadvertently pass it on to their students" (p. 29). The hypothesized cycle of negative teacher affect and negative student affect may provide an added dimension to Ernest's (1989) conceptual framework.

The conceptual framework for the present study used as its base the Ernest (1989) framework with attitude toward mathematics as an added mediating factor (Figure I.2). To illustrate the addition of attitude, we refer back to the scenario of the teacher who finds the reality of the classroom context mediating his or her beliefs about teaching and learning mathematics. A teacher with MA may have less flexibility in his or her beliefs about the nature of mathematics due to the debilitating emotional impact of the anxiety.

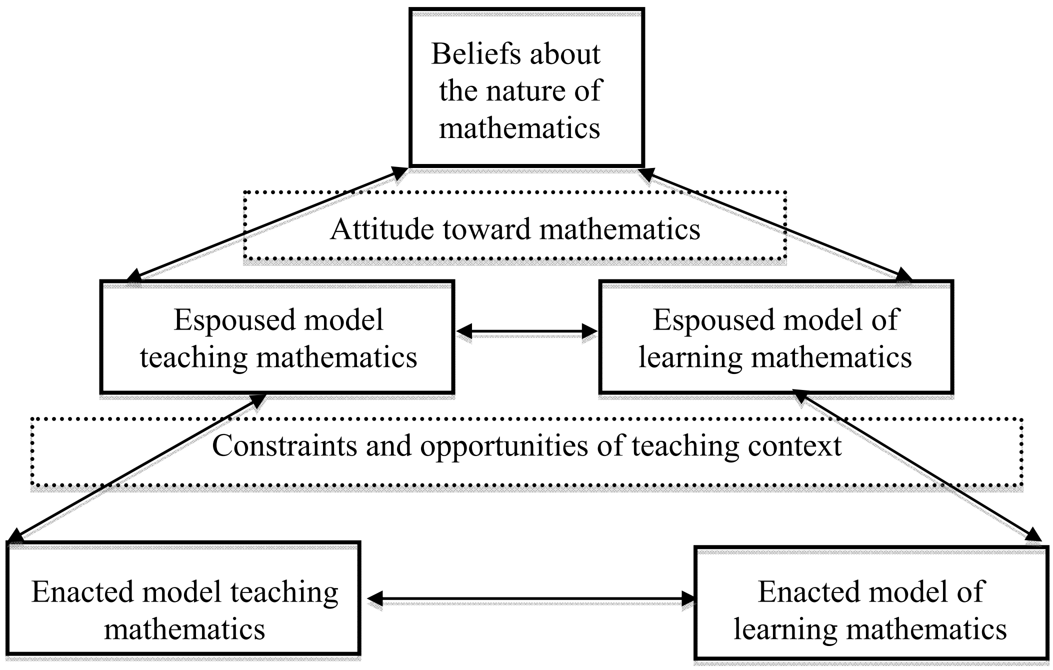

Figure I.2. Conceptual Framework Relating Mathematics' Teacher Beliefs to Teaching Practices With Attitude Toward Mathematics as a Mediating Factor (adapted from Ernest, 1989).

Whereas MA in elementary teachers has been found to correlate to reduced confidence to teach mathematics (Bursal & Paznokas, 2006), the relationship between teacher MA and their beliefs about mathematics is an area with little research. However, in a study of pre-service elementary teachers, Swars, Daane, and Giesen (2006) found that the participants with low MA expressed different perceptions of the nature of mathematics than participants with high MA. Participants with low MA reported mathematics as problem solving and play whereas participants with high MA discussed mathematics as procedural knowledge and memorization.

The relationship between MA and beliefs about the nature of mathematics implied in the previously mentioned studies informed the present study. Of interest to the present study was the role MA plays in special educator beliefs about the nature of mathematics, how mathematics is best learned, and how to teach mathematics.

Conclusion

The evidence is clear that students with SLD are not achieving at levels that are needed for success in postsecondary education and the workforce. Despite efforts to improve overall mathematics achievement in the U. S., persistent gaps exist between U. S. students and their international counterparts and between U. S. students with and without SLD. Special educators are at the nexus of decades of education reforms in the U.S. Reform efforts in special education have resulted in special educators taking on a more prominent role in direct instructional services for students with SLD. At the same time, reform efforts in mathematics have resulted in approaches to teaching and learning mathematics that may challenge the beliefs and attitudes special educators hold.

The research questions for this study are relevant due to a gap in the literature with respect to special education teachers. It is vital that educators and policy makers find ways to improve the mathematical learning experience of students with SLD. The relationships between teacher attitudes and beliefs to instructional practice are worth exploring with special educators who work with the most at risk population, students with SLD. One lever of change may be found in the mathematics attitudes and beliefs of the special educators that serve these at risk students.

CHAPTER 2

REVIEW OF THE LITERATURE

The goal of the present study was to understand the complex phenomena of special education teachers' attitudes about mathematics and teaching mathematics by answering the question: "What is the nature of the beliefs and attitudes held by special educators about the discipline of mathematics, and the teaching and learning of mathematics?" The research question was explored by answering four domain-related questions: (a) what are the attitudes of special educators about mathematics, (b) what are the beliefs of special educators about the discipline of mathematics, (c) what are the beliefs of special educators about teaching mathematics, and (d) what are the beliefs of special educators about learning mathematics? The research question is critical because of the hypothesized link between teacher belief and instructional practice (Jordan & Stanovich, 2004; Thompson, 1984). A better understanding the beliefs and attitudes special educators hold about mathematics, teaching mathematics, and learning mathematics could inform teacher preparation and professional development to better support these educators who provide instruction for a population of students shown to be at risk for failure in mathematics.

In the sections that follow, the literature related the research question for the present study is reviewed and summarized. First, in order to situate the problem that provoked the present study, information about the achievement of U. S. students as a whole and of students with SLD is provided. Next, the context in which special education teachers provide mathematics instruction for students with SLD is explicated. Third, the affective domain with the respective constructs of beliefs and emotions is defined. Fourth, emotions related to mathematics, specifically anxiety, are explored. Finally, an analysis of the literature related to teacher beliefs about the discipline of mathematics and beliefs related to teaching and learning mathematics is presented.

The Context of the Problem

To better understand the significance of the problem explored in the present study, it is important to situate the problem within the context of both special education and mathematics education within the United States. Both special education and mathematics education have undergone significant changes over the past few decades, creating the context for the instructional practice of special educators today. Within special education, changes in educational services for students with disabilities have been driven by federal legislation resulting in a change in the role of special educators. Within mathematics education, a series of reform movements driven by efforts to improve mathematics achievement have challenged beliefs about the nature of mathematics, how mathematics is thought be learned, and how mathematics should be taught. Thus, special educators who support students with SLD in learning mathematics find themselves at the nexus of reform movements within special education and mathematics education.

Mathematics Achievement in the United States

Improving mathematics achievement for all U.S. students has been a national priority, exemplified by the 2006 presidential order issued by George W. Bush to create in a national mathematics panel. The executive order illustrates the importance policy makers place on mathematics:

To help keep America competitive, support American talent and creativity,

> encourage innovation throughout the American economy, and help State, local, territorial, and tribal governments give the Nation's children and youth the education they need to succeed, it shall be the policy of the United States to foster greater knowledge of and improved performance in mathematics among American students. (Exec. Order No. 13398, 2006)

The authors of the final report of the national mathematics panel concluded “the eminence, safety, and well-being of nations have been entwined for centuries with the ability of their people to deal with sophisticated quantitative ideas” (U. S. Department of Education, 2008, p. xi). Evan, Gray, and Olchefske (2006) punctuate the importance of mathematics to all students, not just those who are college bound, in stating that “even jobs that do not require a bachelor’s degree necessitate higher levels of mathematics and science skills from high school graduates” (p. 5).

Achievement of U. S. students as a whole. Despite the rhetoric and policies to strengthen the mathematics preparation of U.S. students, the mathematics achievement of U.S. students remains at or below average when compared with other nations. On the most recent administration of the Programme for International Student Assessment (PISA) mathematics assessment, U.S. students scored significantly lower than the Organisation for Economic Co-operation and Development average (OECD, 2010). On the 2007 Trends in Mathematics and Science Study (TIMSS), U.S. students fared only slightly better than the international average in both fourth- and eighth-grades, with little improvement in both grades since 1995 (Gonzales et al., 2009). In addition to poor performance of U.S. students compared with their international peers, U.S. students are failing to meet college expectations. Evan, et al. (2006) note: “Nationally, 22% of all college freshmen fail to meet the performance levels required for entry-level mathematics courses and must begin their college experience in remedial courses" (p. 8). Whereas the mathematics achievement of U. S. students as a whole is cause for concern, the achievement of students with SLD is alarming.

Students with specific learning disabilities. Students with SLD represent approximately 4.1 to 4.5% of the U.S. student population, or approximately 2,470,000 to 2,960,000 students (U. S. Department of Education, 2007). According to the Individuals with Disabilities Improvement Act of 2004, SLD is defined as "a disorder in 1 or more of the basic psychological processes involved in understanding or in using language, spoken or written, which disorder may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations" (p. 118). Additionally, SLD has been defined as a discrepancy between achievement and intellectual ability meaning that the underperformance of students with SLD cannot be attributed to below average intelligence (Gresham & Vellutino, 2010).

Whereas U. S. students as a whole lag behind their international peers in mathematics, students with SLD lag behind their U. S. counterparts. On average students with SLD perform 3.2 years behind their grade level peers in mathematics (Cortiella, 2011). The National Assessment of Educational Progress (NAEP) mathematics assessment results for 2009 illustrate the performance gaps between students with SLD and those without (National Center for Education Statistics, 2009). As indicated in Table II.1, the mathematics achievement gap between students with and without learning disabilities is troubling; 41% of students with SLD score within the Below Basic range on the NAEP mathematics assessment compared with only 16% of students without disabilities. The gap at eighth-grade is even more startling, where 64% of students with SLD score as Below Basic compared with 24% of students without disabilities.

Table II.1 Comparison of 2009 NAEP Mathematics Results for Students With and Without SLD (National Center for Education Statistics, 2009).

	Students with SLD			Students without SLD		
	Percent Below Basic	Percent Proficient	Percent Advanced	Percent Below Basic	Percent Proficient	Percent Advanced
4th grade	41	17	2	16	35	6
8th grade	64	8	1	24	27	8

In addition to the performance gap between students with SLD and their peers on the NAEP mathematics assessment, studies show that only 13% of high school students with SLD are within grade level, 23% are one to 2.9 years behind, 44% are three to 4.9 years behind, and 20% are five or more years behind (Cortiella, 2011). Clearly, the data signal a crisis in the mathematics preparedness for students with SLD.

Disturbingly, 28% of students with disabilities drop out of high school (Wagner, et al., 2005). Those who remain may not be fully prepared to reach college readiness standards in mathematics. Adequate preparation in mathematics is necessary for both admission to and success in postsecondary institutions (Conley, 2005), yet many U.S. students and most students with SLD are not prepared for postsecondary education options. As illustrated in Table II.2, a review of recent Colorado ACT mathematics performance (Colorado Department of Education, 2011) shows that students with SLD score below the state average and well below the ACT college mathematics readiness benchmark of 22 (ACT, 2010).

Table II.2 Colorado ACT Mathematics Scores, 2007-2011

	2007	2008	2009	2010	2011
State Average ACT Mathematics Score	19.2	19.3	19.5	19.3	20
Average ACT Mathematics Score for Students with SLD	14.2	14.4	14.5	14.4	15

Colorado Department of Education, 2011.

Without having requisite college admission scores, students have more limited postsecondary possibilities. This effect can be seen in the enrollment patterns of students with SLD in postsecondary institutions. Students with SLD attend postsecondary education institutions at a lower rate than their peers, 32.7% compared with 40.5%, with a greater difference at four-year institutions of 9.7% of SLD students compared with 28% of the general population (Wagner, et al., 2005). According to Cortiella (2011),

> Students without disabilities are more than four and one-half times as likely as youth with disabilities to attend four-year institutions. This could be due in large part to the limited ability of students with LD to satisfy admission criteria at four-year colleges and universities. (p. 28)

The long-term economic impact of lower participation in postsecondary education is clear. With the median annual income for college graduates at $43,143 compared with high school graduates at $26,505 (Cortiella, 2011), the cost of not being adequately prepared for college is $16,638 in annual income. Compounded over time, the gap grows exponentially, exacerbating the income disparity. Beyond the income gap, the employment rate for individuals with SLD is lower. According to Cortiella (2011), “In

2005, 55% of adults with LD (ages 18-64) were employed compared to 76% of those without LD, 6% were unemployed vs. 3%, and 39% were not in the labor force vs. 21%" (p. 2). The connection between underachievement in mathematics, lower participation in postsecondary education, and decreased economic opportunities is apparent. Clearly, students with SLD have significant challenges achieving in mathematics and the stakes are high. This population is arguably most at risk for school failure and is in need of educators who can support them in preparing for success in the global economy. The need to improve the educational experiences in mathematics for students with SLD is crucial to their ability to succeed in life.

Special education teachers are uniquely positioned to provide learning supports to students with SLD. The next section of the chapter provides a context for the instructional relationship special educators currently tend to have with students with SLD.

Overview of Issues within Special Education

Since the initiation of legislation designed to support students with disabilities, special educators have played a key role in the education of students with SLD. The role of special educators in the mathematics education of students with SLD has evolved with the progression of legislation related to students with disabilities. As noted earlier, Zigmond, Kloo, and Volonino (2009) contend that special education has been about the *what*, *where*, and *how* of education. Historically, students with disabilities were educated in special education "pullout" classes, which were classes in resource rooms separated from grade level peers (the *where*), taught through a separate curriculum (the *what*), and taught by a special educator (the *who*) using specific teaching strategies (the *how*)

(Zigmond, et al., 2009). A major force that has recently changed this historical model is the concept of inclusion, which seeks to place students with disabilities into mainstream, regular education classes with their grade level peers. What follows is a brief overview of the legislation that has driven educational practices toward greater inclusion of students with disabilities into the general education classroom. This section provides the historic backdrop for the current context in which special educators now serve students with SLD.

Federal policies have been legislative and fiscal drivers of change in special education since the 1970s beginning with the Education for the Handicapped Act (EHA) of 1970. This act was designed to create support for students with disabilities through the creation of special preparation programs for teachers (Katsiyannis, et al., 2001). Prior to this act, the U. S. educated only 20% of students with disabilities (Katsiyannis, et al., 2001).

The reauthorization of the EHA in 1990 came with a renaming of the act to be known as the Individuals with Disabilities Education Act (IDEA), which provided funding to states for programs for students with disabilities along with specific guidance on how the students would be educated (Katsiyannis, et al., 2001). Among the notable components of IDEA are: (a) the zero rejection principle, guaranteeing free access to appropriate public education (FAPE) for students with disabilities, (b) the requirement of evaluation in order to receive special education services, and (c) the principle of least restrictive environment (LRE) (Katsiyannis, et al., 2001). The last point, LRE, requires that students with disabilities be placed with their age level peers to the greatest extent possible (Katsiyannis, et al., 2001), forming the foundation of the concept of inclusion.

The policy shift towards access to education through inclusion was followed by a shift in policy towards accountability for outcomes. The legislative shift from access to accountability for results can be seen through the reauthorization of both the Elementary and Secondary Education Act of 2001, also known as No Child Left Behind, and amendments to IDEA with the passage of the Individuals with Disabilities Education Improvement Act of 2004. The No Child Left Behind Act of 2001 (NCLB) increased the accountability of schools for the academic achievement of all students, including students with SLD. For the first time, students with SLD were expected to meet general education standards (Brownell, Sindelar, & Kiely, 2010). Three years after the passage of NCLB, Congress amended IDEA by removing the requirement to use the discrepancy between a child's achievement and intellectual ability to determine the presence of a SLD. The reauthorized IDEA allowed for the use of an instructional framework called responsive to intervention (RtI) as a method for identifying students with SLD. The RtI framework calls for a comprehensive approach to instruction and intervention for all students, not just students with disabilities, and implies changes in the roles and responsibilities of both special and general educators.

NCLB and IDEA cemented a change in the role of special educators from diagnostician and case manager to direct service provider to students with SLD (Bauer, Johnson, & Sapona, 2004; Chamberlain, 2008; Hoover & Patton, 2008; Smith, Robb, West, & Tyler, 2010), leading to tremendous changes in the both the role and preparation of special educators. Prior to EHA, the role of special educators was primarily that of diagnostician in order to provide students with one-on-one instruction in resource rooms separate from general education classrooms (Baker & Zigmond, 1995). In the 1950s, special educators were trained to serve children according to disability category (e.g., speech impairment or deafness) and were significantly impacted by behavioral psychology (Brownell, et al., 2010). In the 1970s, preparation programs began to emphasize non-categorical training (i.e., service to students with a broad range of disability classifications) while competency-based instructional approaches further entrenched the behavioral tradition for special educators (Brownell, et al., 2010). The 1990s brought a further shift in special education delivery models and preparation programs through inclusion models, where special education students were integrated into general education classrooms (Baker & Zigmond, 1995). The inclusion model "required general and special education teachers to retool in order to adjust to their new roles in schools" (Brownell, et al., 2010, p. 364). Gradually, special educator practice has shifted to support placement of students with disabilities in the LRE through decreasing one-on-one, pullout remedial instruction in favor of increasing specialized instruction in the general education classroom (Baker & Zigmond, 1995).

Despite the increased recent focus on inclusion, instructional delivery models within an RtI framework can still be described along a continuum from full inclusion into a general education classroom to a separate, pullout, special education classroom, ideally dependent on the needs of the student. Each instructional delivery model implies a different role for special educators. For a full inclusion model, special educators collaborate with general education teachers to provide support for students with SLD in the general education classroom. This support can be in the form of (a) co-teaching the class with the general education teacher, (b) parallel teaching, where the special educator teaches the same content to small groups of students within the general education classroom, (c) intervention, where the special educator re-teaches content to small groups of students within the general education classroom, or (d) providing support, where the special educator is present in the general education classroom to lend support to students during the class (Weiss & Lloyd, 2002). Students in this model are expected to learn grade level content with the specialized support of special education teachers. In the middle of the “inclusion-to pullout-continuum” is an instructional delivery model where students with SLD participate in a general education class and receive additional support through an additional intervention class taught by a special educator. In this model, the special educator collaborates with the general educator to identify gaps in learning in order to focus intervention. On the other end of the continuum of instructional delivery is the separate, pullout special education class taught exclusively by a special education teacher.

In order for any instructional delivery model to be successful, special educators must have the necessary content knowledge and differentiation skills across core subjects (Hoover & Patton, 2008). In the mathematics subject area, however, special educators do not often have a strong background (Maccini & Gagnon, 2002) and tend to have a relative lack of understanding of mathematics pedagogy and reform efforts (Maccini & Gagnon, 2002, 2006). The degree to which special educators are comfortable with mathematics and how they approach teaching and learning mathematics have the potential to impact their effectiveness in their role in mathematics instruction. Smith (2010) argues that requisite content and pedagogical knowledge is necessary to support student learning. In mathematics, however, the impact of a teacher’s anxiety (Swars, et al., 2006) and beliefs (Ernest, 1988) are also hypothesized to influence teaching. The present study sought to explore the dimensions of special educators’ mathematics anxiety and beliefs about the nature of mathematics in relation to their beliefs about how mathematics is learned and should be taught. The transformation of the role of special educators has occurred against the background of mathematics reform efforts, placing special educators in the midst of a reform movement that may challenge their attitudes and beliefs.

Overview of Issues within Mathematics Education

Mathematics education in the United States has experienced three major movements in the 20th and 21st centuries. A review of these movements reveals how different paradigms of mathematics have influenced the teaching and learning of mathematics over the past few decades. In the sections that follow, a brief history of reform movements in mathematics education is described and related to perspectives of appropriate mathematics instruction for students with SLD.

Shifting focus in mathematics education. In recent decades, debate about what mathematics should be taught and how it should be taught has shifted among different paradigms, from the New Math movement of the 1960s to the back-to-basics movement of the 1970s to the era of the National Council of Teachers of Mathematics (NCTM) *Standards*. The New Math movement of the 1960s and early 1970s grew out of concern from educators and the public about the preparation of students for an increasingly technological age, an apprehension sparked by the Soviet launch of the Sputnik satellite in 1957 (Herrera & Owens, 2001, p. 6). New Math introduced a mathematics curriculum that was designed around mathematics axioms, guided by how mathematicians view the discipline (Bass, 2005). Despite the intentions of New Math movement, it failed for many reasons, including the abstractness of the mathematics for elementary students

(Woodward, 2004), the frustration of parents at being unable to help their children with mathematics, and the perception that students were not gaining the skills they needed (Herrera & Owens, 2001).

The failure of the New Math movement to achieve its intended outcome prompted a “back-to-basics” movement in the late 1970s, calling for a return to the generally computational focus of the mathematics curriculum. The back-to-basics movement promoted an instructional model that emphasized briskly paced lessons with low level, rapid fire question-answer routines, and a common lesson format consisting of a brief review of learning, presentation of a new lesson, and independent practice (Woodward, 2004).

The next shift in mathematics education was initiated not by policy makers or mathematicians but by mathematics educators as the National Council of Teachers of Mathematics (NCTM) published the first-ever national standards for student learning in mathematics, the *Curriculum and Evaluation Standards for School Mathematics* in 1989. NCTM *Standards*, as the publication came to be called, initiated a wave of reform in mathematics education (Battista, 1994) focused on “increasing conceptual learning, problem solving, and effective mathematical communication for all students” (Simon, 2008, p. 17). The NCTM *Standards* challenged the notions about mathematics, and about teaching and learning mathematics, among educators, mathematicians, policy makers, and the public at large (Battista, 1994; Schoenfeld, 2004). The NCTM *Standards* challenged the back-to-basics approach to teaching mathematics, that critics termed “parrot math” (O'Brien, 1999), by deemphasizing computation and elevating attention to the processes of mathematics: problem solving, communication, representations, reasoning and proof, and connections (NCTM, 1989). The NCTM *Standards* sought to change the mathematics experienced by students by portraying mathematics as a “principled discipline that is based on the conceptual understanding of key ideas” (Berry & Kim, 2008, p. 363). In short, the NCTM *Standards* “have been the centerpiece of mathematics education in the United States … and their influence has been apparent in the growth in reform based research, curricula, and methods of assessment” (Woodward, 2004, p. 16).

Following the publication of the NCTM *Standards*, the federal government, through the National Science Foundation, funded the development of instructional resources based on the NCTM *Standards*. The new NCTM *Standards*-based curricula, although supported by many mathematics educators, were not received with the same enthusiasm by parents, mathematicians, or special educators.

Controversy over the NCTM Standards. The NCTM *Standards* were met with criticism by some parents and mathematicians and skepticism by some special educators. The “math wars” is a term used to “describe the conflicts between mathematicians and educators over the content, goals, and pedagogy of the curriculum” (Bass, 2005, p. 417). The math wars grew out of critiques that the NCTM *Standards*-based materials did not adequately develop basic skills in arithmetic and algebra, encouraged calculator use, emphasized group work and “discovery” learning, and de-emphasized mathematical definitions and proofs (Klein, 2003). The most vicious battle in the math wars came from a group called Mathematically Correct. Mathematically Correct originated from a group of educated parents and university faculty in California that took aim at the California mathematics framework which was based on the NCTM *Standards*, largely due to the decrease in standardized mathematics test scores of California students (Klein, 2003). The group successfully derailed mathematics reform efforts in the state.

The NCTM *Standards* and the associated mathematics reform movement have had differing levels of understanding and acceptance within special education. To begin with, the NCTM *Standards* have not been well known among special educators. In a study of general and special educators, Maccini and Gagnon (2002) found that whereas 95% of general mathematics educators surveyed were familiar with the goals of the NCTM *Standards*, only 55% of special educators reported familiarity. With regard to agreement with these goals, 73% of general educators reported strong agreement or agreement with NCTM goals compared with 50% of special educators.

Furthermore, the NCTM *Standards* have received significant criticism from special educators. The critiques of the NCTM *Standards* from special educators tend to be around three main issues: (a) the lack of references to students with learning disabilities, (b) the lack of research base for the instructional approaches, and (c) the promotion of a constructivist approach to teaching mathematics for all students (Hofmeister, 1993; Rivera, 1997; Simon & Rivera, 2007). Specifically, special education leaders have raised concern over the appropriateness of instructional methods implied by the NCTM *Standards* for students with SLD. Given that students with SLD tend to have memory deficits, attention issues, and can take a passive stance in classes, Miller and Hudson (2007) raised the concern that "these deficits make it difficult for students with SLD to be fully engaged in the types of problem solving promoted in reform-based classes" (Miller & Hudson, 2007, p. 48). Montague (2003) and Griffen, Jitendra, and League (2009) raised similar issues for students with SLD indicating that the characteristics of these students prevent their full participation in mathematics classes.

The instructional approaches promoted by NCTM (hereafter referred to as *reform-based* or *standards-based* mathematics) tended to be at odds with instructional approaches promoted within special education. The instructional model emphasized within special education has been direct or explicit instruction (Jones & Southern, 2003). Sayeski and Paulsen (2010) contended that “Students with math LD require explicit instruction in the desired concept or skill to make these important connections” (p. 18). The direct instruction model is an instructional sequence consisting of (a) an opening of the lesson by gaining student attention, reviewing the previous lesson, and giving the new learning objective, (b) presentation of new material by modeling and checking for understanding, (c) concluding with a summary of what was learned and describing the next lesson, and (d) providing practice with the new skill (Jones & Southern, 2003, p. 6). Interestingly, the direct instruction approach is quite similar to traditional mathematics lessons. In *The Teaching Gap*, Stigler and Hiebert portrayed mathematics instruction in U. S. classrooms as essentially a script consisting of (a) reviewing the previous lesson, (b) checking homework, (c) presenting a new lesson with checks for understanding, and (d) completing seatwork (Stigler & Hiebert, 1999). Thus, the description of direct instruction advocated within special education is almost identical to the traditional mathematics classroom that the NCTM *Standards* were designed to change.

It is within this varied and sometimes contentious context that special educators find themselves as they work to support students with SLD in becoming mathematically proficient. Special educators must reconcile their experience from the research based within special education, that emphasizes rote learning of facts and algorithms and limited emphasis on problem solving (Woodward & Montague, 2002), with other professional development experience, instructional materials, and general education colleagues who have been greatly influenced by constructivism (Rowe, 2006; Steele, 2005). As Van-Garderen et al. (2009) noted, the research bases and professional preparation of general and special educators promote "differing perspectives regarding how students learn, [and] very different ideas as to how mathematics for the struggling learner should be taught are being brought to the classroom" (pp. 71-72).

Summary. The recent history of mathematics education illustrates how deeply connected beliefs about teaching and learning mathematics are held. Whereas there is little dispute that the mathematics achievement of U. S. students must be improved, there are widely differing beliefs about how improvements will be realized. The beliefs that individuals hold about the discipline of mathematics and beliefs about how it should be taught and learned have been at the center of recent reform movements. As Philippou and Christou (2002) noted,

> Beliefs and conceptions of what mathematics is really all about, and what it means to know and learn mathematics is a determinant of the way one views involvement with the subject, that is, the process of developing understanding and competency in doing mathematics. (p. 212)

In the next section, an overview of research related to the attitudes and beliefs of educators related to mathematics is provided. Notably, the subjects of this line of research have been general educators. Special educators have not been the focus of beliefs research creating a noticeable gap in the literature.

The Affective Domain: Emotions, Attitudes, Beliefs, and Values

The research question for the proposed study relates to beliefs and attitudes of special educators. In this section, the terms belief and attitude will be defined within the affective domain.

There are many different definitions of belief and belief systems in mathematics education research (Cooney, 1999; 2002; Pajares, 1992; Pehkonen & Torner, 1999; Torner, 2002; Wilson & Cooney, 2002). The construct of belief has been used many different ways in the literature, including: "attitudes, values, judgments, axioms, opinions, ideology, perceptions, conceptions, conceptual systems, preconceptions, dispositions, implicit theories, explicit theories, personal theories, internal mental processes, action strategies, rules of practice, practical principles, perspectives, repertories of understanding, and social strategy" (Pajares, 1992, p. 309). While some researchers find the plethora of definitions an impediment to research about beliefs (McLeod & McLeod, 2002; Pajares, 1992), others contend that some variation is inevitable (Cooney, 1999). Additionally, the blurring of beliefs with attitudes in the research makes both it both difficult to define the constructs and to further the research base. Because researchers often discuss beliefs and attitudes in tandem (Pajares, 1992), this section will present concepts of beliefs and attitudes together with attention to characteristics that distinguish between them.

Mapping the Affective Domain

Beliefs and attitudes are "intrinsically related" to one another (Leder & Forgasz, 2002, p. 96) and are part of the larger domain of affect (Goldin, 2002). Goldin (2002), suggested that the affective domain consists of the sub-domains of emotions, attitudes, beliefs, and values. Further, McLeod (1988) described dimensions of the affective domain including: (a) the magnitude or intensity of response experienced by the individual, (b) the level of control one has over one's responses, (c) level of consciousness the individual experiences, and (d) the duration of the response. Thus, the sub-domains are differentiated from one another along the dimensions of intensity, control, consciousness, and duration. In the following section, the affective sub-domains of emotions, attitudes, beliefs, and values are defined and elaborated with respect to McLeod's (1988) dimensions of intensity, control, consciousness, and duration.

Emotions

In the literature, the distinguishing features of emotions are their transitory nature, subjectivity to change, and resistance to the influence of cognition. Emotions are "rapidly changing states of feeling, embedded in context" (Goldin, 2002, p. 61). McLeod defined emotion as "a more visceral kind of affect, a response that is quite intense but of relatively short duration" (McLeod, 1988, p. 135). People experience emotions with high intensity and thus have lower levels of response control. Emotions involve lower levels of consciousness. Described along the dimensions of the affective domain, people experience emotions with high intensity for relatively short periods of time and have low levels of control over or consciousness about the emotions. Figure II.3 illustrates emotions along the dimensions of the affective domain.

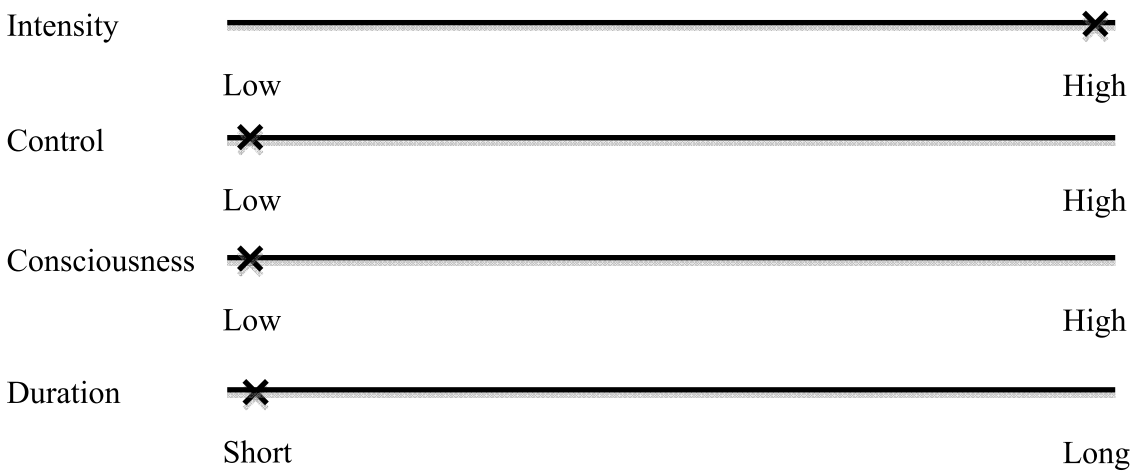

Figure II.3. Dimensions of Emotions.

Attitudes

Statt (1998) defined an attitude as “a stable, long-lasting, learned predisposition to respond to certain things in a certain way” (p. 10). Attitudes are “moderately stable predispositions towards ways of feeling in classes of situations, balanced between affect and cognition” (Goldin, 2002, p. 61). Attitudes are more constant than emotions and tend to be more consciously held. McLeod used the term attitude to describe “less intense affective responses, especially responses that are relatively consistent” (McLeod, 1988, p. 135). Attitudes are smaller in magnitude than emotions (McLeod, 1988) and are held for a longer duration than emotions. Described along the dimensions of the affective domain, people experience attitudes with at a moderate intensity for longer periods of time than emotions. Also, people tend to have greater consciousness of attitudes and experience a greater degree of control over their attitudes. Figure II.4 illustrates attitudes along the dimensions of the affective domain.

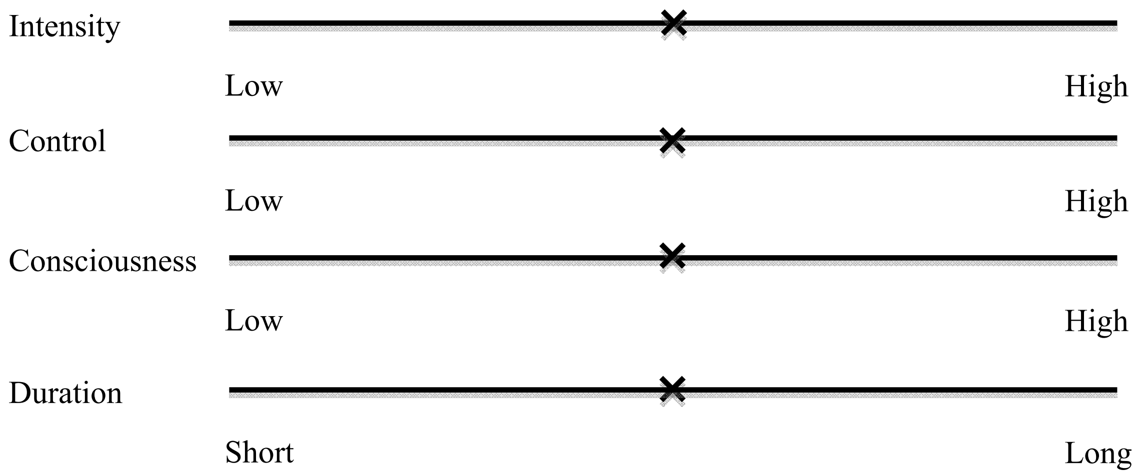

Figure II.4. Dimensions of Attitudes.

Beliefs

Beliefs are “internal representations to which the holder attributes truth, validity, or applicability” (Goldin, 2002, p. 61). Beliefs are more stable than emotions and attitudes, and beliefs tend to be “deeply personal, rather than universal, and unaffected by persuasion” (Pajares, 1992, p. 309). The construct of belief has a stronger cognitive than emotional component (Statt, 1998). Described along the dimensions of the affective domain, people experience beliefs with a relatively low level intensity for relatively long periods of time. People tend to be conscious of their beliefs and hold a relatively low level of control over beliefs they hold. Figure II.5 illustrates beliefs along the dimensions of the affective domain.

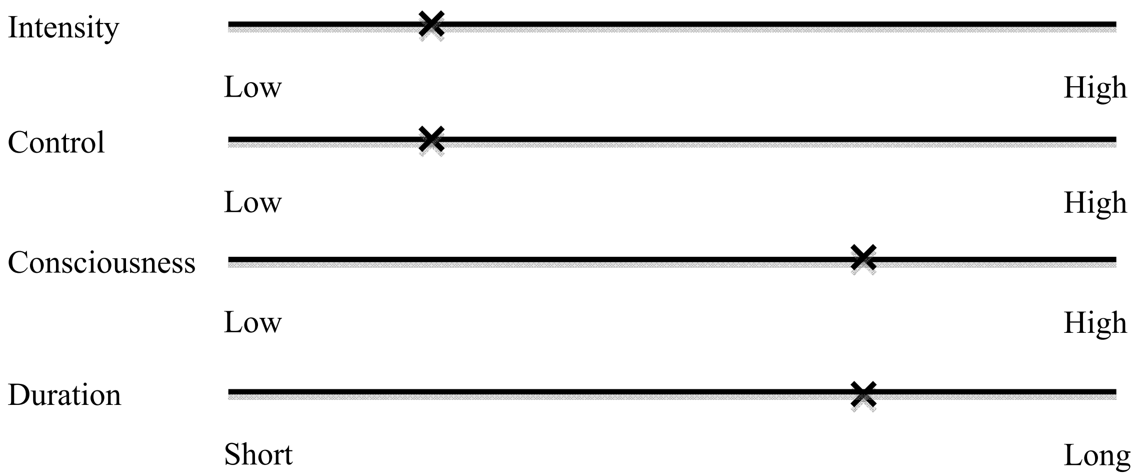

Figure II.5. Dimensions of Beliefs.

Values

Values are the most stable of affective sub-domains. Values are "deeply held preferences and personal truths" (Goldin, 2002, p. 61). Values are held more consciously and have a highly cognitive component. Described along the dimensions of the affective domain, people experience values with low intensity for long periods of time. People have high levels of consciousness and control over their values. Figure II.6 illustrates values along the dimensions of the affective domain.

Figure II.6. Dimensions of Values.

The present study considered the sub-domains of emotions and beliefs related to mathematics.

Emotions and Mathematics Anxiety

Of interest to the present study was the affective sub-domain of emotion as it relates to mathematics, specifically the emotion of anxiety. Using McLeod's (1988) dimension of affect, anxiety can be defined as an intensely negative emotion. Research related to affect in mathematics has been in two areas: affect in general and in the relationship between attitude and achievement (Zan, Brown, Evans, & Hannula, 2006). In this section, mathematics anxiety will be defined and the impacts and causes of mathematics anxiety will be explored.

Mathematics Anxiety: Definition and Impact

Within the literature, mathematics anxiety (MA), often simply termed "math anxiety," has been defined as "a negative reaction to math and to mathematical situations" (Ashcraft & Ridley, 2005, p. 315). Others have described MA emphasizing a greater intensity of emotions, such as "pronounced fear" (Aiken & Dreger, 1961, p. 19), "tension, apprehension, or fear" (Ashcraft, 2002, p. 181), "a feeling of helplessness, tension, or panic" (Gresham, 2007, p. 182) in situations dealing with mathematics or calling for mathematical performance. MA can be manifested as feelings of frustration, anger, and even physical pain when doing mathematics (Carroll, 1994). MA can be debilitating to those who suffer from it (Ho, et al., 2000) impacting memory and mathematics performance (Prevatt, Welles, Li, & Proctor, 2010).

Cognitive and physiological impact. MA is more impactful than its name may imply. MA is not simply a transitory emotion that people who suffer from it experience. MA has been related to cognitive issues such as working memory and numerical processing as well as measurable physiological responses.

One of the cognitive impacts of MA relates to working memory. Baddeley and Logie (1999) have defined working memory as an essential aspect of cognition consisting of a central executive and temporary memory systems. The central executive regulates the memory systems that use phonological and visuospatial information, allowing individuals to "comprehend and mentally represent their immediate environment" (Baddeley & Logie, 1999, p. 29). Essentially, working memory is a temporary cognitive workspace for immediate tasks such as solving problems or forming and acting on current goals.

Working memory has been described as an essential component of mathematical cognition (LeFevre, DeStefano, Coleman, & Shanahan, 2005). For example, the computation 16 x 25 requires a number of subtasks that engage working memory. To carry out the computation, the executive function of working memory calls up information from long-term memory, perhaps the association of the number 25 with quarters. A temporary workspace is then created where reasoning about how the multiplier of 16 is related to the quantity of quarters. Visuospatially, the sixteen quarters would be grouped into four groups making four dollars. The executive function of working memory would then retrieve information related to dollars as a hundred cents, allowing the answer to be translated to four hundred cents, or simply 400.

MA is negatively correlated with working memory (Ashcraft & Kirk, 2001) slowing down or entirely disrupting its function by competing for working memory capacity (Beilock, 2008). Beilock (2008) wrote, “suboptimal math performance in stress-laden situations arises because worries about the situation compete for the working memory (WM) available for performance” (p. 339). Ashcraft and Krause describe the phenomenon as follows: “High math anxiety works much like a dual task setting: Preoccupation with one’s math fears and anxieties functions like a resource-demanding secondary task” (2007, p. 243). Prevatt, et al. (2010) explained the interaction between memory, anxiety, and mathematics performance in terms of processing efficiency: “With regard to math, we would speculate that the individual’s anxiety about their math performance serves as the diversionary stimulus” (p. 45). Prevatt, et al. (2010) speculated that anxiety has a greater impact on memory when mathematical tasks are more complex.

Another cognitive impact of MA relates to numerical processing. Maloney, Ansari, and Fugelsang (2009) examined the numerical processing ability of people with MA. Numerical processing was measured through visual enumeration tasks which involve recording the amount of time it takes for subjects to determine a quantity up to nine. Typically, people can determine a quantity up to four without counting, whereas counting is required for quantities in the range of five and higher (Kaufman, Lord, Reese, & Volkmann, 1949). Their study found that people with high MA performed significantly worse on numerical processing visual enumeration tasks. Thus, MA is generally implicated in the low ability of people to process quantities.

Finally, MA has been found to impact individuals through the release of the stress hormone cortisol (Mattarella-Micke, Mateo, Kozak, Foster, & Beilock, 2011). People with high MA experience a greater release of this stress hormone in situations dealing with mathematics. Interestingly, the release of cortisol has a differential impact related to working memory. Mattarella-Micke, et al. (2011) found that the degree to which people employ working memory in mathematical tasks has a differential effect on cortisol release. Specifically, people who rely more heavily on working memory to complete mathematics tasks experienced greater physiological impact of MA than those who use less working memory. Thus, the greater reliance an individual has on working memory, the greater the release of cortisol, which may provide a physiological explanation for the decrease in working memory capacity for people with MA.

Whereas MA is often described simply in terms of emotion, the cognitive impact of MA is measurable. MA is implicated in the obstruction of working memory and numerical processing, and the release of stress hormones. Thus, MA is more than simply an emotional reaction to mathematics.

Academic impact. The impact of emotions on mathematics performance has also been of interest to researchers (Ma & Kishor, 1997; McLeod, 1994). In fact, Zan et al. (2006) contended: “Arguably the most important problem for research on affect in mathematics is the understanding of the interrelationship between affect and cognition” (p. 117). McLeod (1992) noted that "affective issues play a central role in mathematics learning and instruction" (p. 575). In their meta-analysis of studies relating affect in mathematics to achievement, Ma and Kishor (1997) found a positive but small relationship. Overall, a very small causal relationship was found (0.08) between achievement in mathematics to attitude toward mathematics, which Ma and Kishor (1997) noted was not practically meaningful.

Other studies have found a significant negative relationship between MA and mathematics achievement. Zakaria and Nordin (2008) found that students with high MA had significantly lower mathematics achievement. They also found that students with high MA have lower motivation to learn mathematics. Also, a cross-national study (Ho, et al., 2000, p. 531) of sixth grade students from China, Taiwan, and the United States found that MA was negatively related to mathematics achievement.

Other academic consequences of MA have been found. People with MA avoid mathematics (Ashcraft & Krause, 2007; Ashcraft & Ridley, 2005; Hembree, 1990) and are reluctant to engage in mathematics tasks or to offer their solutions unless they are certain they are correct (Carroll, 1994). In fact, Eccles and Jacobs (1986) found that MA was a predictor of grades and course-taking plans in mathematics. The avoidance of mathematics is likely related to tendency for MA students to have lower mathematics achievement (Ashcraft, 2002).

In conclusion, the cognitive, physiological, and academic impacts of MA are clear. MA negatively impacts the cognition and academic performance of those who suffer from it. The causes of MA are not as clear as its consequences. However, an exploration of the causes of MA does offer insight into possible prevention.

Causes of Math Anxiety

The impact of MA is clearly negative on those who suffer from it. Accordingly, identifying the causes of MA is of interest to researchers. Potential causes that have been explored are past performance in mathematics and the mathematical learning experiences of students. Even so, "little is known about the onset of math anxiety, and even less is known about the factors that either predispose one toward or cause math anxiety"

(Ashcraft, Krause, & Hopko, 2007, p. 341).

Ordering the relationship between mathematics achievement and anxiety. Which comes first, poor mathematics achievement or MA? While it is clear that a relationship exists between MA and mathematics achievement (Zakaria & Nordin, 2008) the ordering of the relationship has not been fully explained. It is known that students with high MA have lower motivation to learn mathematics (Zakaria & Nordin, 2008), but is the MA caused by previous failure in mathematics? The answer is not clear. Hembree (1990) concluded that "There is no compelling evidence that poor performance causes mathematics anxiety" (p. 45). In contrast, the analysis Ma and Xu (2004) conducted on the Longitudinal Study of American Youth found that lower mathematics achievement preceded and was significantly related to higher MA. However, prior high levels of MA showed only a small relationship to subsequent achievement in mathematics in this study.

Whereas the ordering of the relationship between MA and achievement is not clear, it is apparent that MA leads to avoidance of mathematics (Ashcraft & Ridley, 2005; Hembree, 1990). Such avoidance inevitably results in less opportunity to learn mathematics thus leading to lower achievement. The direction of relationship between mathematics achievement and MA may not be fully understood. However, other contributors have been explored, especially the experience students have in learning mathematics.

Role of teachers and instruction in math anxiety. Stodolsky (1985) contended that the attitudes students hold about mathematics relate to the instruction they have experienced. To this point, Stodolsky (1985) noted that the "consequences of instruction in the field of mathematics are evident in the manner in which many adults approach mathematical tasks, in avoidance of math, and in the frequent acceptance of ability as the main determiner of math achievement” (p. 132). Ashcraft (2005) concurred, “We speculate that [unsupportive] teacher attitudes and classroom practices, along with cultural attitudes, generate negativity and anxiety about math” (p. 324). Ashcraft and Krause (2007) described the how MA might be related to stressful or humiliating classroom experiences such as being required to perform mathematics problems on the board and doing poorly.

Evidence for the link between the classroom experiences of students and MA comes from a number of sources. Studies of MA in pre-service elementary teachers frequently point to teachers and instruction as a source for MA. For example, pre-service elementary teachers identified the instructional approaches their teachers used as a factor in their attitudes toward mathematics (Brady & Bowd, 2005). Trujillo and Hadfield (1999) found that pre-service elementary teachers attributed their MA at least in part to their negative experiences in school. Similarly, Bekdemir (2010) studied the MA and past classroom experiences of pre-service elementary Turkish teachers. Asked to reflect on their worst mathematical experience, these teachers frequently reported both the hostile behavior of their instructor and anxiety related to exams. Bekdemir (2010) concluded that “the worst experience and most troublesome mathematics classroom experience are major causes of mathematics anxiety” (p. 324).

Researchers have explored how instructional practices in mathematics potentially contribute to MA, often targeting the general nature of instruction; however, studies of the contribution of specific instructional practices to MA have been inconclusive. One path of inquiry related to instructional practices has consisted of exploring the relationship between MA and traditional or nontraditional mathematics instructional strategies. Citing numerous studies, Gresham (2007) asserted that "'traditional' ways of teaching can be the cause of mathematics anxiety" (p. 182). However, Levine (1993) found no relationship between the MA of pre-service elementary teachers and their reports of the instructional approaches they experienced as students. A brief review of these differing perspectives is offered here.

A description of a traditional mathematics class emerged from an observational study of fifth-grade mathematics and social studies classrooms conducted by Stodolsky (1985). Stodolsky found striking differences in the instructional strategies teachers employed according to subject area. In general, Stodolsky found that mathematics instruction was characterized by (a) the use of skill practice and seatwork, (b) teacher presentation of concepts or procedures, (c) textbook centered instruction, (d) lack of manipulative use, and (e) lack of student interaction. Stodolskly (1985) concluded that,

> Elementary math instruction consists primarily of the teacher introducing new concepts and algorithms to the whole class followed by individual students solving problems at their desks from a textbook or workbook. Essentially, students have one route to learning: teacher explanation and self-paced practice. (p. 169)

The mathematics instruction Stodlosky found was in sharp contrast to social studies instruction by the same teachers. In social studies classes, students spent much greater portions of class time working together. Stodolsky (1985) asserted that students' negative perceptions of mathematics "derive from and are rationales for the consequences of early learning in these areas. The dependence created between the math teacher and math learner over many years is the root problem" (p. 131).

Newstead (1998) studied MA in 5th and 6th grade students in the U.K. characterizing the classroom environment of students as either traditional (focusing on standard, paper-pencil algorithms, and teacher demonstration followed by individual student practice) or alternative (focusing on student problem solving approaches and discussion). Newstead found that students who experienced alternative instructional strategies exhibited less MA. Newstead (1998) concluded that "Mathematics anxiety may therefore be a function of teaching methodologies used to convey basic mathematical skills which involve the mechanical, 'explain-practise-memorise' teaching paradigm, emphasising memorisation rather than understanding and reasoning" (p. 55). Similarly, Swars et al. (2006) found that pre-service elementary teachers with high MA reported their mathematical learning experiences as focused on memorization and procedural knowledge. In contrast, pre-service teachers that had lower MA reported experiences in mathematics that emphasized problem solving.

Sloan et al. (2002) found that MA in pre-service teachers was positively correlated with a "right brain" learning style. According to Sloan, et al. (2002),

> In essence, global or right-brain dominant individuals approach problems in an intuitive manner, whereas most mathematics courses are taught through systematic problem solving in a step-by-step linear fashion. Additionally, mathematics problems are often directed toward finding the one right answer. However, global learners prefer open-ended tasks and approach problems in a divergent manner. (p. 86)

Instructional approaches that emphasize procedures and memorization appear to have a differential impact on students. Ellsworth and Buss (2000) found that while some pre-service elementary teachers reported motivating effects of instructional experiences related to procedures and memorization, others expressed the opposite view, reporting a debilitating effect from such an instructional approach. Furthermore, in a study of students in a college level mathematics survey course, Clute (1984) compared the mathematics achievement of students with low, medium, and high levels of MA after experiencing one of two types of instructional methods, discovery or expository. Clute described the discovery method as presenting the class with a series of problems from simple to most difficult until students discovered solutions. Clute described the expository method as presenting material in lecture format with examples for students to follow and guided practice. Clute found that students with high levels of MA had higher achievement when taught with an expository instructional method. Clute (1984) concluded that "instead of trusting his or her own methods of mastering the material, the highly anxious student needs to rely heavily on a well-structured, controlled plan for learning (i.e., an expository method)" (pp. 56-57).

Overall, classroom experiences appear to affect student learning and attitudes in mathematics. However, as Ashcraft and Ridley (2005) noted: "There appears to be little, if any, direct empirical work on the causes of math anxiety, merely anecdotal evidence and some intriguing possibilities" (p. 324). Whereas the causes of MA are not conclusive, its consequences are quite clear. For those who suffer from it, MA creates intensively negative responses to situations involving mathematics, leads to mathematics avoidance, and is associated with lower mathematics achievement. With the association of MA to the experience students have in mathematics classes, the influence of teachers with MA on their students' affect and achievement in mathematics is worthy of investigation. In short, do teachers with MA produce students with MA? The next section will review the literature related to teacher affect toward mathematics and the hypothesized link between teacher and student affect.

Teacher Affect Towards Mathematics

The vast majority of studies on the affect of teachers toward mathematics have been conducted with pre-service teachers (Ball, 1990; DiMartino & Sabena, 2010; Ellsworth & Buss, 2000; Gresham, 2009; Jackson, 2008; Malinsky, et al., 2006; Peker, 2009; Sloan, et al., 2002; Swars, et al., 2006; Trujillo & Hadfield, 1999; Vinson, 2001). Studies often focus on reducing the level of MA (Gresham, 2007; Vinson, 2001) for the sake of breaking the cycle of instruction that has been presumed to perpetuate the creation of MA (Gresham, 2007). Relatively little research has been conducted on the prevalence of MA in other educator populations. This section will summarize the findings from the literature.

Prevalence and impact of math anxiety on teachers. Although no large-scale studies of the prevalence of MA in elementary pre-service and in-service teachers have been conducted, the existence of the phenomenon has been well documented. For instance, Hembree (1990) found that the college majors with the greatest levels of MA are those preparing to be elementary teachers. In a study of pre-service elementary teachers, Ellsworth and Buss (2000) explored the attitudes of the teachers in relation to mathematics and science. They found a striking difference in the affect towards mathematics and science with 51% of the teachers reporting positive attitudes toward mathematics compared with 81% expressing positive attitudes towards science.

Similarly, in her study of pre-service elementary and secondary teachers, Ball (1990) found that only half of the elementary pre-service teachers reported that they enjoyed mathematics with one-third indicating dislike and avoidance of mathematics. Confirming the relationship between a negative attitude toward mathematics and course-taking patterns, Ball (1990) reported that the most anxious teacher candidates had taken the fewest mathematics courses in high school and college and tended to view mathematical ability as innate.

International studies into MA have also investigated the prevalence of MA in pre-service teachers. A study (DiMartino & Sabena, 2010) of Italian pre-service elementary teachers showed that negative emotions for mathematics were more prevalent than positive or ambivalent feelings. Bekdemir (2010) studied the mathematics anxiety and past classroom experiences of pre-service elementary Turkish teachers. More than half of the participants were rated as moderately math anxious with 6% anxious or high anxious.

The relationship of a teacher's MA to teaching mathematics has also been of interest to researchers. MA has been found to be significantly correlated with anxiety about teaching mathematics (Peker & Ertekin, 2011) and decreased confidence in teaching mathematics (Brady & Bowd, 2005; Gresham, 2009; Swars, et al., 2006). Thus, as Senger (1999) noted: "Millions of elementary school students attempt mathematics tasks daily in the context of thousands of elementary school classrooms under the direction of teachers describing themselves as 'math anxious' as a result of their personal histories in mathematics classes" (p. 199).

The MA of prospective elementary teachers is of concern to those in teacher preparation programs, a point of view expressed by Uusimaki and Nason (2004): "Addressing the causes of negative beliefs held by pre-service primary teacher education students about mathematics therefore is crucial for improving their teaching skills and the mathematical learning of their students" (p. 369). Should the existence of MA in teachers be of concern? Does teacher MA impact student learning? The next section explores these questions and the potential impact of teacher MA on students with learning disabilities.

The cycle of math anxiety. Teachers' positive attitude toward mathematics has been shown to correlate to student achievement in mathematics (Schofield, 1981). Conversely, Geist (2010) contended that "many teachers who have math anxiety themselves inadvertently pass it on to their students" (p. 29). The hypothesized cycle of teacher affect and student affect explains how MA in teachers ultimately has the potential to influence MA in students.

Martino and Sabena's (2010) study of pre-service Italian teachers revealed what the authors termed a "recurrent negative pattern" (p. 9). They contended that teachers' negative experiences as students resulted in insecurity, fear, and disgust at teaching mathematics. Bekdemir (2010) described the remaining part of the MA cycle by speculating:

> If these teacher trainees are mathematically anxious, they have a very good chance of becoming teachers who lack confidence in their own mathematical ability, have a negative attitude towards mathematics itself, and hence teach in ways that develop mathematics anxiety in their own students. Thus, a mathematics anxiety cycle is formed. (p. 313)

A recent study was able to link MA in teachers to MA in students. Beilock, et al. (2010) studied the math anxiety of first- and second-grade teachers and the mathematics achievement of their students. While there was not a relationship between the mathematics achievement and MA at the beginning of the school year, by the end of the school year, female students who were in the classroom of a highly math anxious teacher were more likely to have lower mathematics achievement and tended to ascribe mathematics ability to males.

While further study is needed on the direct link between MA in teachers and MA in students, Ashcraft and Krause (2007) argued that the classroom environment created by teachers with MA is potentially detrimental to students. They contended that "placing an at-risk child into such a teacher's class may be the ideal recipe for creating math anxiety, a hypothesis we are beginning to investigate" (Ashcraft & Krause, 2007, p. 247). Among the most mathematically at-risk students are those with learning disabilities.

Math anxiety in students with specific learning disabilities. Students with learning disabilities may be more susceptible to MA. In their study of children with and without mathematics learning disabilities, Lebens, Graff, and Mayer (2011) found that MA increased with age only for students identified with learning disabilities in mathematics. Also, they found that students with mathematics learning disabilities tend to respond more negatively to their teachers than students without learning disabilities. This suggests that students with learning disabilities are perhaps more at risk academically than originally thought.

Taken together these findings are troubling for students with learning disabilities and punctuate the importance of the attitude of teachers who work with this vulnerable population. To date, studies exploring the MA in teachers have almost exclusively focused on pre-service elementary teachers. The prevalence of MA in special education teachers has not yet been determined.

The potential for MA in teachers to negatively impact students with disabilities by perpetuating negative emotions is a real concern. Of equal interest to the present study were the beliefs teachers hold related to the discipline of mathematics and the teaching and learning of mathematics. The next section provides an overview of the literature related to teacher beliefs as well as beliefs specific to mathematics.

Beliefs

The emotions teachers express toward mathematics have an apparent impact on the classroom experiences of students. As discussed previously, emotions are transitory but intense within the affective domain, whereas beliefs are more stable and deeply held. Just as with emotions, beliefs teachers hold about mathematics are hypothesized to influence teacher instructional practice. In this section, the literature related to characteristics of beliefs in general will be summarized and the relationship between beliefs and instruction related to mathematics will be explored.

Characteristics of Beliefs

Beliefs can be very strong. Pajares (1992) articulated two factors related to the strength of beliefs, the magnitude of importance the individual ascribes to the belief and the degree of certainty with which the belief is held. The greater the importance and certainty, the deeper the belief is held. The strength of an individual’s beliefs can have powerful effects on memory. Deeply held beliefs influence what and how people recall events to the extent of “completely distorting the event recalled in order to sustain the belief" (Pajares, 1992, p. 317). Furthermore, Pajares (1992) contended that beliefs have a tendency to be self-perpetuating. Pajares (1992) wrote "there is the self-fulfilling prophecy—beliefs influence perceptions that influence behaviors that are consistent with, and that reinforce, the original beliefs" (p. 317). Thus, deeply held beliefs are highly resistant to change even in the face of challenge or anomalous information (Chinn & Brewer, 1993; Pajares, 1992).

Beliefs exists within belief systems (Rokeach, 1968). Belief systems store all the beliefs of the individual and help individuals define and make sense of the world and themselves (Pajares, 1992). It is hypothesized that beliefs have a quasi-structure related to the centrality of the belief to the individual (Pajares, 1992; Torner, 2002).

Beliefs are not always based on knowledge and individuals can hold beliefs that are inconsistent with one another (Pajares, 1992). In fact, "individuals tend to hold on to beliefs based on incorrect or incomplete knowledge, even after scientifically correct explanations are presented to them" (Pajares, 1992, p. 325).

Of greatest interest to the conceptual framework of this dissertation is Pajares' contention that beliefs "play a critical role in defining behavior", in this case, the teaching behavior, or instructional strategies, of mathematics teachers. According to Pajares (1992), "all teachers hold beliefs, however defined and labeled, about their work, their students, their subject matter, and their roles and responsibilities" (p. 314). The beliefs teachers hold are important and impact the experience of students in the classroom.

Beliefs of Teachers

Interest in teacher beliefs gained greater prominence in research in the 1970s with the advent of cognitive psychology. Attention expanded throughout the 1980s into beliefs and belief systems (Thompson, 1992). Early research examined teachers' attitudes towards mathematics and teaching (Cooney, 1999), beliefs in terms of factors impacting teacher performance (McLeod & McLeod, 2002), and how beliefs impact teachers' decision making processes (Cooney, 1999). The rationale for focusing on teacher beliefs was the potential for impacting teacher education and ultimately instructional practice (Pajares, 1992). Expanding out from investigations of teacher beliefs, researcher interest began to turn to affect (McLeod & McLeod, 2002).

Multiple educational researchers have studied the influence of beliefs on mathematics teachers' instructional practice in recent decades (Chapman, 2002; Cooney, 1999; Cooney, Shealy, & Arvold, 1998; Cross, 2009; Debellis & Goldin, 2006; DeSimone & Parmar, 2006; Furinghetti & Pehkonen, 2002; Gates, 2006; Gill, Ashton, & Algina, 2004; Goldin, 2002; Handal, 2003; Hart, 1999; Jordan & Stanovich, 2004; Leder & Forgasz, 2002; Lerman, 1999; McLeod & McLeod, 2002; Perry, et al., 2006; Stipek, Givvin, Salmon, & MacGyvers, 2001; Swan, 2007; Thompson, 1984; Wilkins, 2008). Researchers have contended that the relationship between beliefs and instructional practice has a considerable impact on the experience of students (Dossey, 1992; Jones, Wilson, & Bhojwani, 1997; Pehkonen & Torner, 1999; S. Wilson, 1999). Specifically, Pehkonen and Torner (1999) asserted that "the connection between a teacher's beliefs and his teaching practice is well-documented" (p. 5).

Rationale for Studying Beliefs: Links to Instruction

Teacher beliefs are important because beliefs are hypothesized to influence instruction (Jordan & Stanovich, 2004; Thompson, 1984), which directly impacts the mathematical learning experience of students. McLeod and McLeod (2002) contended that teacher beliefs are a key idea in understanding factors that contribute to achievement in mathematics. As Pajares (1992) noted, “Few would argue that the beliefs teachers hold influence their perceptions and judgments, which, in turn, affect their behavior in the classroom” (p. 307). Pehkonen and Torner (1999) maintained the importance of teacher beliefs in establishing the classroom experience of students: “Since the teacher is the central influential factor as an organizer of learning environments, his beliefs are also essential. Therefore, teachers’ and pupils’ mathematical beliefs play a key role when trying to understand their mathematical behavior” (p. 4).

Researchers have hypothesized that the way teachers approach mathematics teaching is related to the beliefs teachers hold about how mathematics is best learned and ultimately to their beliefs about the discipline of mathematics itself (Ball, 1990; Dossey, 1992; Ernest, 1989b; Schoenfeld, 1992; Thompson, 1992). According to Barkatsas and Malone (2005), “mathematics teachers’ beliefs have an impact on their classroom practice, on the ways they perceive teaching, learning, and assessment, and on the ways they perceive students’ potential, abilities, dispositions, and capabilities” (p. 71). Pehkonen and Torner (1999) illustrated how beliefs may impact the learning experience of students: “If a teacher thinks that the learning of mathematics happens at its best by doing calculation tasks, his teaching will concentrate on doing as many calculations as possible” (p. 5).

The role of beliefs as a primary mediating factor in mathematics teacher instructional practice (Thompson, 1984) has been represented in conceptual frameworks (Cross, 2009; Ernest, 1985). The next section will describe the Cross (2009) conceptual framework for relating teacher beliefs to instructional practice.

Conceptual Frameworks: Relating Beliefs to Instructional Practice

Cross (2009) proposed a conceptual framework to illustrate the relationship between beliefs about the nature of mathematics to teacher beliefs about teaching and learning. Beginning with beliefs about the nature of mathematics, Cross (2009) suggested that these beliefs directly relate to a teacher's conception of mathematical expertise (i.e., what it means to do mathematics) and to beliefs about teaching and learning mathematics. Cross (2009) wrote: "The hypothesized models presented demonstrate how these teachers' beliefs about the nature of mathematics, mathematics teaching, and mathematics learning were organized in a derivative manner where beliefs about teaching and learning appeared to stem from beliefs about the epistemology of mathematics" (p. 338). This derivative relationship is illustrated in Figure II.7.

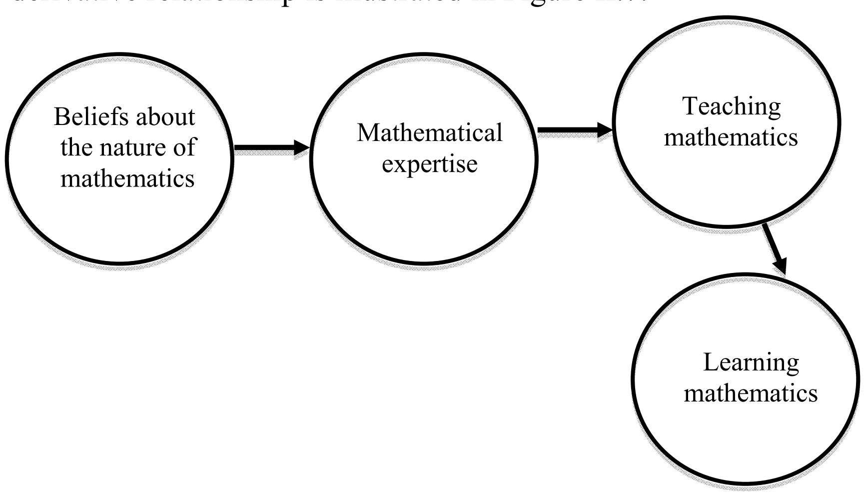

Figure II.7. Hypothesized Relationship among Beliefs about the Nature of Mathematics, Mathematical Expertise, and Teaching Learning Mathematics (adapted from Cross, 2009).

The relationship between teacher beliefs and instructional practice in mathematics frequently begins with a question about the nature of the discipline of mathematics itself. As Thompson (1992) wrote:

> One's conceptions of what mathematics *is* affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it….The issue, then, is not, What is the best way to teach? But, what is mathematics really all about? [emphasis in original]. (p. 127)

Because teacher beliefs about the nature of mathematics are hypothesized to be at the origin of instructional practice, explicating beliefs about the nature of mathematics, that is, what is at the heart of the discipline, has been of interest to researchers. A synthesis of the commonly held views follows.

The Nature of Mathematics: Perspectives and Implications

What is mathematics? This question is at the heart of characterizing the nature of mathematics. The nature of mathematics knowledge, what is means to do mathematics, is foundational to frameworks for understanding teacher instructional practice in mathematics (Cross, 2009; Ernest, 1985). Pehkonen and Torner (1999) acknowledge there are many possible answers to the question "What is mathematics?", and a number of researchers have attempted to classify beliefs that teachers hold about the nature of mathematics (Ernest, 1985, 1989a, 1989b; Jordan & Stanovich, 2004; Lerman, 1990; Lloyd, 2005; Nisbet & Warren, 2000; Pajares, 1992; Stipek, et al., 2001; Szydlik, Szydlik, & Benson, 2003; Thompson, 1992). In this section, common perspectives on beliefs about the nature of mathematics and the implications of these beliefs for instruction will be described.

Continuum of mathematics beliefs. Researchers have classified mathematics beliefs into as few as two categories (Cooney, 1999) up to as many as five categories (Ernest, 1985). The numerous categories of mathematics beliefs can be thought of as continuum from traditional to nontraditional (Raymond, 1997). Drawing on Raymond (1997), Figure II.8 illustrates a continuum with the traditional end portraying mathematics as memorization of rules, facts, and procedures and the nontraditional end portraying mathematics as a dynamic, problem-driven discipline.

Traditional	Even Mix	Nontraditional
Mathematics is fixed, predictable; consisting of rules, facts, and procedures.	Mathematics is a static, unified body of knowledge, both predictable and surprising.	Mathematics is a dynamic, problem-driven, continually expanding discipline.

Figure II.8. Continuum of Mathematics Beliefs (based on Raymond, 1997).

The next section will further refine and explicate the broad categories of traditional to nontraditional beliefs proposed by Raymond (1997) in order to clarify how beliefs about the nature of mathematics relate to instructional practice.

Categorizing beliefs and relating to instructional practice. Whereas Raymond (1997) proposed broad language for categorizing beliefs about the nature of mathematics, researchers have used various terminology to refine these categories (Cooney, 1999; Ernest, 1989b; Kuhs & Ball, 1986; Lerman, 1990; Raymond, 1997; Skemp, 2006; Swan, 2007). Ernest's (1985) foundational exploration into the relationship between mathematics beliefs and instructional practice provided categories for beliefs about the nature of mathematics that are useful organizers for the multiple terms used across studies. Ernest (1988) proposed three broad categories of beliefs: instrumentalist, Platonic or discovery, and problem solving. Teachers who view mathematics from an instrumentalist perspective see mathematics as a series of rules and procedures used for specific tasks. Teachers holding a Platonic or discovery view of mathematics consider mathematics to be a unified body of knowledge existing outside of cultural contexts that people discover through inquiry. Finally, teachers who hold a problem solving view of mathematics see mathematics as a dynamic body of knowledge, ever growing through inquiry and invention, and intricately interwoven into everyday living. In the section that follows, the literature related to each of the different views of mathematics is summarized and the Cross (2009) framework is used to relate the respective belief to instructional practice.

The instrumentalist perspective of mathematics. A number of researchers (Cooney, 1999; Ernest, 1989b; Kuhs & Ball, 1986; Lerman, 1990; Raymond, 1997; Skemp, 2006; Swan, 2007) describe a perspective of mathematics that can be characterized as an instrumentalist view (Ernest, 1989b). This perspective has been termed product-oriented (Ernest, 1989a), instrumental (Skemp, 2006), traditional (Raymond, 1997), transmission (Swan, 2007), absolutist (Lerman, 1990), content focused with emphasis on performance (Kuhs & Ball, 1986), and dualistic (Cooney, 1999). From an instrumentalist perspective, mathematics is "a discipline characterized by accurate results and infallible procedures, whose basic elements are arithmetic operations, algebraic procedures, and geometric terms and theorems" (Thompson, 1992, p. 127). Mathematics viewed from an instrumental point of view is a body of knowledge that comprised of rules, procedures, facts, and skills (Raymond, 1997; Swan, 2007). Mathematics is absolute and value-free (Lerman, 1990), consisting of a hierarchy of skills and concepts (Kuhs & Ball, 1986).

Table II.3 summarizes the literature of the various descriptions of mathematics that can be classified as from an instrumentalist perspective. The table includes the terminology, researcher, definition of the nature of mathematics, and the related perspectives on teaching and learning mathematics. A review of each category within the table shows commonalities among the different components.

Table II.3 The Instrumentalist Perspective of Mathematics, Teaching, and Learning

Terminology	Nature of mathematics	Perspective of teaching mathematics	Perspective of learning mathematics
Instrumentalist or Product-oriented (Ernest, 1989b) (Ernest, 1989a)	Mathematics consists of following rules. (Ernest, 1985)	Teaching mathematics involves emphasis on skill mastery and accurate performance of procedures (Ernest, 1989b). The role of teacher is the arbiter of truth and demonstrator of appropriate methods (Ernest, 1989a). Teaching involves demonstrating a single, correct method. Errors are to be avoided; accuracy is the goal of teaching. (Ernest, 1989a).	Learning consists of gaining knowledge of mathematical facts, rules, and methods (Ernest, 1989b). Learning mathematics involves compliance and mastery of skills (Ernest, 1989b). Learning involves practice of routine tasks after demonstration by the teacher (Ernest, 1989a).

Table II.3 (Continued)

Instrumental (Skemp, 2006)	Mathematics is about rules without concern for reasoning (Skemp, 2006).	Teaching involves demonstrating what to do in order to get correct answers (Leinwand & Fleischman, 2004).	Learning involves "memorizing and routinely applying procedures and formulas" (Leinwand & Fleischman, 2004, p. 88).
Traditional (Raymond, 1997)	Mathematics is an "unrelated collection of facts, rules, and skills; mathematics is fixed, predictable, absolute, certain, and applicable" (pp. 556-557).	The role of the teacher is to dispense knowledge, seek correct answers, and ensure mastery and memorization of skills and facts.	Learning mathematics involves passively receiving knowledge through demonstration, memorization, and mastery of algorithms.
Transmission (Swan, 2007)	Mathematics is a body of knowledge, consisting of standard procedures. Mathematics is a set of universal truths and rules to be conveyed to students.	The role of the teacher is to present a sequential curriculum to students, provide explanations, check for understanding, and correct misunderstandings.	Learning mathematics is and individual activity consisting of watching teacher demonstration, listening to explanations, and imitating procedures until fluent.
Absolutist (Lerman, 1990)	Mathematics is an absolute, value-free, consistent body of knowledge.	The role of the teacher is to share knowledge and algorithms discovered by mathematicians.	Not defined.
Content focused with emphasis on performance (Kuhs & Ball, 1986)	Mathematics consists of a hierarchy of skills and concepts.	The role of the teacher is to sequence the presentation of skills and concepts to students through demonstration, explanation, and definitions. Teaching mathematics involves mastery of rules and procedures.	Learning mathematics involves listening to teacher explanations, responding to teacher questions, following procedures to complete exercises.
Dualistic (Cooney, 1999)		Teaching mathematics involves an emphasis on product, telling, and certainty.	Learning involves acquisition of procedures without attention to meaning.

Table II.3 (Continued)

Summary	Mathematics is a body of knowledge consisting of facts, rules, and procedures.	Mathematics teaching involves conveying rules and demonstrating procedures to students.	Mathematics learning involves acquisition of rules and procedures through demonstration and practice.

According to an instrumentalist perspective, mathematics consists of following rules and procedures (Ernest, 1985) even when doing so may not have meaning or make sense (Cooney, 1999; Skemp, 2006). Examples of instrumental portrayals abound from the characterization of regrouping in subtraction as “borrowing” to the proceduralized approach to division of fractions called “invert and multiply” (Skemp, 2006), as summarized in the chant “ours is not to reason why just invert and multiply” (Wilensky, 1991).

The instrumentalist perspective of mathematics has implications for how mathematics is taught and learned. For instance, if mathematics consists of rules and procedures, then mathematics instruction consists of demonstrating methods and having students memorize facts and practice procedures. Stipek, Givvin, Salmon, and MacGyvers (2001) described the role of teachers with instrumental or traditional beliefs as follows: “[Instrumentalist] beliefs about mathematics confer upon teachers the responsibility of transmitting those rules to students. Consistent with this conception of mathematics and mathematics learning, the teacher is in control” (p. 214). Thus, the role of the teacher is that of arbiter of truth and demonstrator of appropriate methods (Ernest, 1989a), dispenser of knowledge and procedures discovered by mathematicians (Lerman, 1990; Raymond, 1997).

Further, the role of students from an instrumentalist perspective is to receive knowledge through demonstration, memorize facts, and accurately follow procedures (Leinwand & Fleischman, 2004; Raymond, 1997; Swan, 2007). Thus, developing expertise in mathematics consists of being able to accurately and efficiently apply rules and procedures.

Figure II.9, based on Cross (2009), illustrates the relationship between beliefs about the nature of mathematics, mathematical expertise, teaching mathematics, and learning mathematics from an instrumental perspective.

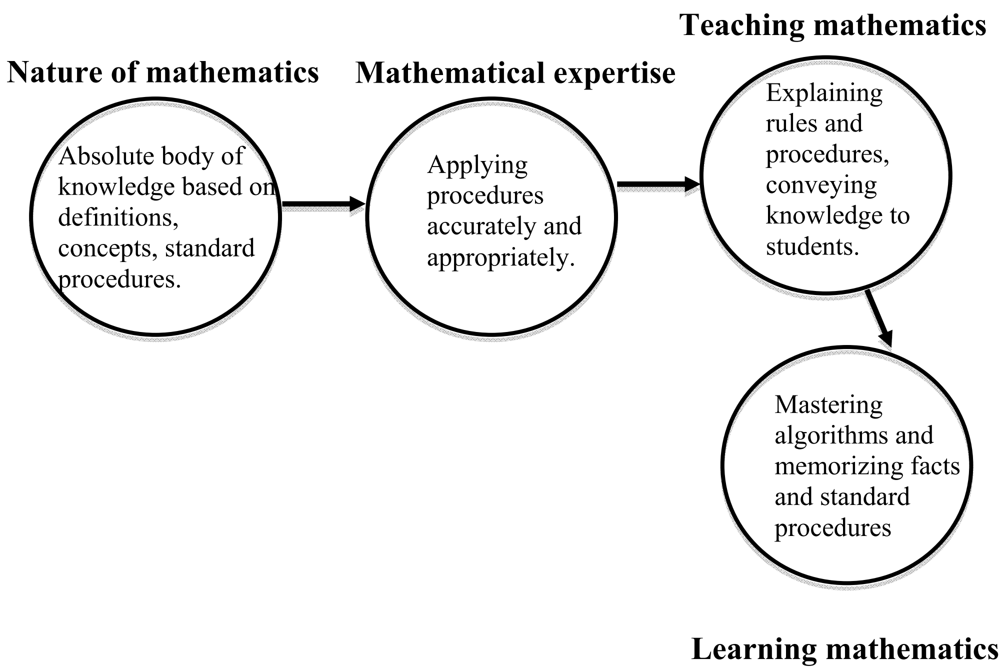

Figure II.9. The Relationship among Instrumental Beliefs about Mathematics, Mathematical Expertise, Teaching Mathematics, and Learning Mathematics (adapted from Cross, 2009).

From an instrumentalist perspective, mathematics is an absolute body of knowledge and universal truth consisting of a hierarchy of definitions, concepts, and standard procedures. Teaching mathematics consists of explaining rules and procedures, and the role of the teacher is to convey knowledge and skills to students through demonstration and explanation. Learning mathematics consists of mastering algorithms and memorizing facts and procedures that student receive through listening to teacher explanations and following demonstrated procedures.

The discovery perspective of mathematics. Another perspective of nature of mathematics can be classified as discovery view (Swan, 2007). Also termed a Platonist view (Ernest, 1989b), perception-based (Simon, Tzur, Heinz, & Kinzel, 2000), or content-focused with emphasis on conceptual understanding (Kuhs & Ball, 1986), the discovery view is based on a Platonist perspective where mathematics exists in an ideal realm, external to the human mind, able to be discovered through inquiry (Ernest, 1985). A perception-based perspective is based on the assumptions that mathematics is an interconnected and understood body of knowledge that exists independent of human activity; knowing mathematics involves firsthand experience in discovering the math, and mathematics is perceived the same by each individual. Again, referring to Ernest (August, 1988), this view can be considered a discovery perspective.

Table II.4 summarizes the literature of the various descriptions of mathematics that can be classified as from a discovery perspective. The table includes the terminology, researcher, definition of the nature of mathematics, and the related perspectives on teaching and learning mathematics. A review of each category within the table shows commonalities among the different components.

Table II.4 The Discovery Perspective of Mathematics, Teaching, and Learning

Terminology	Nature of mathematics	Perspective of teaching mathematics	Perspective of learning mathematics
Platonist view (Ernest, 1989b)	Mathematics is a consistent, connected and objective structure (Ernest, 1989b). Mathematics exists in an ideal realm that can be discovered (Ernest, 1985).	Teaching mathematics involves assisting learners in discovering mathematical truths. The role of the teachers is that of explainer or guide (Ernest, 1989b).	Learning mathematics involves developing a conceptual understanding and unified knowledge of mathematics truths (Ernest, 1989b).
Platonist (Dossey, 1992)	Mathematics objects exist beyond the mind in the external world.		
Perception-based perspective (M. Simon, et al., 2000, p. 594).		Teaching mathematics involves creating opportunities for students conceptualize mathematical relationships.	
Discovery (Swan, 2007)	Mathematics is a creative discipline.	Teaching mathematics involves assessing when students are ready to learn, providing a stimulating environment to facilitate exploration, and avoiding misunderstandings by the careful sequencing of experience.	Learning mathematics involves individual activity through exploration and reflection.

Table II.4 (Continued)

Content-focused with an emphasis on conceptual understanding (Kuhs & Ball, 1986)		Teaching mathematics involves a balance between the content of mathematics and learners. Teaching mathematics involves developing conceptual understanding in students.	Learning mathematics involves making sense of the material presented by the teacher through presentations, demonstrations, and discovery-based activities.
Relational (Skemp, 2006)		Teaching mathematics involves emphasis the why of learning. Teaching mathematics involves explaining, reasoning, and using multiple representations (Leinwand & Fleischman, 2004).	Learning mathematics involves developing one's own understanding of content.
Summary	Mathematics is dynamic discipline that exists external to human beings and can be discovered.	Teaching mathematics involves guiding learners to discover mathematical concepts, emphasizing why mathematical relationships exist.	Learning mathematics involves developing one's own conceptual understanding of mathematical concepts and relationships.

From a discovery perspective, mathematics is a discipline in which people can uncover concepts that underlie mathematical rules and procedures through the assistance of a knowledgeable other (Ernest, 1989b). In contrast to an instrumentalist perspective where memorization is important and procedures without connections to meaning are emphasized, a discovery perspective emphasizes the meaning or concepts behind mathematics rules and procedures (Stein, Grover, & Henningsen, 1996). An example of the discovery approach would be using different colored counting chips to represent positive and negative numbers. Because positive and negative numbers cancel one another out, adding positive and a negative numbers actually results in subtraction.

The discovery perspective of mathematics has implications for how mathematics is taught and assumed to be learned. Teaching mathematics involves assisting students in discovering the mathematical truths that underlie mathematical procedures (Ernest, 1989b) and creating opportunities for students to develop conceptual understanding (Kuhs & Ball, 1986) by emphasizing the "why" of learning (Leinwand & Fleischman, 2004). Thus teaching mathematics consists of using multiple representations of ideas (Leinwand & Fleischman, 2004) and careful sequencing of facilitated exploration (Simon, et al., 2000) to assist students in discovering mathematical relationships and truths.

In the discovery view, the teacher is the necessary mediator between mathematics and the learner (Ernest, 1989b), one responsible for creating opportunities for students to conceptualize mathematical relationships (Simon, et al., 2000). From a discovery perspective, learning mathematics involves deepening one's own understanding of content (Skemp, 2006) by making sense of material and activities orchestrated by the teacher (Kuhs & Ball, 1986; Swan, 2007).

Figure II.10, based on Cross (2009), illustrates the relationship between beliefs about the nature of mathematics, mathematical expertise, teaching mathematics, and learning mathematics from a discovery perspective.

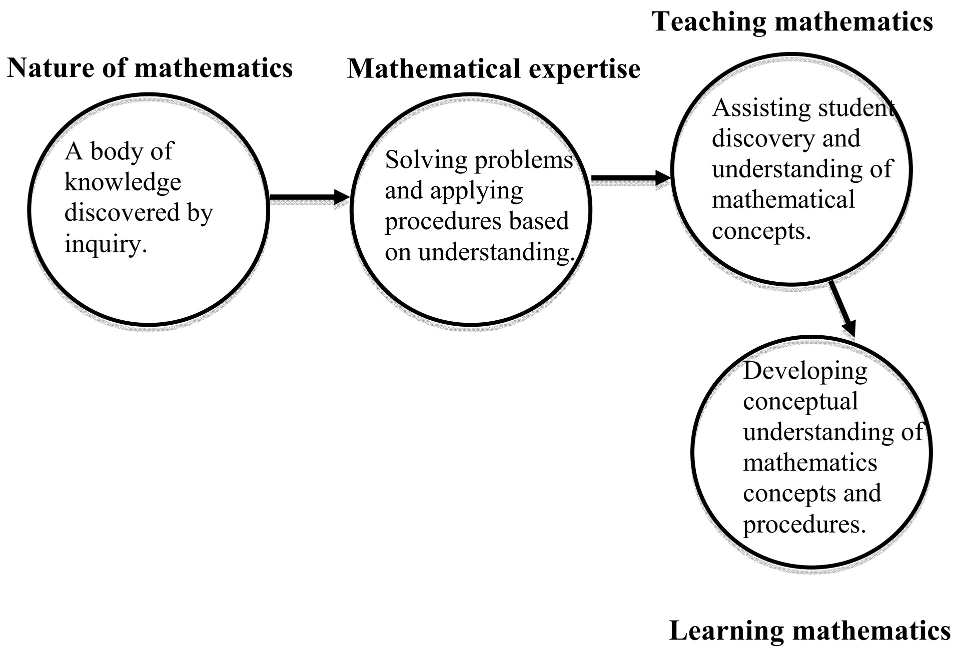

Figure II.10. The Relationship among Discovery Beliefs about Mathematics, Mathematical Expertise, Teaching Mathematics, and Learning Mathematics (adapted from Cross, 2009).

In summary, from a discovery perspective, mathematics is a body of knowledge separate from human experience that can be discovered through inquiry. Teaching mathematics involves assisting students in the discovery of mathematical concepts and truths, and learning mathematics involves coming to understand mathematics concepts.

The problem solving perspective of mathematics. A final category of commonly held beliefs about the nature of mathematics can be called a problem solving view, which characterizes the nature of mathematics as a dynamic discipline, created through human activity (Ernest, 1989b; Simon, et al., 2000). Others have called this perspective nontraditional (Raymond, 1997), fallibilist (Lerman, 1990), conceptions-based (Simon, et al., 2000), connectionist (Swan, 2007), and learner focused (Kuhs & Ball, 1986).

A problem solving perspective is based on the assumptions that mathematics is created through human activity, what individuals see is constrained by their current conceptions, and mathematics learning is a process of transforming one's existing ideas into more sophisticated ways of knowing (Simon, et al., 2000). From this perspective, mathematics is: (a) a method of inquiry or way of thinking (Kuhs & Ball, 1986), (b) contextually and culturally bound (Ernest, 1989b; Lerman, 1990), (c) created through discussion (Swan, 2007), and (d) subject to change and ever expanding through human contribution (Lerman, 1990; Raymond, 1997).

Table II.5 summarizes the literature of the various descriptions of mathematics that can be classified as from a problem solving perspective. The table includes the terminology, researcher, definition of the nature of mathematics, and the related perspectives on teaching and learning mathematics. A review of each category within the table shows commonalities among the different components.

Table II.5 The Problem Solving Perspective of Mathematics, Teaching, and Learning

	Nature of mathematics	Perspective of teaching mathematics	Perspective of learning mathematics
Problem solving view (Ernest, 1989a, 1989b)	Mathematics is a dynamic discipline in a social and cultural context (Ernest, 1989b)	The role of the teacher is that of facilitator of problem solving (Ernest, 1989b). Teaching mathematics involves confident problem posing and encouraging multiple solution methods, including those that are student generated (Ernest, 1989a).	Learning mathematics involves active construction of understanding (Ernest, 1989b). Learning mathematics consists of engaging in empirical testing of conjectures and reasoning through discourse (Ernest, 1989a).
Nontraditional (Raymond, 1997)	Mathematics is dynamic, problem driven discipline. Mathematics is ever expanding, relative, and aesthetic.	The role of the teacher is to guide learning, pose challenging questions, and promote knowledge sharing.	The role of the student is that of autonomous explorer through active learning. Learning mathematics involves problem solving and explaining understanding rather than memorization and algorithms.
Fallibilist (Lerman, 1990)	Mathematics is a compendium of the accumulated experience of human thought. Mathematics is a social construction, relative to time and place, and subject to change.	Teaching mathematics involves facilitating student development of knowledge.	Learning mathematics consists of students engaging in problem-posing and problem solving.

Table II.5 (Continued)

Conceptions-based perspective (Simon, et al., 2000)		Teaching mathematics involves understanding students' conceptions and determining activities to transform, build, modify, or re-create these conceptions.	
Connectionist (Swan, 2007)	Mathematics is an interconnected body of ideas created together through discussion.	Teaching mathematics involves dialogue between teacher and students in which meanings and connections are explored verbally.	Learning mathematics is an interpersonal activity in which students are challenged and arrive at understanding through discussion.
Learner-focused (Kuhs & Ball, 1986)	Mathematics is a method of inquiry or a way of thinking.	Teaching mathematics involves focusing on students' personal construction of mathematical knowledge. Teaching mathematics consists of stimulating student learning through questioning and experiences to reveal the inadequacy of inappropriate conceptions.	Learning mathematics is a process of constructing understanding through inquiry.
Summary	Mathematics is a dynamic discipline that is contextually bound. Mathematics is a way of thinking, a discipline of inquiry.	Teaching mathematics involves understanding student conceptions of mathematics and facilitating modifications of student conceptions through problem posing and discourse.	Learning mathematics involves active construction of understanding by the learner through problem solving, inquiry, and discourse.

From a problem solving perspective, teaching mathematics involves facilitating student learning by posing interesting and challenging problems, encouraging multiple solution methods, and promoting knowledge sharing (Ernest, 1989b; Lerman, 1990; Raymond, 1997). Teaching mathematics focuses on students' personal construction of mathematical knowledge (Kuhs & Ball, 1986) by understanding students' current conceptions and determining instructional activities to modify, transform, or build new conceptions (Simon, et al., 2000). Thus, the role of teachers is that of facilitator of student understanding (Ernest, 1989b).

Learning mathematics from a problem solving perspective involves active construction of understanding on the part of the student (Ernest, 1989b) through inquiry, discourse, problem-posing, and problem solving (Ernest, 1989b; Kuhs & Ball, 1986; Lerman, 1990).

Figure II.11, based on Cross (2009), illustrates the relationship between beliefs about the nature of mathematics, mathematical expertise, teaching mathematics, and learning mathematics from a problem solving perspective.

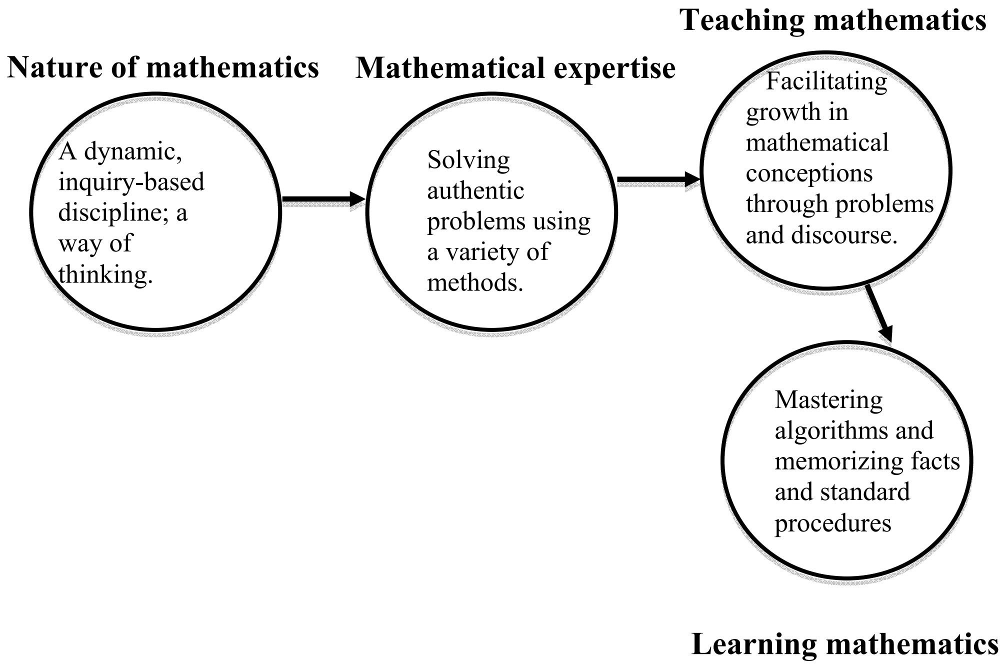

Figure II.11. The Relationship among Problem Solving Beliefs about Mathematics and Mathematical Expertise, Teaching Mathematics, and Learning Mathematics (adapted From Cross, 2009).

In summary, the problem solving perspective portrays mathematics as a dynamic, inquiry based discipline and a way of thinking. Teaching mathematics involves understanding student conceptions of mathematics and facilitating modification and development of student conceptions through problem posing and discourse. Learning mathematics involves active construction of understanding by the learner through problem solving, inquiry, and discourse.

The preceding discussion demonstrates that hypothesized relationships about the role of beliefs in teaching and learning mathematics have been clearly articulated in the literature. Beginning with one's beliefs about the nature of mathematics, the link to how mathematics is best taught and learned seems to naturally follow. However, there are limitations and constraints associated with the hypothesized relationships between beliefs and educational practices/outcomes that must be recognized.

Limitations of Conceptual Frameworks

A review of the literature would be incomplete without acknowledging that there are limitations to the direct connection between beliefs and instructional practices enacted in the classroom that the Cross (2009) framework supposes. Ernest (1989) has proposed another conceptual framework (Figure II.12) that suggests how beliefs about mathematics ultimately relate to classroom practice. This framework indicates that a teacher's instructional practice begins with a personal philosophy of mathematics (what it means to do mathematics), which in turn influences their conceptions about teaching and learning mathematics. Here Ernest distinguishes between espoused beliefs and enacted beliefs, suggesting that espoused beliefs are influenced by the constraints and opportunities afforded by the social context and realities of teaching, in turn becoming enacted beliefs that translate into classroom practices.

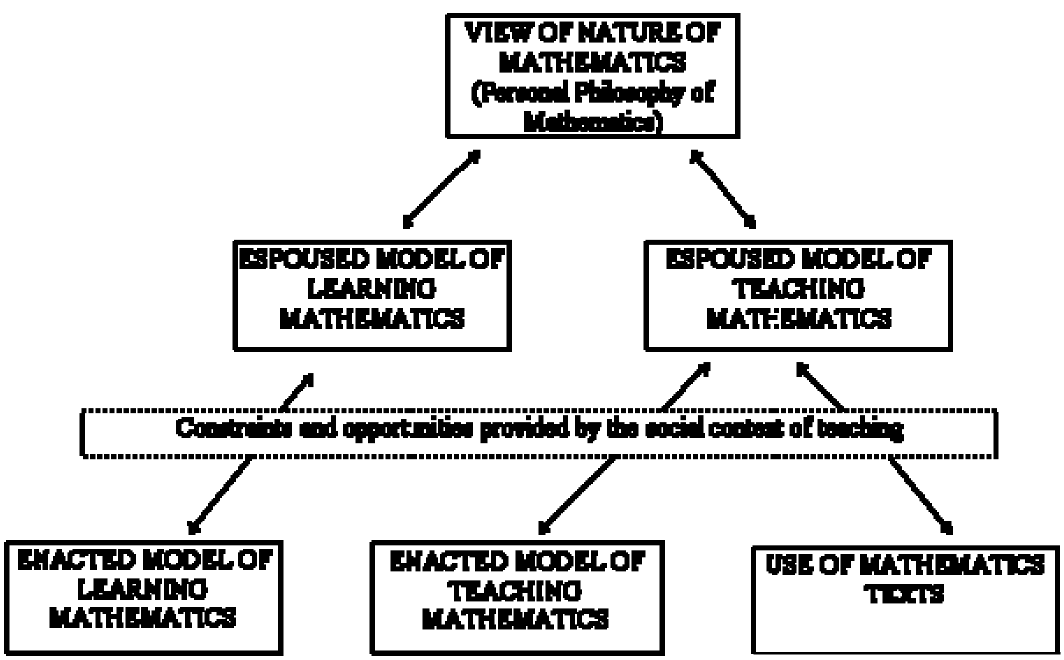

Figure II.12. Conceptual Framework Relating Mathematics' Teacher Beliefs to Teaching Practices (based on Ernest, 1989).

To Ernest, the link between beliefs and practices is not as simplistic as the Cross (2009) framework might suggest. Similarly Handal (2003) indicated that there is not a one–to-one correspondence between beliefs and practice, noting the mediating effects of school and classroom culture. Referencing Clark and Peterson (1986), Handal (2003) suggested that beliefs are a filter through which instructional decisions are made. Lerman (1999) and Skott (2009) questioned the distinction between espoused and enacted beliefs, suggesting instead that beliefs are contextualized and proposing a situated view of the relationship between beliefs and practice. Thus, the relationship between beliefs and practice is likely more complex than the framework suggested by Cross.

Special Educators and Beliefs about Mathematics

The literature cited thus far has largely involved mathematics teachers and mathematics teaching. However, a review of the literature related to students with learning disabilities in mathematics indicates that the vast majority of studies relating to the acquisition of basic skills and a strong endorsement of direct instruction implying specific beliefs about the nature of mathematics (Kroesbergen & Van-Luit, 2003). Studies from special education tend to emphasize computational fluency and solving routine problems (Bryant, Bryant, & Hammill, 2000; Calhoon, et al., 2007; Fuchs et al., 2005; Geary, Brown, & Samaranayake, 1991; Simon & Hanrahan, 2004; Woodward, 2006). A strong argument could be made that the perspective of special education research questions and methodologies reflect a view of mathematics as a collection of procedures and rules.

With much of the literature in special education related to mathematics reflecting a procedural, utilitarian view, a question to ask is whether the views of special educators differ from those of mathematics teachers. In their study, Gagnon and Maccin (2007) sought to clarify the relationship between teacher beliefs, educational background, and instructional practices. This survey study of mathematics and special education teachers indicated no significant difference between these teachers' perceptions of mathematics. The most commonly reported definitions of mathematics were (a) a necessary tool for life, (b) a language, and (c) a means of logical thought. The authors found an unanticipated low correlation between teacher beliefs about mathematics and their reported use of empirically based instructional strategies. The authors suggest further research into the role of instructional setting, student characteristics, and teacher knowledge, reinforcing the mediating effect of the social context of teaching highlighted in Ernest's framework.

Whereas the literature suggests that special educators tend to hold more instrumentalist views of mathematics, there is simply not sufficient data to draw conclusions about the influences of beliefs and attitudes about mathematics. The present study sought to address this gap in the literature.

Conclusion

Research suggests a link between teacher beliefs and attitudes about mathematics and their instructional practice. The mathematics achievement of students is directly related to the instruction they receive. Students with SLD are arguably most at risk for academic failure in mathematics; therefore, inquiry into the beliefs and attitudes that special education teachers hold about mathematics may offer insight into how to better support student learning. Research related to the beliefs of general education mathematics teachers indicates that there are commonly held views about the nature of mathematics and how mathematics is learned and how it should be taught. The literature has not yet been inclusive of special education teachers. The present study seeks to address the gap in the literature related to the nature and impacts of the beliefs and attitudes held by special educators about the discipline of mathematics, teaching mathematics, and learning mathematics.

CHAPTER 3

METHODS

The goal of this study was to understand the beliefs and attitudes special education teachers hold about the discipline of mathematics and the teaching and learning of mathematics. The study is important because of the hypothesized link between the beliefs and attitudes teachers hold about mathematics and their instructional practices (Ernest, 1988; Pajares, 1992; Thompson, 1992). Students with specific learning disabilities (SLD) persistently underachieve in mathematics (Cortiella, 2011), thus the instruction students with SLD experience is of great importance to their success. Given the absence of research related to beliefs and attitudes of special educators relative to mathematics and teaching and learning mathematics, the research question for the current study was: what is the nature of the beliefs and attitudes held by special educators about the discipline of mathematics and the teaching and learning of mathematics? The research question was explored through four sub-questions: (a) what are the attitudes of special educators about mathematics, (b) what are the beliefs of special educators about the discipline of mathematics, (c) what are the beliefs of special educators about teaching mathematics, and (d) what are the beliefs of special educators about learning mathematics? The study explored the question using a mixed method design to provide insight into the approaches special educators take to teaching an at-risk student population.

The goal of the mixed method study was to begin to understand the complex phenomena of special education teachers' beliefs and attitudes about mathematics and the teaching and learning of mathematics. The research objective was exploration (Johnson & Christensen, 2008). Exploratory research entails "generat[ing] information about unknown aspects of a phenomenon" (Teddlie & Tashakkori, 2009, p. 25) as opposed to explanatory research which seeks to test hypotheses and theories (Johnson & Christensen, 2008). As demonstrated in Chapter 2, the research base related to the mathematics attitudes and beliefs of special educators is limited; therefore, this exploratory study has the potential to generate information upon which future studies may build. The purpose for using a mixed method research design was for complementarity, which "capitalizes on the inherent method strengths and counteracting inherent biases in methods" (Greene, Caracelli, & Graham, 1989, p. 259).

The study utilized quantitative measures of beliefs and attitudes about mathematics complemented by qualitative measures that provided "elaboration, enhancement, illustration, clarification of the results" (Greene, et al., 1989, p. 259). The quantitative strand of the study examined the degree of alignment of participant beliefs with reform-based mathematics and the degree of mathematics anxiety the participants experienced. The qualitative strand of the study complemented the quantitative phase by further examining participant beliefs and attitudes through a semi-structured interview. Results of the qualitative and quantitative strands were mixed to examine what conclusions or meta-inferences might be made (Teddlie & Tashakkori, 2009).

A complete description of the study methods is provided in this chapter. The description includes the rationale for the appropriateness of a mixed method research design for answering the research question, the multiple phases of the study, data analysis within a fully mixed design, and the ways in which inferences were drawn to answer the research question.

Study Design

The study utilized a fully mixed, sequential, qualitative dominant mixed method study design or quan --> QUAL using Leech and Onwuegbuzie's (2006) notation. A mixed method research design incorporates both quantitative and qualitative approaches to form research questions, determine research methods, collect data, analyze data, and make inferences (Teddlie & Tashakkori, 2009). Mixed method research designs "allows the researcher to use the strengths of both quantitative and qualitative analysis techniques so as to understand phenomena better" (Onwuegbuzie & Teddlie, 2003, p. 353).

Quantitative measures were administered to ascertain the level of attitudes and beliefs of the study sample followed by qualitative data collection from a sub-sample of the full study sample to understand these attitudes and beliefs more deeply. Data collection was richer by using quantitative measures of attitudes and beliefs augmented by qualitative data about these attitudes and beliefs from the participants.

Collins, Onwuegbuzie, and Sutton (2006) proposed that mixed method research follow a thirteen step process. Those steps include:

1. determining the goal of the study,
2. formulating the research objective(s),
3. determining the research/mixing rationale(s),
4. determining the research/mixing purpose(s),
5. determining the research question(s),
6. selecting the sampling design,

7. selecting the mixed-methods research design,
8. collecting the data,
9. analyzing the data,
10. validating/legitimating the data and data interpretations,
11. interpreting the data,
12. writing the final report, and
13. reformulating the research question(s) (Collins, et al., 2006, pp. 69-70).

The current study adhered to the steps delineated by Collins, Onwuegbuzie, and Sutton (2006) with relevant aspects, steps six through thirteen, described in this chapter.

Mixed method study designs are described along three dimensions: (a) the level of mixing, either partially or fully mixed, (b) the time orientation, either concurrent or sequential, and (c) the emphasis of the research approach, with either equal status given to both qualitative and quantitative approaches or with one approach dominating (Leech & Onwuegbuzie, 2006). This study was fully mixed, which involves "mixing of quantitative and qualitative techniques within one or more stages of the research process or across these stages" (Leech & Onwuegbuzie, 2006, p. 267). With respect to time orientation, the study utilized a sequential design, a design in which the data collected and analyzed in one phase of the study were used to inform the next phase (Onwuegbuzie & Collins, 2007). Specifically, Phase One involved quantifying the mathematics anxiety (MA) and beliefs related to teaching and learning mathematics of special educators. Demographic data were collected also in Phase One, such as gender, educational background, teaching experience, and teaching experience in mathematics, to inform the selection of a sub-sample to participate in Phase Two the study.

Phase Two of the study was in the form of a phenomenological study involving the sub-sample of special educators from Phase One of the study. A phenomenological study, as defined by Creswell (2007), “describes the meaning for several individuals of their *lived experiences* of a concept or phenomenon” (p. 57, emphasis in original). Results from Phase One were not only used to determine the sub-sample for Phase Two qualitative inquiry, but also quantitative data from Phase One was integrated with qualitative data from Phase Two to better understand the phenomenon of special educators’ beliefs and attitudes about mathematics and teaching and learning mathematics.

The study was conducted in three phases and is illustrated in Figure II.13. Phase One consisted of quantitative data collection and analysis to quantify participants’ mathematics anxiety and beliefs related to reform oriented mathematics teaching and learning. Phase Two consisted of qualitative inquiry to explore participants’ attitudes related to mathematics, their beliefs about the nature of mathematics, and their beliefs about how mathematics is learned and should be taught. Phase Three of the study consisted of data interpretation, in which data from both phases were integrated and conclusions were drawn and verified. With respect to emphasis of research approaches, qualitative data analysis was given greater weight, as it was used to triangulate quantitative findings and to explore the phenomena of mathematics attitudes and beliefs.

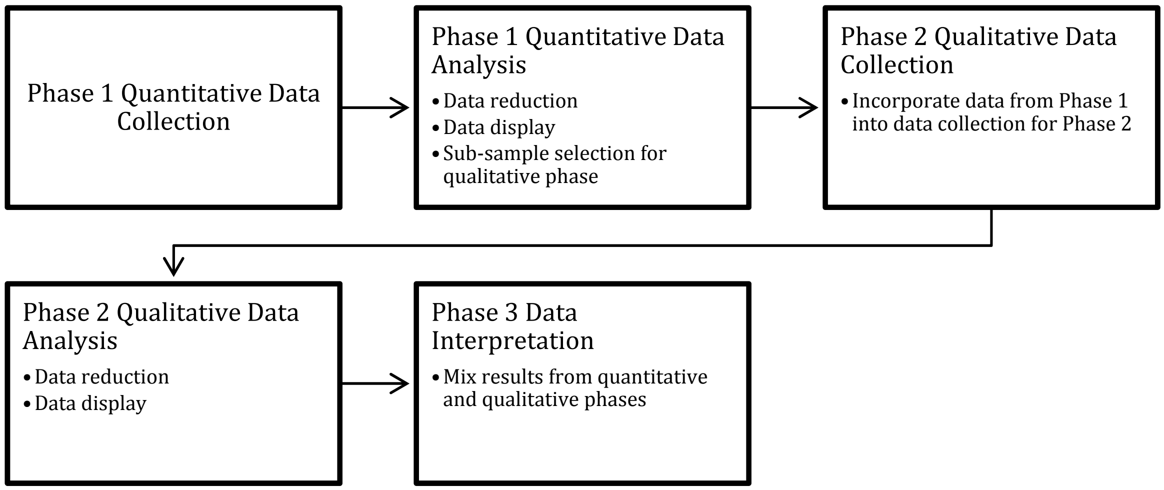

Figure II.13. Study design.

Sampling Design

Choices about participants (the study sample) for a research project are critical to the outcomes of the study (Onwuegbuzie & Collins, 2007; Onwuegbuzie & Leech, 2007a). In this section, the sampling design (inclusive of sample size and sampling schemes) is described.

A nested sampling design was applied consisting of two sequential phases. A nested sampling design is one in which "one or more members of the subgroup represent a sub-sample (e.g., key informants) of the full sample" (Onwuegbuzie & Leech, 2007a, p. 240). Participants for the full study sample were selected through purposeful sampling, which involves selecting "information-rich cases whose study will illuminate the questions under study" (Patton, 2002, p. 230). Study participants were required to be special education teachers who provide instruction in mathematics for students with SLD. A sub-sample of participants from Phase One of the study was selected for Phase Two of the study using extreme case sampling (Miles & Huberman, 1994) achieved by selecting participants from Phase One with the highest and lowest levels of MA and alignment of mathematics beliefs.

Onwuegbuzie and Leech (2007a) contend that the sample design and sampling scheme(s) for a study must be logically tied to the type of generalization researchers intend to make. The types of generalizations this study attempted to make were internal statistical generalization and analytic generalization. Onwuegbuzie and Leech (2007a) define internal statistical generalization as "making generalizations or inferences on data extracted from one or more representative or elite participants to the sample from which the participant(s) was drawn" (p. 240). Internal statistical generalization relates to the sample itself, not to the population from which the sample is drawn, thus avoiding the common interpretation error of attributing generalizations from a sample to the population (Onwuegbuzie & Daniel, 2003). Onwuegbuzie et al. (2009) suggest that in order for internal statistical generalization to be possible, the subsample on which the generalizations will be made must be representative of the study sample. In the case of this study, the subsample was a subset of the study sample. An analytic generalization involves generalizing to a theory, not to a population (Firestone, 1993). According to Leech and Onwuegbuzie (2010), "small and purposive samples tend to facilitate analytic generalizations" (p. 64). Thus, the sampling scheme for this study was consistent with the overall sample design and the study design.

A priori power analysis was conducted to determine the sample size to ensure adequate power for statistical analyses of Phase One. The analysis was conducted using the G*Power online calculator developed by Faul, Erdfelder, Lang, & Buchner (2007).

According to the analysis, the full sample size needed to be greater than or equal to 47 in order to ensure adequate power for the quantitative analysis in Phase One; a total of 48 special educators participated in Phase One of the study. From the full sample of participants for Phase One of the study, a smaller subsample was selected along two dimensions, level of math anxiety and degree of alignment of beliefs with NCTM reforms, for Phase Two of the study, a phenomenological case study. A subsample size of 6 to 10 is considered appropriate for a phenomenological design of the second phase of the study (Onwuegbuzie & Collins, 2007).

Purposive sampling is not used for external generalization but to obtain insights and to “maximize understanding of the underlying phenomenon” (Onwuegbuzie & Collins, 2007, p. 287). The method for identifying the purposeful sample through the stratification and classification process is fully described subsequent sections.

Participants

This section describes the recruitment process and demographic information for the study sample that participated in the study.

Recruitment

The study participants were 48 elementary, middle, and high school special education teachers from Colorado who whose teaching assignment at the time of the study involved teaching, co-teaching, or supporting mathematics instruction for students with specific learning disabilities (SLD). Participants volunteered to participate through a recruitment process that utilized professional educator organizations and networks in Colorado, including the Colorado Council for Learning Disabilities (CCLD), the Colorado Metro Math Intervention Team (CoMMIT), the Colorado Math Leaders (comath), and the Colorado Council of Teachers of Mathematics (CCTM). These professional educator networks consist of email distribution lists of district and school leaders who have contact with special educators in Colorado school districts.

Multiple recruitment emails were used to solicit names and email addresses of prospective participants. Recruitment emails described the purpose of the study, the research question to be answered, requirements for participation, the two-phase study design, and nature of data to be collected. All recruitment communications can be found in Appendix A. Recipients of the recruitment emails were asked to forward the message to prospective participants. Recruitment for Phase One of the study lasted from April through May 2012 and was ceased once the minimum of 47 (to achieve power for data analysis) participants completed the online survey. A total of 48 special education teachers ultimately participated.

Demographic Information

The Phase One survey included collection of demographic information (see Appendix B). A summary of data can be found in Table III.6. The sample consisted of 44 females (91.67%) and four males (8.3%). The sample was relatively diverse with 32 White (not Hispanic) participants (66.67%), eight White/Hispanic participants (16.67%), one Hispanic participant (2.08%), three African-American/White/Hispanic participants (6.25%), and two African-American/Hispanic/Pacific Islander/White participants (4.17%). Two participants (4.17%) did not provide information on ethnicity.

The sample was highly educated with over 79.17% (n = 38) indicating coursework beyond a Bachelor's degree. Specifically, 64.58% (n = 31) reported having a Master's Degree, 10.42% (n = 5) reported having more than one Master's Degree, and one participant reported being in a PhD program. Of the 48 participants, six (25%) reported teaching a pullout mathematics class for students with SLD at the time of the study. Fourteen participants (29.17%) reported serving students only in a general education classroom. Twenty-eight (58.33%) reported serving students in both models. One participant reported not currently delivering services in either model

Information on participant geographic distribution was not part of the Phase One survey but such information could be gleaned from email addresses that participants provided. Forty-six of the 48 participants provided an email address associated with their school district. The study sample included a geographically diverse representation of educators from across the state of Colorado with 23 (47.92%) of participants from school districts along Colorado's front-range, which consists of cities located on the Interstate 25 corridor, the most populous part of the state. Nine participants (18.75%) were from mountain area school districts, which are located in the mountainous areas of the state. Five participants were from rural districts (10.42%) located mainly on the eastern planes of the state. Eight participants were from western slope districts (16.67%), which are located on the western part of the state. One participant (2.1%) was from an online school. Two participants (4.17%) did not provide a school district identifiable email address. Distribution of participants across grade bands was varied with 10 participants (20.83%) reporting teaching at the elementary level only, three (6.25%) reporting an assignment including both elementary and middle school, 11 (22.92%) teaching middle school only, three (6.25%) teaching at the middle and high school, 16 (25%) reporting teaching only high school, and five (10.42%) with assignments at all three levels (elementary, middle, and high school).

Table III.6 Demographic Information about Study Participants

	N	Percentage
Sex		
Female	44	91.67%
Male	4	8.33%
Ethnicity		
African-American/White/Hispanic	3	6.25%
African-American/Hispanic/Pacific Islander/White	2	4.17%
Hispanic	1	2.10%
White/Hispanic	8	16.67%
White (not Hispanic)	32	66.67%
Did not provide information	2	4.17%
Highest Degree Attained		
Bachelor's degree	10	20.83%
Graduate hours beyond Bachelor's	2	4.17%
Master's degree	31	64.58%
More than one Master's degrees	5	10.42%
PhD (in progress)	1	2.10%
Geographic Region		
Front-range	23	47.92%
Mountain	9	18.75%
Online	1	2.10%
Rural	5	10.42%
West-slope	8	16.67%
Unknown	2	4.17%
Level		
Elementary school only	10	20.83%
Elementary and middle school	3	6.25%
Middle school only	11	22.92%
Middle and high school	3	6.25%
High school only	16	25%
All levels	5	10.42%
Service Delivery Model		
Pullout	6	25%
General education only	14	29.17%
Both models	28	58.33%
Neither	1	2.10%

Measures

The first phase of data collection involved administration of the *Math Anxiety*

Rating Scale: Short Version (*MARS-SV*) (see Appendix C) and the *Mathematics Beliefs Instrument* (*MBI*) (see Appendix D) to the study sample. The two instruments were combined into one online survey that also included questions related to the demographics of the respondents. The second phase of data collection involved a semi-structured interview (see Appendix E) with questions designed to explore participants' attitudes and beliefs about mathematics, as well as relevant demographic information. Specific information about data collection tools is detailed next.

Mathematics Anxiety

The Math Anxiety Rating Scale (*MARS*) is the most frequently used instrument to measure mathematics anxiety (Capraro, Capraro, & Henson, 2001), initially developed in the 1970s as a 98-item Likert scale survey with questions designed to gauge the respondent's level of anxiety related to mathematics tasks (Richardson & Suinn, 1972). The *MARS* was originally designed to measure mathematics anxiety in adults. A shortened 30-item survey, the *Mathematics Anxiety Rating Scale: Short Version* (*MARS-SV*), was developed to reduce the length of the 98-item survey (Suinn & Winston, 2003). The 30-item *MARS-SV* is simple to administer and can be completed in under twenty-minutes (Suinn & Winston, 2003).

MARS-SV items were designed to measure the anxiety respondents have toward activities that involve mathematics, such as performing calculations (e.g., question 1), experiences in mathematics classes (e.g., questions 2 and 3), and using mathematics in everyday life (e.g., question 4). Respondents rate their anxiety on a five-point scale with descriptors of: (a) not at all, (b) a little, (c) a fair amount, (d) much, or (e) very much. Sample questions include:

1. Dividing a five-digit number by a two-digit number in private with pencil and paper.
2. Taking an examination (final) in a math course.
3. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.
4. Totaling up the dues received and the expenses of a club you belong to.

The original reliability and validity study for the *MARS* was conducted with undergraduate students (Richardson & Suinn, 1972). Reliability of *MARS* scores is high (Capraro, et al., 2001) with a test-retest reliability coefficient of .78 (Richardson & Suinn, 1972). Discriminant construct validity for the *MARS* was established by correlating high ratings on the *MARS* with lower performance on a measure of mathematics achievement, the *Differential Aptitude Test* (*DAT*) (Richardson & Suinn, 1972). The correlation between performance on the *MARS* and the *DAT* was -.35 indicating that higher mathematics anxiety was associated with lower performance on the mathematics achievement test (Richardson & Suinn, 1972).

The *MARS-SV* was developed in the early 2000s (Suinn & Winston, 2003). Internal consistency of the *MARS-SV* was measured against the *MARS*. A Cronbach alpha of .96 was found, an indication of high internal consistency, and a test-re-test reliability of .91 was found (Suinn & Winston, 2003). Concurrent validity of the *MARS-SV* with the *MARS* was conducted using a Pearson correlation with $r = .92$ indicating a high correlation (Suinn & Winston, 2003).

Mathematics Beliefs

Investigations of mathematics beliefs tend to consist of instruments developed specifically for a research study such as the Indiana Mathematical Beliefs Scale (Kloosterman & Stage, 1992), an instrument by Wilkins (2008), and an instrument by Ambrose and Clement (2004). Thus, selection of a measure of beliefs for the present study required more research into the purposes for the different measures. Given that the present study was intended to explore the beliefs of educators in relation to reform-based mathematics, the *Mathematics Beliefs Instrument* (*MBI*) was selected. The *MBI,* and its predecessor the *Standards Beliefs Instrument* (*SBI*), have been used in the greatest number of studies related to mathematical beliefs and were designed specifically to measure the degree of alignment of education beliefs related to practices advocated within the NCTM *Standards* (Futch & Stephens, 1997; Hart, 2002; Wilkins & Brand, 2004).

The *MBI* was first published in as an extension of the *SBI* (Hart, 2002). The *SBI* is a 16-item instrument developed in the 1990s to assess teachers' beliefs about the NCTM *Standards* (Zollman & Mason, 1992). Statements in the *SBI* are either direct quotes from the *Standards* or their inverses, with eight of the items representing statements consistent with the NCTM *Standards* and eight of the items inconsistent with the NCTM *Standards* (Zollman & Mason, 1992). Items for the *SBI* were developed as single sentence statements utilizing both positive or negative statements designed to "avoid a socially desirable (or correct according to the *Standards*) pattern of responding" (Zollman & Mason, 1992, p. 359). The content validity of the *SBI* was evaluated by a panel of seventeen mathematics educators who were involved in developing, writing, or editing the NCTM *Standards* (Zollman & Mason, 1992). Reliability of the *SBI* was determined to be adequate at with a Spearman-Brown correlation coefficient of .65

(Zollman & Mason, 1992). In addition to the SBI items, the *MBI* incorporates items from a student mathematics beliefs survey from Schoenfeld (1992).

The *MBI* is a 28-item instrument designed to assess the consistency of a teacher's beliefs related to the instructional practices advocated by the NCTM *Standards* and has been "used to assess change in teachers' beliefs about teaching and learning mathematics within and outside the school setting" (Hart, 2002, p. 7). Sixteen items on the instrument require binary responses (agree or disagree) with the remaining 12 items involving a scaled response with these options: (a) true, (b) more true than false, (c) more false than true, and (d) false. Like the *MARS-SV*, the *MBI* is simple to administer and takes less 30-minutes to complete. Unlike the *SBI*, reliability studies have not been conducted on the *MBI*.

MBI items were designed to measure the beliefs respondents have toward teaching and learning mathematics, such as whether learning mathematics is an active or passive process (e.g., questions 1 and 2), how mathematics teaching should be approached (e.g., questions 3 and 4), the role of reasoning and individual sense-making in learning mathematics (e.g., questions 5-7), and the respondent's efficacy related to mathematics and teaching mathematics (Hart, 2002). Sample questions include:

1. Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.
2. Learning mathematics must be an active process.
3. Mathematics should be taught as a collection of concepts, skills and algorithms.

4. To solve most math problems you have to be taught the correct procedure.
5. A demonstration of good reasoning should be regarded even more than students' ability to find correct answers.
6. In mathematics something is either right or it is wrong.
7. In mathematics you can be creative and discover things by yourself

(Hart, 2002).

The *MBI* does not utilize a standardized scoring protocol. Instead, each item on the instrument indicates the topic measured within the item and the direction of the response most aligned with reform-based beliefs (Hart, 2002). The author of the *MBI* recommended assigning numerical values to each item in order to quantify responses (Hart, personal communication, May 6, 2012). Conferring with the author, a scoring protocol was developed based on the notes included with the instrument and assignment of points to each response. On binary responses, a score of one was assigned to responses that were not aligned with reform-based practices and a score of two was assigned to responses that were aligned. On the four point scale responses, a score of one was assigned to responses that were least aligned with reform-based practices, a score of two was assigned for the response with the next highest degree of alignment, up to a score of four for most aligned response. Total *MBI* scores were calculated by summing the scores for each question on the *MBI*.

In addition to the lack of a standardized scoring protocol, the *MBI* is limited by the absence of investigation into the reliability and validity of the instrument, which has the potential to impact the quality of the study (Gliner, Morgan, & Leech, 2009).

Demographic Survey

In addition to data collected to answer the research questions, demographic data was collected to inform the selection of the sub-sample participants and the nature of the semi-structured interview questions used in Phase Two of the study. Demographic survey questions relevant to the research question are listed here. The full demographic survey is included in Appendix B. Relevant questions include:

1. Your highest degree.
2. Degree major and minor.
3. Approximate number of credit hours of mathematics content courses included in undergraduate study.
4. Counting this year, how many years in total have you been teaching?
5. Counting this year, how many years in total have you taught or supported teaching mathematics?
6. Do you currently teach a pullout mathematics for students with SLD? Do you currently support students with SLD in general education classes?
7. Number of years (including the current year) where teaching assignment involved teaching or providing support in mathematics.
8. What level students do you teach? Check all that apply.
    a. Elementary
    b. Middle school
    c. High school
9. Ethnicity (check all that apply)
    a. African-American
    b. American Indian or Alaskan Native

c. Asian

d. Hispanic

e. Pacific Islander

f. White (not Hispanic origin)

g. Other (Describe)

10. Which of these commonly held views about the nature of mathematics most accurately fits your perspective:

    a. Mathematics consists of rules and procedures to be memorized and practiced.

    b. Mathematics is a tool to use to solve problems and/or find solutions.

    c. Mathematics is a discipline of logic and reasoning.

Questions 6 and 7 were included to ensure that respondents qualified for the study. Question 8 was included to elicit participant perceptions of mathematics based on the conceptual framework of the study.

The complete Phase One data collection instrument includes the demographic survey, questions from the *MARS-SV*, and questions from the *MBI* (see Appendix F).

Semi-structured Interview

Phase Two data collection involved a semi-structured interview protocol designed to explore the attitudes and beliefs of participants. Interview questions were developed through a review of qualitative studies related to the mathematics attitudes and mathematical beliefs of teachers (Barkatsas & Malone, 2005; Bekdemir, 2010; Beswick, 2007; Carroll, 1994; Dogan, 2011; Foss & Kleinsasser, 1996; Gresham, 2007) as well as questions unique to this study. Questions related to attitude include:

1. How do you feel about mathematics (Gresham, 2007)?
2. What do you think contributed to your attitude toward mathematics (Gresham, 2007)?
3. What do you think about the way that you have been taught mathematics? What do you remember best about learning mathematics in school (Foss & Kleinsasser, 1996)?

Questions related to beliefs about the nature of mathematics, teaching mathematics, and learning mathematics included:

4. In your online questionnaire, you chose [insert response] as the descriptor that matches your view about the nature of mathematics. Can you tell me more about this?
5. Please describe an ideal mathematics classroom (Beswick, 2007).
6. How do children learn mathematics?
7. What is most important about teaching mathematics?

Following data collection and analysis of Phase One data, three Phase One participants who were not selected to participate in Phase Two were involved in a pilot of the interview protocol. Responses from the pilot interviews were not used in the final analysis for the study. Instead, based on the quality of responses from participants in the pilot, an additonal question was added to explore how teaching mathematics had influenced the participant's beliefs and attitudes about mathematics.

Mixing of data from Phase One of the study with Phase Two of the study occurred through the use of participant responses from the online survey to customize the semi-structured interview questions. Data collected from Phase One of the study was used to either customize or inform follow-up, probing questions for the semi-structured interview. For instance, question 2 read differently for participants with low MA than with high MA. For low MA participants, the question read "What do you think contributed to your comfort with mathematics?" whereas the question read "What do you think contributed to your anxiety in mathematics?" Furthermore, the interview protocol was augmented with responses from the Phase One survey in order to provide the interviewer with data to probe responses from participants. For instance, question 7 of the interview protocol related to how children learn mathematics. Two statements from the *MBI* relate to how children learn mathematics, thus the statements and participant responses were inserted into the interview protocol to prompt follow-up questions to expand upon participant responses as needed:

a. Learning mathematics must be an active process.
b. Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.

Mixing the responses from Phase One and Phase Two of the study was consistent with the study purpose of significance enhancement, which permits the researcher to expand the interpretation of findings from qualitative and quantitative strands of a study to enhance, compare, and clarify across methods (Collins, et al., 2006).

Data Collection and Analysis

Phase One of the study consisted of collecting and analyzing quantitative data using the *MARS-SV*, *MBI*, and demographic survey information from the full study sample of 48 teachers. The primary purpose of Phase One data collection was to quantify participants' level mathematics anxiety level and degree of alignment with reform based approaches to teaching and learning mathematics.

Phase One Data Collection

All items from the *MARS-SV*, *MBI*, and demographic questions (Appendix F) were administered together through the Internet utilizing Google Forms. Google Forms is an online tool that allows for secure collection of questionnaire responses.

As individuals agreed to participate in the study, their names and email addresses of were entered into a spreadsheet in order to track the completion of the online questionnaire. Respondents were emailed a link to the online questionnaire with information related to the purpose and design of the study as well as instructions on how to complete the survey (Appendix A). Once respondents were provided the link to access to the online survey, they were able complete the survey at a time and location that suited their individual schedules. The estimated time to complete the survey was less than one hour. Respondents who had not completed the survey within the communicated timeframe were sent a maximum of three reminder emails until the survey was completed or it was determined that the respondent was not interested in participating. Once the minimum number of respondents needed for the study had completed the online survey, recruitment of participants was discontinued and survey data was exported to a Microsoft Excel spreadsheet to facilitate analysis. Once data was in Excel, the information was transferred to SPSS for analysis.

Phase One Data Analysis

Before data were analyzed, exploratory data analysis was conducted to determine whether there were problems with the data (i.e., missing or incorrect values, outliers).

Three participants submitted surveys with incomplete data. Each was contacted by email requesting responses to skipped questions. All participants promptly provided missing responses and these data were added into the survey spreadsheet. Separate descriptive analyses was conducted on the *MARS-SV* and *MBI* portions of the survey, including frequencies, normality (skewness), range, min/max, and standard deviation. Assumptions of normality of data distributions were violated for the *MBI* item analysis, thus a non-parametric test was selected (Morgan, Leech, Gloeckner, & Barrett, 2011). Assumptions for the Wilcoxon signed ranks test were checked and met.

Phase One data analysis involved determining the reliability and validity of the data produced through the *MARS-SV* and *MBI*. A Cronbach's alpha was computed for each of the instruments to determine internal consistency of responses. Factor analyses for the *MARS-SV* and *MBI* were not possible given the absence of information related to the subconstructs for the instruments. Thus, assessment of validity of the data produced was limited. To avoid common errors of quantitative analysis (Onwuegbuzie & Daniel, 2003), tests for appropriate assumptions were conducted and causal inferences were avoided, such as attributing high MA to specific beliefs or vice versa.

Phase Two Participant Selection

Descriptive results were used to stratify participants along two dimensions, degree of math anxiety and degree of alignment of mathematics beliefs with the NCTM *Standards*. Participants classified in the high or low categories on both measures were stratified along the two dimensions represented in the matrix (Table III.7) in order to identify extreme cases (Miles & Huberman, 1994) from which to draw participants for Phase Two of the proposed study. Participants were classified as low alignment of mathematics beliefs, high mathematics anxiety (I); high alignment of mathematics beliefs, high mathematics anxiety (II); low alignment of mathematics beliefs, low mathematics anxiety, (III); and high alignment of mathematics beliefs, low mathematics anxiety (IV).

Table III.7 Variable Dimension Matrix: Phase One of Research Design

Mathematics Anxiety Level		Mathematical Belief Alignment: Low	Mathematical Belief Alignment: High
	High	I	II
	Low	III	IV

Selection of participants for Phase Two of the study involved stratifying participants according to level of MA and level of mathematical belief alignment. Stratification of participants according to MA initially followed the definitions of high and low MA put forth by Ashcraft and Kirk (2001) who defined high MA as those who score one standard deviation above the grand mean and low MA as those who score one standard deviation below the grand mean. Initially, a scatter plot was created to plot participant scores on the *MARS-SV* against participant scores on the *MBI* (Figure II.14). The scatter plot illustrates the distribution of scores on both instruments with the scales set to the full range of possible scores on each instrument and solid lines indicating median scores of the study sample. Dotted horizontal lines represent one standard deviation below and above the grand mean on the *MARS-SV*, distinguishing participants with high MA (points above the top dotted line) and low MA (points below the lower dotted line).

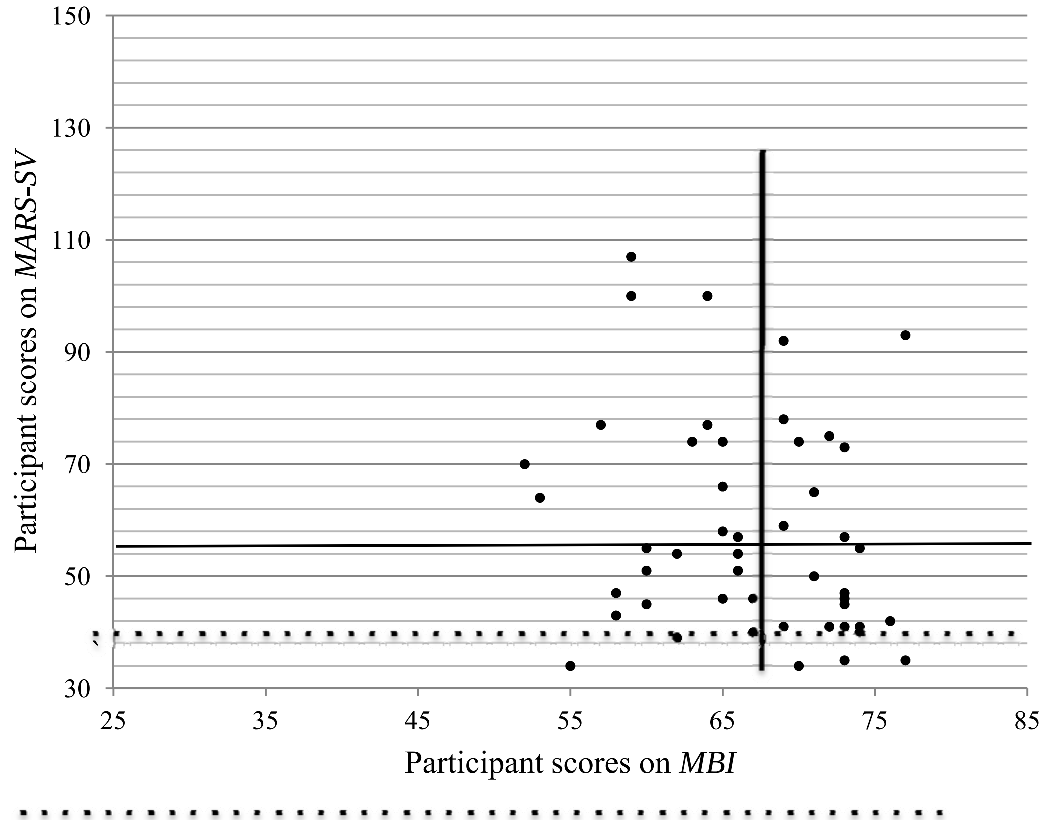

Figure II.14. Relationship between Participant Mathematics Beliefs and Mathematics Anxiety.

Only two participants (Participants 48 and 22) qualified as low MA, with *MARS-SV* scores of 34 and 39, respectively, whereas six participants (Participants 7, 10, 13, 19, 21, and 23) qualified as high MA, with *MARS-SV* scores of 100, 78, 100, 93, 92, and 100, respectively. Similar statistical studies have not been conducted using the *MBI*, thus, the differentiation between high and low alignment of mathematics beliefs were in relation only to the sample and determined using the top and bottom quartile of participant scores on the *MBI*. Thirteen participants were in the bottom quartile; thirteen participants were in the top quartile.

Comparing the eight participants defined as either low MA or high MA with the participants considered as high alignment of beliefs or low alignment of beliefs resulted in only five participants in the sub-sample for Phase Two. In order to expand the data set for Phase Two, criteria for high and low MA was broadened to the top and bottom quartile of MA of the *MARS-SV*. Expanding the criteria allowed for a greater pool of candidates with lower mathematics anxiety to be included in the study. Table III.8 illustrates the expanded pool of candidates for Phase Two based on the broadened MA criteria.

Table III.8 Variable Dimension Matrix: Stratification of Participants along the Dimensions of Mathematics Anxiety and Alignment of Mathematical Beliefs

<table>
<tr><td rowspan="2">Mathematics Anxiety Level</td><td>High</td><td>Cell I
Participants 13 and 23</td><td>Cell II
Participant 19</td></tr>
<tr><td>Low</td><td>Cell III
Participants 22, 38, 42, and 48</td><td>Cell IV
Participants 11, 27, 30, 36, 37, 40, and 46</td></tr>
<tr><td></td><td></td><td>Low</td><td>High</td></tr>
<tr><td></td><td></td><td colspan="2">Mathematical Belief Alignment</td></tr>
</table>

Once subjects were stratified along the two dimensions (math anxiety and mathematical belief alignment), further analysis of survey data was conducted to select specific subjects from Cells III and IV to include in Phase Two of the study. Participant selection from Cells I and III was limited by the willingness and availability of participants to participate in Phase Two of the study. Participant 38 was unwilling to participate in Phase Two of the study and provided no contact information in the Phase One survey. Participants 22 and 23 provided email and phone contact information; however, neither participant responded to multiple email and phone inquiry invitations to participate in Phase Two data collection. Thus, only Participant 23 was selected to represent the subsample of participants with high MA and low alignment of beliefs, and Participants 42 and 48 were selected to represent the subsample of participants with low MA and low alignment of beliefs. Participant 48 met the Ashcraft and Kirk (2001) criteria of low MA whereas the MA score of Participant 42 at 45 fell just below the 25th percentile of 46.

Selecting participants from Cell IV involved narrowing the candidates by scores on the two instruments. The range of *MARS-SV* scores for participants in Cell IV were within one standard deviation (*SD* = 19.07) ranging from 35 to 46. The range of *MBI* scores for participants in Cell II were also within one standard deviation (*SD* = 6.52) ranging from 73 to 77. The candidates with the combination of the lowest *MARS-SV* scores and highest *MBI* scores were selected for Phase Two, participants 40, 46, and 36. Pseudonyms of final participants selected for Phase Two are included in Table III.9.

Table III.9 Variable Dimension Matrix: Participants for Phase Two of Study

	High	Cell I Natalie	Cell II Callie
Mathematics Anxiety Level	Low	Cell III Steven Tammy	Cell IV Carson Sally Betty
		Low	High
		Mathematical Belief Alignment	

Ideally, in order to fully explore differences among the individual cells, the sub-sample for Phase Two of the study was to include two subjects from each cell in the variable dimension matrix illustrated in Table III.7. However, the MA and MB profiles of the study sample revealed through the stratification process necessitated an adjustment to the data analysis process. As Onwuegbuzie and Collins (2007) recommend a subsample size of 6 to 10 for the phenomenological study, the qualification of seven participants for Phase Two of the study was appropriate. However, data saturation for cells of the matrix could not be achieved with only one participant in Cells I and II respectively. Thus, the analysis of data for Phase Two of the study was limited to the variables of MA and MB. Specifically, analysis of MA data for Phase Two included participants identified with high MA, Natalie and Callie, and low MA, Steven, Tammy, Carson, Sally, and Betty as displayed in Table III.10.

Table III.10 Variable Dimension Matrix: Mathematics Anxiety

Mathematics Anxiety Level	High	Cells I and II Natalie Callie
	Low	Cells II and IV Steven Tammy Carson Sally Betty

Accordingly, analysis of MB data for Phase Two included participants identified with high MB (Callie, Carson, Sally, and Betty) and low MB (Natalie, Steven, and Tammy) as displayed in Table III.11.

Table III.11 Variable Dimension Matrix: Mathematical Belief Alignment

Cells I and II	Cells II and IV
Natalie Steven Tammy	Callie Carson Sally Betty
Low	High

Mathematical Belief Alignment

In summary, the MA and MB profiles of the participants from the full study sample limited the analysis of data to the dimensions of MA and MB instead of the intended analysis according to the cells in the variable dimension matrix. Additionally, because the selection criteria for Phase Two was expanded to include a broader range of participants along the MA scale, inferences drawn about the mathematics attitudes of participants must be tempered.

Phase Two Data Collection

The focus of the second phase data collection was exploring and relating the attitudes and beliefs special educators have about the nature of mathematics and how mathematics is learned and should be taught. Data collection for Phase Two was in the form of semi-structured interviews conducted either in-person at an agreed upon local restaurant or by telephone with qualified Phase One participants. Interviews lasted from 25 to 65 minutes and were recorded using a digital recording device with the advance permission of the subjects. The questions in the semi-structured interview protocol, found in Appendix E, were augmented and customized based on the respective participant responses on the Phase One questionnaire, allowing for mixing of data from both phases of the study. Following the recorded interviews, all interviews were transcribed into verbatim transcripts for analysis.

Phase Two Data Analysis

Phase Two qualitative data analysis consisted of data reduction, data display, and drawing conclusions (Miles & Huberman, 1994). Data analysis for Phase Two of the study included techniques for reducing and displaying the qualitative data. To avoid data overload during the data reduction and data display processes, a priori codes derived from the conceptual framework and research questions were utilized (Miles & Huberman, 1994). The process used for data reduction and data display is illustrated in Figure III.15 and described in detail next.

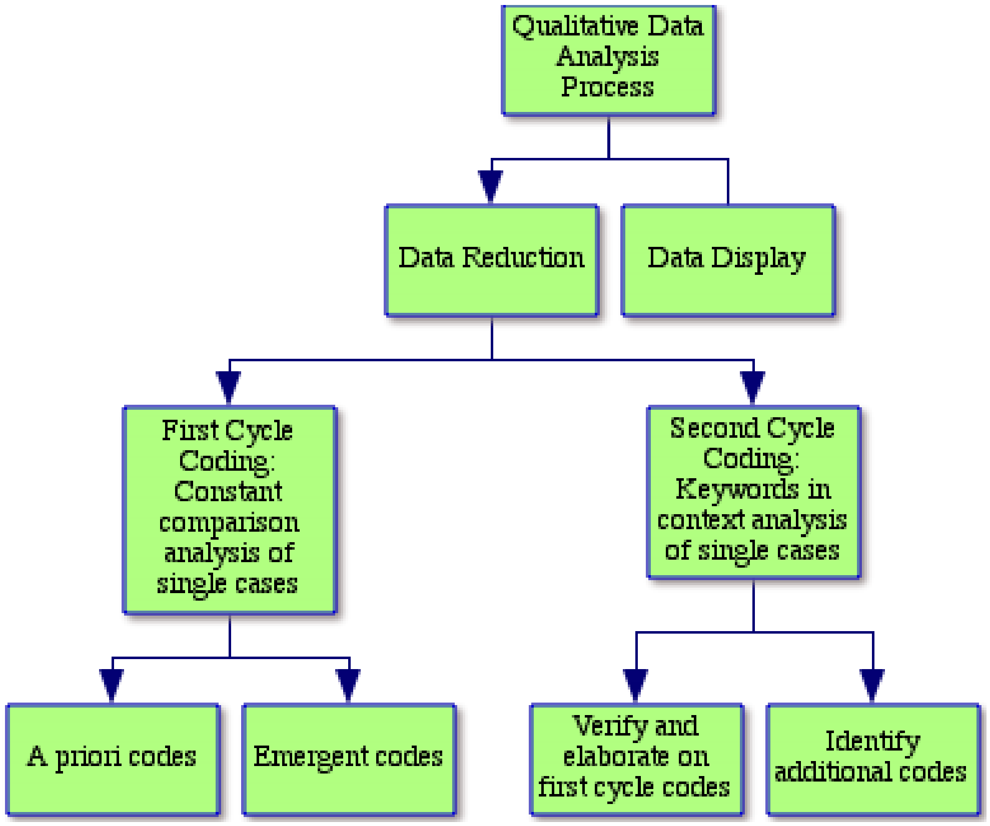

Figure III.15: Overview of Qualitative Data Analysis Process.

Data reduction. Data reduction began with the selection of coding techniques. For research questions that ask “what does Y mean”, Leech and Onwuegbuzie (2007) recommend four techniques: keywords in context (KWIC), constant comparison, domain analysis, and taxonomic analysis. The current study was essentially an inquiry into what mathematics means to the participants, thus analysis techniques were drawn from those suggested. Furthermore, Leech and Onwuegbuzie (2008) note that both constant comparative analysis and keywords-in-context are appropriate for data involving talk.

Saldaña’s codes to theory model for qualitative theory provided a framework for the study’s qualitative analysis (Saldaña, 2009). Saldaña (2009) contended that the qualitative data analysis is cyclical in nature involving coding and recoding in order to define and refine categories and themes that emerge within the data. As such, data reduction for the study at hand consisted of two cycles of coding represented in Figure III.15. Data reduction for each single case interview consisted of two iterative coding cycles utilizing constant comparative analysis for first cycle coding and keywords in context (KWIC) for second cycle coding. A priori start codes (Miles & Huberman, 1994) were used based on the conceptual framework, research questions, and key variables in both analysis techniques. A single coder, the researcher, was involved in the coding the data.

First cycle coding utilized constant comparative analysis of each interview transcript applying a priori start codes (Appendix G) based on the study's research question and conceptual framework (i.e., beliefs, attitudes, and role of the teacher) and identification of emergent codes. Constant comparison analysis is a qualitative data analysis technique used to "identify underlying themes presented throughout the data" (Onwuegbuzie & Leech, 2007). Constant comparative analysis is useful when a researcher will be utilizing an entire data set as in the case of this study (Leech & Onwuegbuzie, 2007). According to Leech and Onwuegbuzie, "Constant comparison analysis is a method of choice when the researcher wants to answer general, or overarching, questions of the data" (2007, p. 576).

The process for coding consisted of sequential coding of each interview transcript. Codes from both CCA and KWIC analysis were inserted into the transcripts using the comment function in Microsoft Word. An example of coded data is displayed in Figure III.16. Codes of ATTITUDE and NATURE OF MATHEMATICS are CCA codes denoting responses related to the research question and conceptual framework. Codes labeled as KEYWORDS denote responses where the participant utilized frequently used terms by the participant.

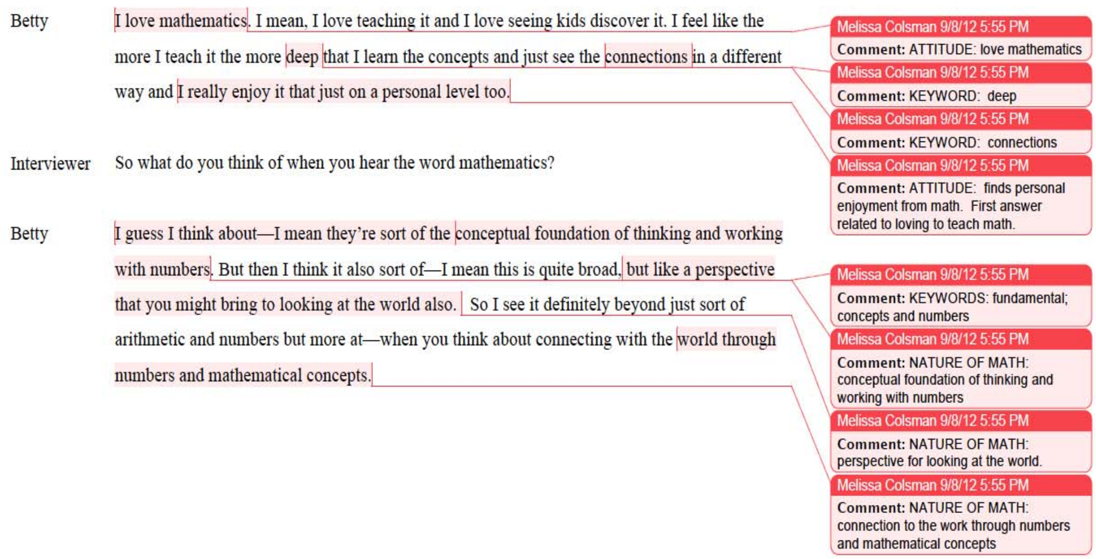

Figure III.16. Example of Coded Text Utilizing Both CCA and KWIC Coding.

Second cycle coding for single case interview data applied KWIC to validate and elaborate on first cycle codes and identify additional codes. KWIC analysis is a technique used to determine how people use words in the context of other words, assuming that different people use words in different ways (Leech & Onwuegbuzie, 2008). KWIC was appropriate as it is "particularly useful when analyzing short responses to unstructured or structured questions" (Leech & Onwuegbuzie, 2007, p. 576). The KWIC analysis process consists of three stages: (a) identifying frequently or uniquely used words, (b) listing the words that precede or follow the word, and (c) using the context of the word to interpret meaning (Leech & Onwuegbuzie, 2007). The analysis process was recorded in table display (Table III.12).

Table III.12 Sample Display for Second Cycle KWIC Analysis

Keyword	Keyword-in-context	Interpretation
Frequently used term from semi-structured interview	Direct quotes from the interview transcript	Interpretation of the meaning the participant ascribes to the term

An example of the KWIC second cycle coding of interview data can be found in Table III.13. The table shows Steven's use of the word "practice" or "practicing" throughout the interview. For each use of the term, the context of the term and meaning of its use was interpreted and summarized. From the analysis of Steven's use of the term "practice", the central role that Steven places on practice in learning mathematics can be surmised.

Table III.13 Sample of KWIC Analysis Display Interpreting Steven's Use of *Practice*

Keyword	Keyword-in-context	Interpretation
Practice	I remember him being by the chalk board and teaching us this is exactly how you do it and give us lots of different examples and interacting, going back and forth, you know, and then practice, we'd practice for a while. I remember having to practice, doing a lot of math problems.	Practice follows explicit teaching by the teacher. Practice involves doing lots of examples and is part of learning math.
	Role of students in learning math is to listen and practice, in my opinion. You know, listen, follow along and practice, practice the problems and, you know, take that and be able to problem solve. And yeah, I mean I would say that's the role of the students is- is to receive to- to listen and practice.	The role of students is to practice the problems.
	And I think that's how I learned math, its practice and production and doing it.	Children learn math by practicing.

Combined, the two cycle coding and analysis afforded by CCA and KWIC "increase[d] understanding of the data" (Onwuegbuzie & Leech, 2007, p. 563) and triangulated the qualitative analyses thereby increasing trustworthiness (Leech & Onwuegbuzie, 2008). Also, to increase the power of the qualitative analysis, data analysis processes were fully described in order to make the process public (Anfara, Brown, & Mangione, 2002).

Data display. A series of data displays were utilized during qualitative analysis to further reduce and simplify the data (Onwuegbuzie & Teddlie, 2003). Displays of qualitative data are especially valuable in extracting themes in order to draw valid conclusions from often bulky, poorly ordered data (Miles & Huberman, 1994). Furthermore, displaying data is a necessary part of an audit trail for data analysis enhancing the legitimacy of inferences (Onwuegbuzie & Leech, 2007b).

Data displays were first used to identify and refine single case themes then utilized to identify and refine cross case themes. To identify and revise single case themes, codes from the transcripts were reviewed to determine overall trends in the data and draw conclusions (Miles & Huberman, 1994). Single case data display involved creation of a theme display to record themes for each participant according to aspects of the study's conceptual framework (Table III.14). The themes recorded in the single case conceptual framework display emerged from analyzing and refining first and second round codes.

Table III.14 Single Case Conceptual Framework Theme Display

Nature of mathematics	Perspective of teaching mathematics	Perspective of learning mathematics
Themes related to the beliefs and attitudes about the nature of mathematics	Themes related to the beliefs and attitudes about approaches to teaching mathematics	Themes related to the beliefs and attitudes about how mathematics is learned

Table III.15 illustrates one theme from the single case theme display for Betty. All responses related to themes of *isolation* and *connection* were displayed according the relevant aspect of the conceptual framework. In response to the question, what do you think of when you hear the word mathematics, Betty replied that "mathematics is a perspective one brings to the world". Later in the interview, Betty provided responses that illustrated how this perspective of mathematics related to her beliefs about teaching and learning mathematics. Betty contrasted an isolated approach to mathematics teaching and learning with a "deep" approach where people make connections among ideas, procedures, and the world. Displaying the data in this manner allowed connections to be made across interview questions related to the research questions and conceptual framework.

Table III.15 An Example Of a Single Case Conceptual Framework Theme Display

Nature of mathematics	Perspective of teaching mathematics	Perspective of learning mathematics
Mathematics is about <u>connecting</u> with the <u>world</u> through numbers and mathematical concepts <u>A way of looking at the world</u>	Contrast of deep learning with <u>isolated nuggets</u> and procedures lacking understanding. The teacher's role is to ensure students understand the <u>rationale</u> for what they are learning	Depth of learning involves concepts and connections. Depth of learning increases enjoyment. Depth of understanding is exciting and is where math makes sense for kids. Deep level understanding relates to why <u>procedures work.</u> Concepts <u>can be discovered</u>, involves flexibility, play, and <u>experimentation</u>.

As initial themes were recorded into the table display, each theme was refined and verified by comparing against the first round constant comparative analysis codes and second round KWIC analysis. Hyperlinks (shown as underlined text within Table III.15) between coded interview transcripts, KWIC tables, and theme display tables were created in order to cross reference codes and verify themes.

Themes within and across cases were identified and refined through an iterative process illustrated in Figure III.17. Each single case conceptual framework theme display was used to identify single case themes. As each single case theme display was developed, it was compared against the other theme displays to identify initial cross case themes. These initial themes were then used to refine single case themes.

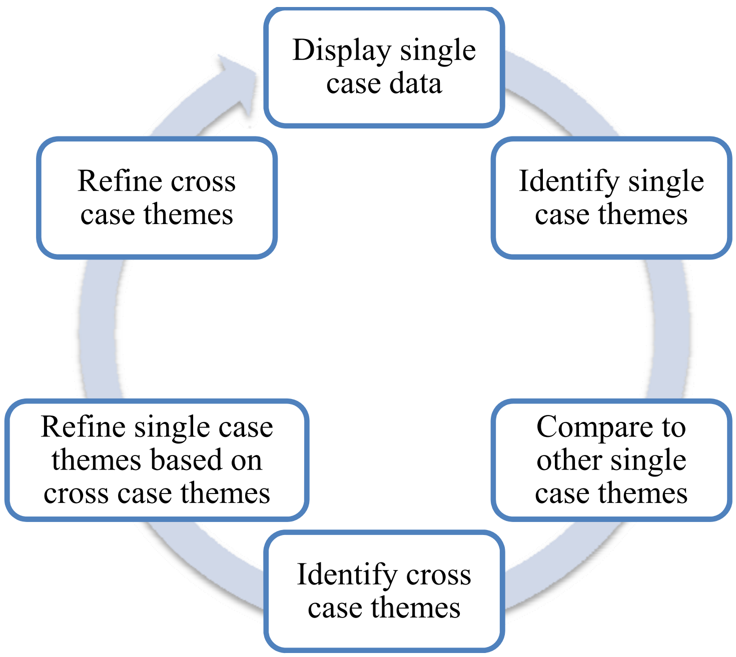

Figure III.17. Data Display and Theme Identification Process.

Cross-case themes were identified through cross-case analysis of single case displays. Themes that were common across multiple cases were analyzed to determine trends across the participants. An example of a cross-case theme common to all of the participants was the role of teacher biography in the formation of beliefs about teaching. Through the interview process, each participant described aspects of their mathematics learning experience as students. They also shared their beliefs about teaching mathematics. A relationship between participant responses related to their learning experiences and their beliefs about teaching was apparent for each participant. The common theme display is summarized in Table III.16 from each single case into a cross-case display. Underlined text represents where hyperlinks existed between the different data analysis displays.

Table III.16 Sample Cross-case Theme Related to Biographical Approach to Teaching

Participant	Participant Experience Learning Mathematics	Participant Ideal of Teaching Mathematics
Natalie	Math was hard for her. She could not memorize. She was not prepared for Algebra 1.	Kids need a safe environment. Some kids cannot memorize. Kids need to be prepared. Frequent comment about students not ready for Algebra 1.
Callie	Didactic approach she experienced was not motivating, very sterile; struggled in writing and makes connection to kids who struggle.	Started teaching the way she was taught; found it too sterile and not motivating. Changed to problem solving approach; now thinks problem solving is motivating.
Sally	Recognition of her comfort with math related to teaching. Math part of life growing up; attributes comfort with math to having math integrated with daily life.	Knows that her comfort with math is not shared by her students. Believes that students need to make the connection of math to their lives.
Betty	Recalls learning math in isolated nuggets; now sees the connections.	Believes in the importance of students seeing the connections.
Tammy	Struggled with reading growing up. Intervention consisted of practice.	Believes that kids who struggle in math need practice.
Carson	Always enjoyed math, played games. Could not see in three-dimensions and struggled in geometry.	Believes that playing games is important to learning math. Relates her own inability to see in three-dimensions to the struggles her students have in math.
Steven	Appreciated his middle school math teacher's approach to teaching. Liked the sequential, structured approach.	Replicates in his middle school math teacher's practices. Ordered, sequential way easiest to teach, easiest to learn.

As cross-case themes were identified, the themes were recorded into a cross-case theme matrix (Table III.17). The cross-case theme matrix allowed for comparison of themes according to the profiles of the participants. Themes that emerged within a particular subject's data were indicated with a mark in the table. Because the table was organized according to participate profile identified within the variable dimension matrix, themes were able to be classified as applying to (a) all or most subjects (Theme 1), (b) subjects with high or low anxiety (Themes 4 and 5), (c) subjects with high or low alignment of beliefs (Themes 2 and 3), or (d) subjects without regard to study variables (Theme 6). The cross-case theme matrix identified themes according to study variables enabling data interpretation.

Table III.17 Sample Cross-Case Theme Matrix

	I: Low MB, High MA	II: High MB, High MA	III: Low MB, Low MA		IV: High HB, Low MA		
	Natalie	Callie	Steven	Tammy	Carson	Sally	Betty
Theme 1	X	X	X	X	X	X	X
Theme 2	X		X	X			
Theme 3		X			X	X	X
Theme 4	X	X					
Theme 5			X	X	X	X	X
Theme 6		X		X		X	X

Once themes were identified according to variables of MA and mathematics belief alignment, common themes according to the each variable were able to be identified. Themes common to participants with high and low MA were identified (Table III.18) and high and low MB were identified (Table III.19).

Table III.18 Display of Themes Identified by Variable: Mathematics Anxiety Level

Mathematics Anxiety Level	High	Common Themes: High MA Cells I and II	
	Low	Common Themes: Low MA Cells III and IV	
		Low	High
		Mathematical Belief Alignment	

Table III.19 Display of Themes Identified by Variable: Mathematical Beliefs Alignment

Mathematics Anxiety Level	High Low	Common Themes: Low MB Cells I and III	Common Themes: High MB Cells II and IV
		Low	High
		Mathematical Belief Alignment	

The iterative nature of the analysis process involved analysis of single case data and cross-case data to identify and verify themes related to the research question and conceptual framework. The analysis process involved interpretation at multiple stages, between first and second cycle coding of single cases, and at the cross-case analysis stage. Thus, analyses and interpretation were significantly intertwined.

Interpretation

The interpretation process consisted of three components initiated within the data analysis process: (a) data correlation, (b) data consolidation, and (c) data comparison (Collins, Onwuegbuzie, & Sutton, 2006). Data correlation involved relating the qualitative and quantitative data through the identification of single case themes. Data consolidation involved combining data from both quantitative and qualitative sources during the analysis process. Data comparison occurred through the cyclical analysis process described previously.

Data Integration

Mixed method data analysis ultimately involves seamlessly integrating qualitative and quantitative analytic techniques (Teddlie & Tashakkori, 2009). Thus, the next stage of data analysis consisted of data integration. Data integration involves combining the data into a coherent whole or two separate coherent wholes (Onwuegbuzie & Teddlie, 2003). Data integration occurred at multiple stages of the study. First, the quantitative data from Phase One of data collection informed the development of the semi-structured interviews for the qualitative strand of the study. Second, data integration occurred through an analysis of the combined quantitative and qualitative data using the variable dimension matrix as a framework for analysis as described previously. This analysis involved drawing inferences from all of the data sources (Teddlie & Tashakkori, 2009).

Data Validation

Onwuegbuzie and Johnson (2006) recommend using the term legitimation to refer to the validity in mixed method research, as data validation in mixed method research encompasses the issues of validity in both quantitative and qualitative research and the issues of validity unique to mixed research methods. The primary method the study used for legitimation of data is weakness minimization (Onwuegbuzie & Johnson, 2006) where the weaknesses in one approach is compensated by the strengths of another. In the current study, the potential for weakness in the quantitative strand was compensated for in the qualitative data strand.

In a mixed method study, legitimation of data occurs at the design, collection, analysis, and interpretation stages (Onwuegbuzie, 2003). An overview of how data validation was considered throughout the stages of the study is illustrated in Figure III.18 and is described herein. In the quantitative data collection stage, legitimation was accomplished by confirming the internal validity of the instruments and considering power and sample size. Data collection in the quantitative strand involved the use of two instruments, the *MARS-SV* and *MBI*, which as noted earlier have sufficient internal validity. Legitimation of quantitative analysis addressed what Onwuegbuzie and Daniels (2003) identified as the quantitative paradigm specific errors of failure to test for violation of statistical assumptions. Thus, all tests for statistical assumptions were conducted in the quantitative data analysis stage. Validation of data from the quantitative strand was critical to the selection of participants and the quality of data collected in the qualitative strand of the study.

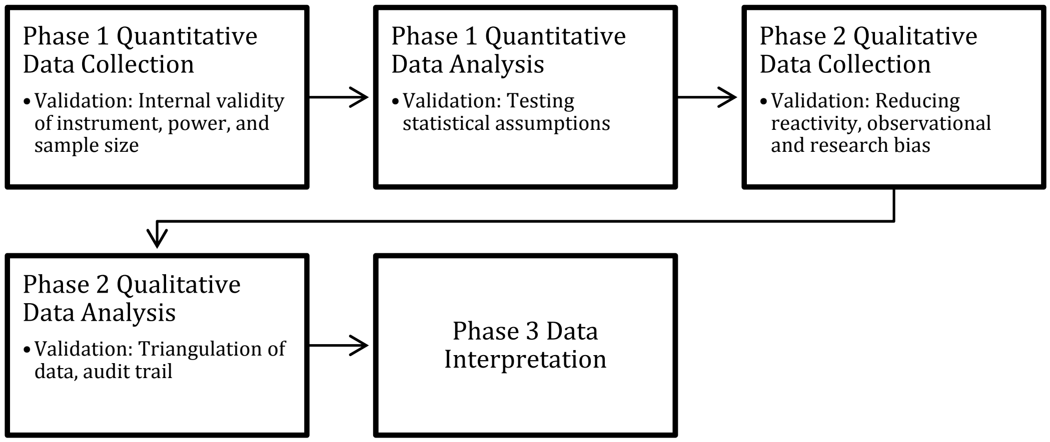

Figure III.18. Legitimation Considerations Within Study Design.

In the qualitative data collection phase, potential threats to legitimation were observation bias, researcher bias, and reactivity. Observational bias occurs when insufficient data is obtained (Onwuegbuzie, 2003). The design of this study utilized collection of data through semi-structured interviews allowing the researcher to probe participant's responses to gather rich descriptions of their beliefs. Researcher bias occurs when the researcher has a preference for one intervention over another (Onwuegbuzie, 2003). Researcher bias can be active or passive and result in the researcher influencing the data, through asking leading questions or making statements that reveal researcher preferences (Onwuegbuzie & Leech, 2007b).

While no intervention was at play in the present study, the researcher needed to be aware of her bias. The researcher was a former middle school mathematics teacher who had extensive experience collaborating with special educators to support students with SLD in mathematics. In addition, the researcher had provided professional development to special education teachers related to mathematics instructional practice. The experience of working with special education teachers in the context of mathematics instruction created biases within the researcher. The biases included the beliefs that special educators tended to (a) work from an instrumentalist perspective of mathematics, (b) be relatively anxious about the discipline, and (c) possess skepticism of the appropriateness of reform-based approaches to teaching mathematics. One way to minimize bias is through bracketing (Creswell, 2007), that is setting aside one's own experience. The researcher actively practiced bracketing her experiences by employing a researcher journal to record instances of biased thinking. The researcher employed the journal following the semi-structured interviews to bracket her personal responses to the participants' responses. In addition to bracketing, the use of a standard protocol of interview questions and follow-up questions reduced reactivity. Minimizing the potential to use leading questions or revealing researcher preference minimized subject reactivity.

To bolster legitimation of the qualitative analysis, the two-cycle analysis process described earlier allowed for triangulation of data analysis. Using two data reduction techniques allows for triangulation between analyses. Triangulation is recommended to increase the rigor and trustworthiness of findings (Leech & Onwuegbuzie, 2007). In addition the use of data displays and complete analysis descriptions will create an audit trail (Onwuegbuzie & Leech, 2007b). Finally, peer review and member checking will enable inside-outside legitimation (Onwuegbuzie & Johnson, 2006). Peer review occurred through the process of oversight from my dissertation committee. Member checking occurred through exchanges with study participants via email. Study participants were sent relevant portions of the data analysis and interpretation to determine the accuracy of interpretation. All participants responded to the researcher inquiry about the accuracy of interpretation with only one participant, Nancy, offering clarification of one point.

Limitations and Delimitations

The most apparent limitation of the present study is external validity. Care needs to be taken to avoid a major qualitative interpretative error by not generalizing the results beyond the study participants (Onwuegbuzie & Daniel, 2003). Also because the study relied on accessible and willing participants, a random sample was not possible (Onwuegbuzie, 2003). The voluntary, non-random nature of participant recruitment may have limited the sample to participants who have a lower level of mathematics anxiety and higher degree of alignment of beliefs with reform based approaches to mathematics. Other limitations include the limited engagement with participants based on the length of interviews. This limitation relates to study design and limits the ability to craft rich descriptions.

The delimitations relate to the anticipated sample (i.e., special educators involved in delivering mathematics instruction) and what is intended to be accomplished in the study, an analysis of study participants' beliefs about mathematics, beliefs about teaching and learning mathematics, and their level of anxiety about mathematics. The study was delimited to only the stated aspects of participant beliefs. Out of scope for the study were the level of mathematics teaching efficacy the teachers have, their student mathematics achievement, or an analysis of instructional methods the teachers employ. The study was limited to beliefs about the nature of mathematics and the teaching and learning, in order to explicate beliefs the educators hold without diffusing the outcomes with these related, important factors.

CHAPTER FOUR

PHASE ONE RESULTS

The purpose of the current study was to explore the beliefs and attitudes special education teachers hold about the discipline of mathematics and the teaching and learning of mathematics. The research question for the study was, "What is the nature of the beliefs and attitudes held by special educators about the discipline of mathematics and the teaching and learning of mathematics?" The research question was investigated using mixed method research design exploring four domain-related questions: (a) what are the attitudes of special educators about mathematics, (b) what are the beliefs of special educators about the discipline of mathematics, (c) what are the beliefs of special educators about teaching mathematics, and (d) what are the beliefs of special educators about learning mathematics?

The study was conducted in two phases with the first phase consisting of quantitative data collected from the full study sample using the *Math Anxiety Rating Scale: Short Version* (*MARS-SV*), the *Mathematics Beliefs Instrument* (*MBI*), and relevant demographic data through an online survey. The second phase of the study involved collecting qualitative data through a semi-structured interview from a sub-sample selected from the full study sample. Results from Phase One of the study are reported in Chapter Four; results from Phase Two of the study are reported in Chapter Five.

In this chapter, the findings from Phase One of the study will be described. First, the mathematics attitudes of full study sample will be reported. Next, the mathematics beliefs of the full study will be described with respect to beliefs about the nature of mathematics, beliefs about teaching mathematics, and beliefs about learning mathematics. Finally, results of additional statistical tests used to determine whether relationship existed among the study variables will be summarized.

Full Study Sample Attitude about Mathematics

In Phase One of the study, mathematics attitude was defined in terms of participants' anxiety toward mathematics. Mathematics anxiety (MA) was measured using the *Math Anxiety Rating Scale: Short Version* (*MARS-SV*). The *MARS-SV* is a 30-item instrument consisting of questions about activities that involve mathematics, such as performing calculations, experiences in mathematics classes, and using mathematics in everyday life (Appendix C). Respondents rate their anxiety for each item on a five-point scale with descriptors of: (a) not at all, (b) a little, (c) a fair amount, (d) much, or (e) very much. Descriptive analyses were conducted on the full study sample *MARS-SV* results to explain the MA of the study sample and answer one domain of the research question, "What is the nature of the attitudes of special educators toward mathematics?"

Reliability and Validity of Data

Analysis of data from the *MARS-SV* included an assessment of the reliability and validity of the data. To assess whether the *MARS-SV* items formed a reliable scale, Cronbach's alpha was computed. The alpha for the *MARS-SV* items was .96, which is considered high internal consistency (Cohen, 1988).

Given the absence of information related to subconstructs for the *MARS-SV*, analysis of whether the data produced conformed to the structure of the instrument was not possible. However, as Gliner, Morgan, and Leech (2009) noted, "measurement validity is concerned with establishing evidence for the use of a particular measure or instrument in a particular setting with a particular population for a specific purpose" (p.

255). The use of the *MARS-SV* for the purpose of measuring the MA of study participants was consistent with the intended purpose of the instrument and produced results consistent with the normative sample. The results from the study sample were compared with the normative sample used for the *MARS-SV*. Only percentile data were available for the *MARS-SV* normative sample thus the medians of the study sample and normative sample were compared using a Wilcoxon signed ranks test. The results indicated that the medians of the two samples are not significantly different (p = .39). These results suggest that the *MARS-SV* results for the study sample do not differ from that of the normative sample thus establishing limited evidence of validity.

Results

The sample included 48 special education teachers involved in providing instruction to students with specific learning disabilities (SLD). Descriptive results are provided here and summarized in Table IV.20. The mean MA level for the study sample was 58.08, and the standard deviation was 19.07. The minimum MA level was 34, which is equivalent to the 5th percentile of normative data for the *MARS-SV* (Appendix H). The maximum MA level was 107, which is slightly lower than the equivalent to the 95th percentile of normative data for the *MARS-SV*. The median of the sample was 54, which was five points lower than normative data for the *MARS-SV*. Furthermore, the 25th percentile of the study sample was 42.25, lower than the normative sample of 46. The 75th percentile of the study sample was 73.75, lower than the normative sample of 78. The skewness for the MA of the study sample is .89, which is within the range assumed for a normal distribution (Leech, et al., 2008).

Table IV.20 *Math Anxiety Rating Scale: Short Version* Results for Study Sample

Statistic	Study Sample	Normative *MARS-SV* Data
N	48	--
Mean	58.08	--
SD	19.07	--
Minimum	34	--
Maximum	107	--
Median	54	59
Quartile 1	42.25	46
Quartile 3	73.75	78
Skewness	.89	--

In order to better understand the distribution of *MARS-SV* scores across the full study sample, a box and whisker plot (Figure IV.19) and histogram (Figure IV.20) were created. The box and whisker plot (Figure IV.19) of the *MARS-SV* scores shows a concentration on scores on the lower end of the distribution.

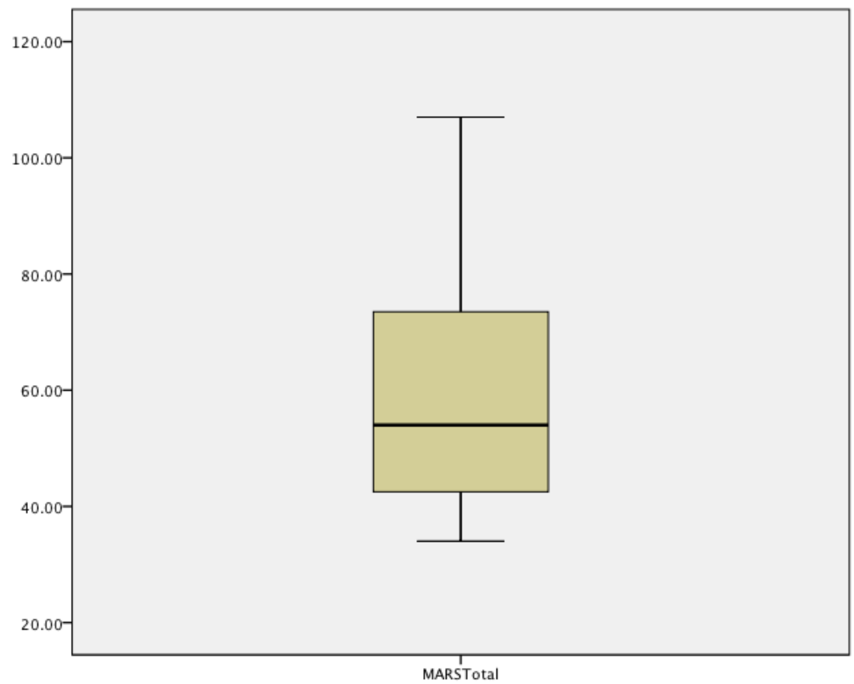

Figure IV.19. Box and Whisker Plot Displaying the Distribution *MARS-SV* Scores for Full Study Sample.

A similar concentration of *MARS-SV* scores among the study sample on the lower end of the distribution is illustrated in the histogram (Figure IV.20).

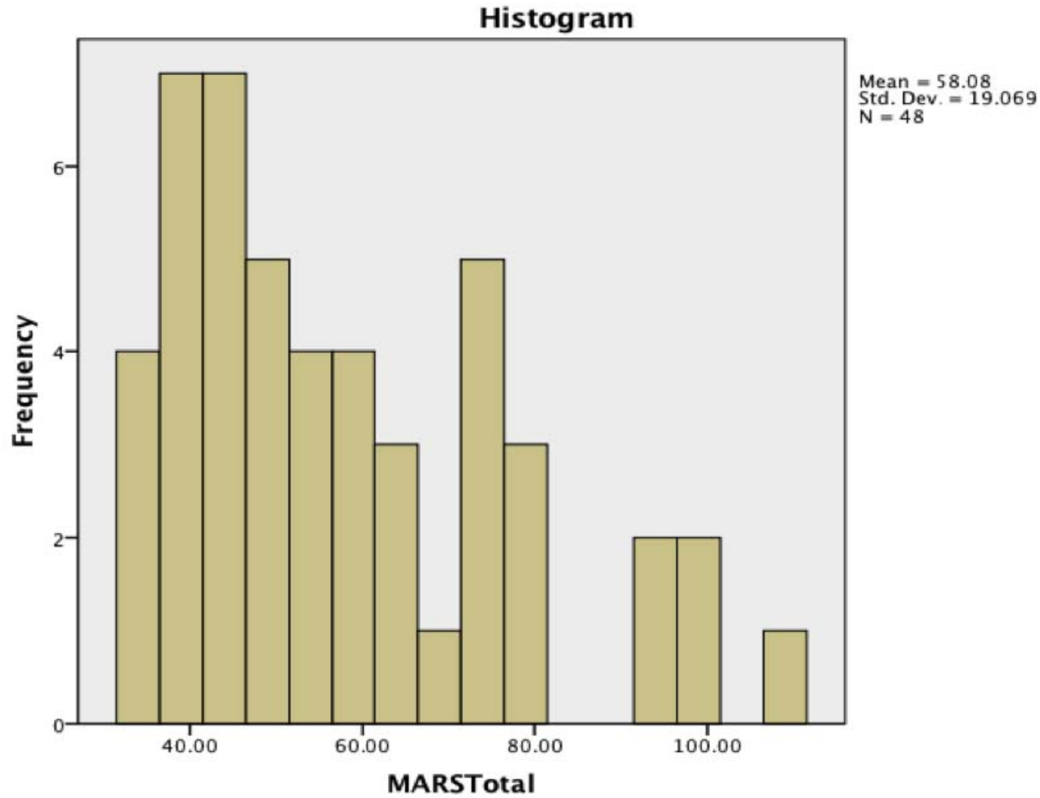

Figure IV.20. Histogram Displaying the Distribution of *MARS-SV* Scores For Full Study Sample.

Analysis of *MARS-SV* data indicated that the sample results are normally distributed despite a relative concentration on the lower end of the distribution.

Full Study Sample Mathematics Beliefs

In Phase One of the study, the mathematics beliefs of the full study sample were measured using the *Mathematics Beliefs Instrument* (*MBI*). The *MBI* provided data relevant for answering the domain questions (b) what are the beliefs of special educators about the discipline of mathematics, (c) what are the beliefs of special educators about teaching mathematics, and (d) what are the beliefs of special educators about learning mathematics?

The *MBI* is a 28-item instrument designed to measure the beliefs respondents have toward mathematics, teaching mathematics, and learning mathematics. The *MBI* includes items requiring binary responses (agree or disagree) and scaled responses. Responses to *MBI* items were scored using a 2- or 4-point scale. Descriptive results are provided here.

Reliability and Validity of Data

Analysis of data from the *MBI* included an assessment of the reliability and validity of the data. To assess whether the *MBI* items formed a reliable scale, Cronbach's alpha was computed. The alpha for the *MBI* items was .84, which is considered high internal consistency (Cohen, 1988). Like the *MARS-SV*, the *MBI* does not have information related to subconstructs underlying the items, thus, analysis of whether the data produced conformed to the structure of the instrument was not possible.

Descriptive Statistics For The Full Study Sample MBI Results

Descriptive statistics and distribution of the *MBI* scores is illustrated in Table IV.21. Results indicated that the data represented a normal distribution with the skewness of -.433, which is within the range assumed for a normal distribution (Leech, et al., 2008). Because the *MBI* does not utilize standardized scoring protocol, there is no available normative data for comparison with the full study sample.

Table IV.21 Descriptive Statistics of the Study Sample on the *MBI*

Statistic	Study Sample
N	48
Mean	66.73
SD	6.52
Minimum	52
Maximum	77
Median	67
Quartile 1	62
Quartile 3	73
Skewness	-.43

To visualize the distribution of *MBI* data, a histogram was created (Figure IV.21). The display shows a concentration of scores on the higher end of the score range; however, the skewness of -.43 indicates the data are normally distributed.

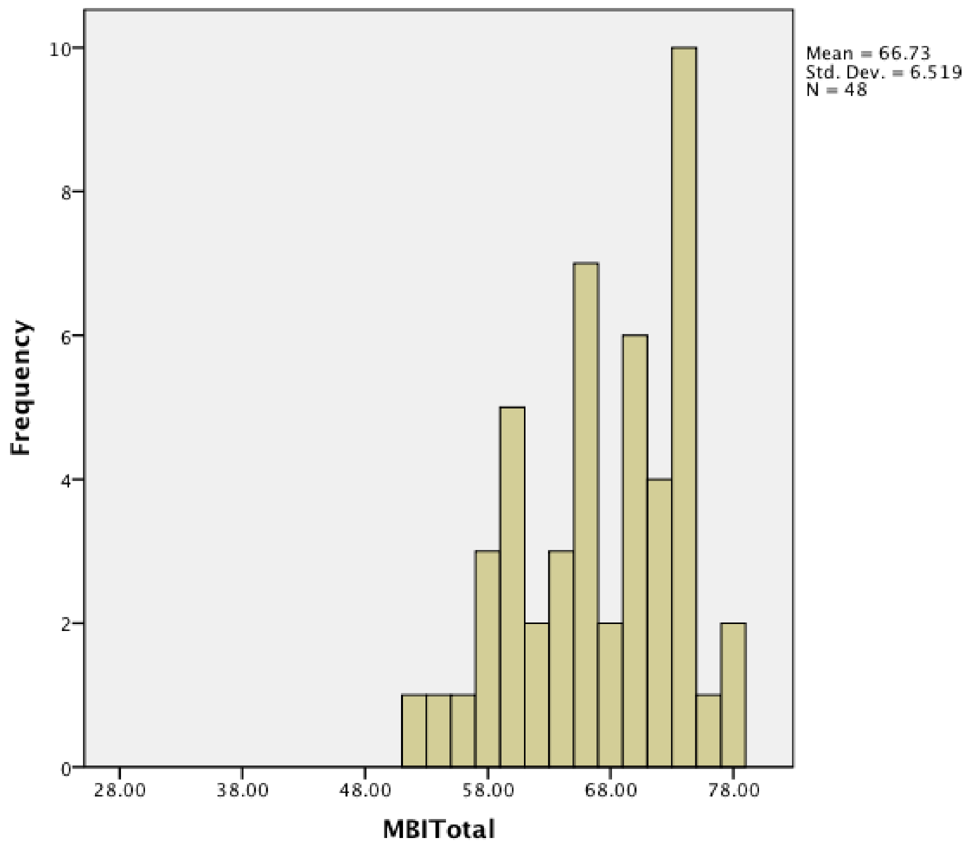

Figure IV.21. Distribution of Participant Scores on the *MBI*.

Given the absence of normative data for the *MBI*, further analysis of the *MBI* was conducted in order to make meaning of the results. Participant responses for each item were analyzed to better understand the beliefs of participants with high and low MB.

Item Analysis of the MBI

As noted previously, the *MBI* does not utilize standardized scoring protocol, and there is no available normative data for comparison. However, responses for each question contained in the *MBI* can provide information related study participant beliefs on particular aspects of mathematics, teaching mathematics, and learning mathematics. Thus, data from each item were analyzed to determine the degree of alignment with practices and assumptions promoted by reform-based mathematics.

To begin with, results on a number of questions were skewed toward reform-based beliefs. Table IV.22 illustrates the skewness of each item. A key to the abbreviated *MBI* questions used in Table IV.22 is provided in Appendix I.

Table IV.22 Skewness of *MBI* Responses

MBI Question	Skewness
ShowMany	-3.367
GoalPower	-3.113
ClueWords	2.676
EthnicBetter	-2.526
MathStrands	-2.342
Reasoning	-2.072
MalesBetter	-1.765
MathKnown	-1.509
JustSoln	-1.483
CorrectOne	-1.458
MathCollec	1.192
ProbSolv	-1.065
SolveQuickly	-1.040

In order to understand trends in participants' responses according to the conceptual framework that guided this study, analyses of participants' responses to *MBI* questions were categorized according to the domains of (a) beliefs about the nature of mathematics, (b) beliefs about teaching mathematics, and (c) beliefs about learning mathematics. The categorization of the *MBI* questions according to the research question domains is provided in Table IV.23.

Table IV.23 *Mathematics Beliefs Instrument* Statements Categorized By Research Question Domains

Beliefs about the nature of mathematics	Beliefs about teaching mathematics	Beliefs about learning mathematics
Problem solving should be a separate, distinct part of the mathematics curriculum.	A major goal of mathematics instruction is to help children develop the belief that they have the power to control their own success in mathematics.	Students should share their problem solving thinking and approaches with other students.
Mathematics can be thought of as a language that must be meaningful if students are to communicate and apply mathematics productively.	Children should be encouraged to justify their solutions, thinking, and conjectures in a single way.	The study of mathematics should include opportunities of using mathematics in other curriculum areas.
The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation.	In K-5 mathematics, increased emphasis should be given to reading and writing numbers symbolically.	In K-5 mathematics, skill in computation should precede word problems.
A demonstration of good reasoning should be regarded even more than students' ability to find correct answers.	In K-5 mathematics, increased emphasis should be given to use of clue words (key words) to determine which operation to use in problem solving.	Learning mathematics must be an active process
In mathematics something is either right or it is wrong.	Mathematics should be taught as a collection of concepts, skills, and algorithms.	Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.

Table IV.22 (Continued)

Everything important about mathematics is already known by mathematicians.	Appropriate calculators should be available to all students at all times.	Children enter kindergarten with considerable mathematical experience, a partial understanding of many mathematical concepts, and some important mathematical skills.
Math problems can be done correctly in only one way.	Good mathematics teachers show students lots of different ways to look at the same question.	Some people are good at mathematics and some aren't.
	Good math teachers show you the exact way to answer the math question you will be tested on.	In mathematics you can be creative and discover things by yourself.
		To solve most math problems you have to be taught the correct procedure.
		The best way to do well in math is to memorize all the formulas.
		Males are better at math than females.
		Some ethnic groups are better at math than others.
		To be good in math you must be able to solve problems quickly.

In order to determine the degree to which the mathematics beliefs of the study sample beliefs aligned with reform-based beliefs, one-sample Wilcoxon signed ranks tests were conducted for each item on the *MBI* to assess whether the median of the study sample results significantly differed from hypothesized median for the item. For the analysis of two-point scale responses on the *MBI*, the hypothesized median was set at 1.5. For the analysis of the of the four-point scale responses, the hypothesized median was set at 2.5. Assumptions of independent data are independent and continuity of data from low to high in the dependent variable were checked and met (Morgan, Leech, Gloeckner, & Barrett, 2011).

In the sections that follow, item analyses of each of the questions of the *MBI* is provided according to the research question domains related to understand participants' beliefs about the nature of mathematics, beliefs about teaching mathematics, and beliefs about learning mathematics.

Full Study Sample Beliefs about the Nature of Mathematics

Seven items on the *MBI* provided insight into the beliefs that the study sample held about the nature of mathematics. The following *MBI* items reflect the nature of mathematics. Included are the abbreviations given for each item that appear in Table IV.22.

1. Problem solving should be a separate, distinct part of the mathematics curriculum (ProbSolv).
2. Mathematics can be thought of as a language that must be meaningful if students are to communicate and apply mathematics productively (MathLang).
3. The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation (MathCurric).
4. A demonstration of good reasoning should be regarded even more than students' ability to find correct answers (Reasoning).
5. In mathematics something is either right or it is wrong (RightWrong).
6. Everything important about mathematics is already known by mathematicians (MathKnown).
7. Math problems can be done correctly in only one way (CorrectOne).

Table V.24 displays the results of descriptive analyses and one sample Wilcoxon signed ranks tests used to determine whether the median of the study sample participant responses were significantly different from an hypothesized median for each item.

Table IV.24 Results of Wilcoxon Signed Ranks Tests for *MBI* Items Related to Beliefs about the Nature of Mathematics

Variable	*M*	*SD*	*Median*	*p*
2 Point Responses				
ProbSolv	1.73	.45	2	.001
MathLang	2.00	-	2	-
MathCurric	2.00	-	2	-
4 Point Responses				
Reasoning	1.85	.36	2	<.001
RightWrong	2.27	.76	2	.03
MathKnown	3.63	.64	4	<.001
CorrectOne	3.71	.50	4	<.001

Problem solving. Item one on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Problem solving should be a separate, distinct part of the mathematics curriculum. A reform-based approach would not be in agreement with this statement and would yield a score of 2. Alternately, a non-reformed based approach would yield a score of 1. Among the study sample, 27.1% (n = 13) did not agree with this statement whereas 72.9% (n = 35) agreed. For this question, the population was skewed (-1.065) away from a reform-based approach to mathematics. The average score on a two-point scale was 1.73 with a standard deviation of .45. The median response for this item was 2, which was significantly different than the hypothesized median (p = .001) suggesting that the study sample was not in alignment with reform-based beliefs on this statement.

Mathematics as a language. Item three on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Mathematics can be thought of as a language that must be meaningful if students are to communicate and apply mathematics productively. All participants agreed with the statement indicating alignment of the study sample with a reform-based belief about the nature of mathematics.

Teaching mathematics strands in isolation. Item seven on the *MBI* asked participants for a binary response (agree or disagree) to this statement: The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation. A reform-based approach would not be in agreement with this statement and would yield a score of 2. Alternately, a non-reformed based approach would yield a score of 1. Among the study sample, 87.5% (n = 42) did not agree with this statement whereas 12.5% (n = 6) agreed. For this question, the population was skewed (-2.34) toward a reform-based approach to mathematics. The average score on a two-point scale was 1.88 with a standard deviation of .33. The median response for this item was 2, which was significantly different than the hypothesized median (p = .001) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Demonstrating reasoning. Item thirteen on the *MBI* asked participants for a binary response (agree or disagree) to this statement: A demonstration of good reasoning should be regarded even more than students' ability to find correct answers. A reform-based approach would be in agreement with this statement and would yield a score of 2. Alternately, a non-reform based approach would yield a score of 1. Among the study participants, 85.4% ($n = 41$) agreed with this statement whereas 14.6% ($n = 7$) disagreed. The average score on a two-point scale was 1.85 with a standard deviation of .36 and was skewed toward a reform-based approach (-2.07). The median response for this item was 2, which was significantly different than the hypothesized median ($p < .001$) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Mathematics answers are right or wrong. Item seventeen on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: In mathematics something is either right or it is wrong. A reform-based approach would not be in agreement with this statement and would yield a score of 4. A non-reformed-based response would yield a score of 1. Among the participants, 6.3% ($n = 3$) answered true, 54.2% ($n = 26$) answered more true than false, 27.1% ($n = 13$) answered more false than true, and 6.3% ($n = 3$) answered false. The average score on a four-point scale was 2.27 with a standard deviation of .76. The median response for this item was 2, which was significantly different than the hypothesized median ($p = .03$) suggesting that the study sample was not in alignment with reform-based beliefs on this statement.

All mathematics is known by mathematicians. Item twenty-one on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: Everything important about mathematics is already known by mathematicians. A reform-based approach would not be in agreement with this statement and would yield a score of 4. A non-reformed-based response would yield a score of 1.

None of respondents answered true, 8.3% ($n = 4$) answered more true than false, 20.8% ($n = 10$) answered more false than true, and 70.8% ($n = 34$) answered false. The average score on a four-point scale was 3.63 with a standard deviation of .64 and was skewed toward a reform-based approach (-1.51). The median response for this item was 4, which was significantly different than the hypothesized median ($p < .001$) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Mathematics problems are solved in single way. Item twenty-three on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: Math problems can be done correctly in only one way. A reform-based approach would not be in agreement with this statement and would yield a score of 4. Alternately, a non-reformed based approach would yield a score of 1. No respondents answered true, 2.1% ($n = 1$) answered more true than false, 25% ($n = 12$) answered more false than true, and 72.9% ($n = 35$) answered false. The average score on a four-point scale was 3.71 with a standard deviation of .50 and responses were skewed toward a reform-based approach (-1.46). The median response for this item was 4, which was significantly different than the hypothesized median ($p < .001$) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Full Study Sample Beliefs About Teaching Mathematics

Eight items on the *MBI* provided insight into the beliefs that the study sample held about teaching mathematics. The following *MBI* items reflect the teaching. Included are the abbreviations given for each item that appear in Table IV.22.

1. A major goal of mathematics instruction is to help children develop the belief that they have the power to control their own success in mathematics (GoalPower).
2. Children should be encouraged to justify their solutions, thinking, and conjectures in a single way (JustSoln).

3. In K-5 mathematics, increased emphasis should be given to reading and writing numbers symbolically (ElemSym).
4. In K-5 mathematics, increased emphasis should be given to use of clue words (key words) to determine which operation to use in problem solving (ClueWords).
5. Mathematics should be taught as a collection of concepts, skills, and algorithms (MathCollec).
6. Appropriate calculators should be available to all students at all times (Calc).
7. Good mathematics teachers show students lots of different ways to look at the same question (ShowMany).
8. Good math teachers show you the exact way to answer the math question you will be tested on (ShowExact).

Table IV.25 displays the results of descriptive analyses and the one sample Wilcoxon signed ranks tests used to determine whether the median of the study sample participant responses were significantly different from an hypothesized median for each item.

Table IV.25 Results of Wilcoxon Signed Ranks Tests for *MBI* Items Related to Beliefs about Teaching Mathematics

Variable	*M*	*SD*	*Median*	*p*
2 Point Responses				
GoalPower	1.92	.28	2	<.001
JustSoln	1.79	.41	2	<.001
ElemSym	1.42	.50	1	.25
ClueWords	1.10	.31	1	<.001
MathCollec	1.25	.44	1	.001
Calc	1.40	.49	1	.15
4 Point Responses				
ShowMany	3.88	.39	4	<.001
ShowExact	3.21	.87	3	<.001

Goal of mathematics instruction. Item four on the *MBI* asked participants for a binary response (agree or disagree) to this statement: A major goal of mathematics instruction is to help children develop the belief that they have the power to control their own success in mathematics. A reform-based approach would be in agreement with this statement and would yield a score of 2. Alternately, a non-reform based approach would yield a score of 1. Among the study participants, 91.7% (n = 44) agreed with this statement whereas 8.3% (n = 4) disagreed. The average score on a two-point scale was 1.92, which was skewed (-3.11) toward a reform-based approach to mathematics. The median response for this item was 2, which was significantly different than the hypothesized median (p <.001) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Justifying solutions. Item four on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Children should be encouraged to justify their solutions, thinking, and conjectures in a single way. A reform-based approach would not be in agreement with this statement and would yield a score of 2. Alternately, a non-reformed based approach would yield a score of 1. Among the study participants, 79.2% (n = 38) disagreed with this statement whereas 20.8% (n = 10) agreed. The average score on a two-point scale was 1.79, which was skewed (-1.48) toward a reform-based approach to mathematics. The median response for this item was 2, which was significantly different than the hypothesized median (p < .001) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Elementary mathematics emphasis. Item eight on the *MBI* asked participants for a binary response (agree or disagree) to this statement: In K-5 mathematics, increased emphasis should be given to reading and writing numbers symbolically. A reform-based approach would not be in agreement with this statement and would yield a score of 2. Alternately, a non-reformed based approach would yield a score of 1. Among the participants, 41.7% (n = 20) disagreed with this statement whereas 58.3% (n = 28) agreed. The average score on a two-point scale was 1.42 with a standard deviation of .50. The median response for this item was 1, which was not significantly different than the hypothesized median.

Use of clue words. Item nine on the *MBI* asked participants for a binary response (agree or disagree) to this statement: In K-5 mathematics, increased emphasis should be given to use of clue words (key words) to determine which operation to use in problem solving. A reform-based approach would not be in agreement with this statement and would yield a score of 2. Alternately, a non-reformed based approach would yield a score of 1. Among the participants, 10.4% (n = 5) disagreed with this statement whereas 89.6% (n = 43) agreed. The average score on a two-point scale was 1.10, which was skewed away (2.68) from a reform-based approach to mathematics. The median response for this item was 1, which was significantly different than the hypothesized median (p < .001) suggesting that the study sample was not in alignment with reform-based beliefs on this statement.

Mathematics as a collection of concepts, skills, and algorithms. Item twelve on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Mathematics should be taught as a collection of concepts, skills and algorithms. A reform-based approach would not be in agreement with this statement and would yield a score of 2. Alternately, a non-reformed based approach would yield a score of 1. Among the participants, 75% (n = 36) agreed with this statement whereas 25% (n = 12) disagreed. The average score on a two-point scale was 1.25 with a standard deviation of .44 and was skewed toward a reform-based approach (1.19). The median response for this item was 1, which was significantly different than the hypothesized median (p = .001) suggesting that the study sample was not in alignment with reform-based beliefs on this statement.

Availability of calculators. Item fourteen on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Appropriate calculators should be available to all students at all times. A reform-based approach would be in agreement with this statement and would yield a score of 2. Alternately, a non-reform based approach would yield a score of 1. Among the participants, 39.6% (n = 19) agreed with this statement whereas 60.4% (n = 29) disagreed. The average score on a two-point scale was 1.39 with a standard deviation of .49. The median response for this item was 1, which was not significantly different than the hypothesized median.

Teachers show multiple strategies. Item eighteen on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: Good mathematics teachers show students lots of different ways to look at the same question. A reform-based approach would be in agreement with this statement and would yield a score of 4. Alternately, a non-reform based approach would yield a score of 1. Among the participants, 89.6% (n = 43) answered true, 8.3% (n = 4) answered more true than false, 2.1% (n = 1) answered more false than true, and no respondents answered false. The average score on a four-point scale was 3.88 with a standard deviation of .39, which was skewed toward a reform-based approach (3.37). The median response for this item was 4, which was significantly different than the hypothesized median ($p < .001$) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Teachers show exact ways to answer. Item twenty on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: Good math teachers show you the exact way to answer the math question you will be tested on. A reform-based approach would not be in agreement with this statement and would yield a score of 4. Alternately, a non-reformed based approach would yield a score of 1. Among the participants, 4.2% (n = 2) answered true, 16.7% (n = 8) answered more true than false, 33.3% (n = 16) answered more false than true, and 45.8% (n = 22) answered false. The average score on a four-point scale was 3.21 with a standard deviation of .87. The median response for this item was 3, which was significantly different than the hypothesized median ($p < .001$) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Full Study Sample Beliefs About Learning Mathematics

Thirteen items on the *MBI* provided insight into the beliefs that the study sample held about learning mathematics. The following *MBI* items reflect the teaching. Included are the abbreviations given for each item that appear in Table IV.22.

1. Students should share their problem solving thinking and approaches with other students (ShareThink).
2. The study of mathematics should include opportunities of using mathematics in other curriculum areas (MathCurric).
3. In K-5 mathematics, skill in computation should precede word problems (CompPrec).
4. Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement (LearnAbsorb).
5. Learning mathematics must be an active process (Active).

6. Children enter kindergarten with considerable mathematical experience, a partial understanding of many mathematical concepts, and some important mathematical skills (EnterK).
7. Some people are good at mathematics and some aren't (GoodNot).
8. In mathematics you can be creative and discover things by yourself (DiscoverSelf).
9. To solve most math problems you have to be taught the correct procedure (TaughtProc).
10. The best way to do well in math is to memorize all the formulas (Memorize).
11. Males are better at math than females (MalesBetter).
12. Some ethnic groups are better at math than others (EthnicBetter).
13. To be good in math you must be able to solve problems quickly (SolveQuickly).

Table IV.26 displays the results of descriptive analyses and the one sample Wilcoxon signed ranks tests used to determine whether the median of the study sample participant responses were significantly different from an hypothesized median for each item.

Table IV.26 Results of Wilcoxon Signed Ranks Test for *MBI* Items Related to Beliefs about Learning Mathematics

Variable	*M*	*SD*	*Median*	*p*
2 Point Responses				
ShareThink	2.00	-	2	<.001
MathCurric	2.00	-	2	<.001
CompPrec	1.54	.50	2	.56
LearnAbsorb	1.50	.51	1.5	1
Active	2.00	-	2	<.001
EnterK	1.56	.50	2	.39
4 Point Responses				
GoodNot	2.73	.89	3	.09
DiscoverSelf	3.50	.65	4	<.001
TaughtProc	2.67	.93	3	.23
Memorize	3.25	.73	3	<.001
MalesBetter	3.75	.48	4	<.001
EthnicBetter	3.71	.65	4	<.001
SolveQuickly	3.50	.68	4	<.001

Students should share their thinking. Item two on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Students should share their problem solving thinking and approaches with other students. There was unanimous agreement with this statement indicating alignment of the study sample with a reform-based belief about learning mathematics.

Mathematics should be studied across the curriculum. Item six on the *MBI* asked participants for a binary response (agree or disagree) to this statement: The study of mathematics should include opportunities of using mathematics in other curriculum areas. There was unanimous agreement with this statement indicating alignment of the study sample with a reform-based belief about learning mathematics.

Computation precedes word problems. Item ten on the *MBI* asked participants for a binary response (agree or disagree) to this statement: In K-5 mathematics, skill in computation should precede word problems. A reform-based approach would not be in agreement with this statement and would yield a score of 2. Alternately, a non-reformed based approach would yield a score of 1. Among the participants, 54.2% (n = 26) disagreed with this statement whereas 45.8% (n = 22) agreed. The average score on a two-point scale was 1.10. The median response for this item was 2, which was not significantly different than the hypothesized median.

Learning mathematics through absorption. Item eleven on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement. A reform-based approach would not be in agreement with this statement and would yield a score of 2. Alternately, a non-reformed based approach would yield a score of 1. Among the study participants, 50% (n = 24) disagreed with this statement whereas 50% (n = 24) agreed. The average score on a two-point scale was 1.50 with a standard deviation of .51. The median response for this item was 1.5, which was not significantly different than the hypothesized median.

Learning mathematics as an active process. Item fifteen on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Learning mathematics must be an active process. There was unanimous agreement with this statement indicating alignment of the study sample with a reform-based belief about learning mathematics.

Children enter kindergarten with mathematical knowledge. Item sixteen on the *MBI* asked participants for a binary response (agree or disagree) to this statement: Children enter kindergarten with considerable mathematical experience, a partial understanding of many mathematical concepts, and some important mathematical skills. A reform-based approach would be in agreement with this statement and would yield a score of 2. Alternately, a non-reform based approach would yield a score of 1. Among the participants, 56.3% (n = 27) agreed with this statement whereas 43.8% (n = 21) disagreed. The average score on a two-point scale was 1.56 with a standard deviation of .50. The median response for this item was 2, which was not significantly different than the hypothesized median.

Innate mathematics ability. Item seventeen on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: Some people are good at mathematics and some aren't. A reform-based approach would not be in agreement with this statement and would yield a score of 4. Alternately, a non-reformed based approach would yield a score of 1. Among the participants, 20.8% (n = 10) answered true, 39.6% (n = 19) answered more true than false, 31.3% (n = 15) answered more false than true, and 8.3% (n = 4) answered false. The average score on a four-point scale was 2.73 with a standard deviation of .89. The median response for this item was 3, which was not significantly different than the hypothesized median.

Mathematics creativity and discovery. Item twenty-two on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: In mathematics you can be creative and discover things by yourself. A reform-based approach would be in agreement with this statement and would yield a score of 4. Alternately, a non-reform based approach would yield a score of 1. Among the participants, 58.3% (n = 28) answered true, 33.3% (n = 16) answered more true than false, 8.3% (n = 4) answered more false than true, and no respondents answered false. The average score on a four-point scale was 3.50 with a standard deviation of .65. The median response for this item was 4, which was significantly different than the hypothesized median ($p < .001$) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Teaching procedures. Item twenty-four on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: To solve most math problems you have to be taught the correct procedure. A reform-based approach would not be in agreement with this statement and would yield a score of 4. Alternately, a non-reformed based approach would yield a score of 1. No respondents answered true, 2.1% (n = 1) answered more true than false, 25% (n = 12) answered more false than true, and 72.9% (n = 35) answered false. The average score on a four-point scale was 2.67 with a standard deviation of .93. The median response for this item was 3, which was not significantly different than the hypothesized median.

Memorization. Item twenty-four on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: The best way to do well in math is to memorize all the formulas. A reform-based approach would not be in agreement with this statement and would yield a score of 4. Alternately, a non-reformed based approach would yield a score of 1. Among the participants, 2.1% (n = 1) answered true, 10.4% (n = 5) answered more true than false, 47.9% (n = 23) answered more false than true, and 39.6% (n = 19) answered false. The average score on a four-point scale was 3.25 with a standard deviation of .73. The median response for this item was 3, which was significantly different than the hypothesized median (p < .001) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Superiority of males in mathematics. Item twenty-six on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: Males are better at math than females. A reform-based approach would be in agreement with this statement and would yield a score of 4. Alternately, a non-reform based approach would yield a score of 1. No participants answered true, 2.1% (n = 1) answered more true than false, 20.8% (n = 10) answered more false than true, and 77.1% (n = 37) answered false. The average score on a four-point scale was 3.75 with a standard deviation of .48, which was skewed in the direction of a reform-based approach (-1.77). The median response for this item was 4, which was significantly different than the hypothesized median (p < .001) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Superiority of ethnic groups in mathematics. Item twenty-seven on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: Some ethnic groups are better at math than others. A reform-based approach would not be in agreement with this statement and would yield a score of 4. Alternately, a non-reformed based approach would yield a score of 1. Among the participants, 2.1% ($n = 1$) answered true, 4.2% ($n = 2$) answered more true than false, 14.6% ($n = 7$) answered more false than true, and 79.2% ($n = 38$) answered false. The average score on a four-point scale was 3.71 with a standard deviation of .65, which was skewed in the direction of a reform-based approach (-2.53). The median response for this item was 4, which was significantly different than the hypothesized median ($p < .001$) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Solving problems quickly. Item twenty-eight on the *MBI* asked participants for a scaled response (true, more true than false, more false than true, false) to this statement: To be good in math you must be able to solve problems quickly. A reform-based approach would not be in agreement with this statement and would yield a score of 4. Alternately, a non-reformed based approach would yield a score of 1. No participants answered true, 10.4% ($n = 5$) answered more true than false, 29.2% ($n = 14$) answered more false than true, and 60.4% ($n = 29$) answered false. The average score on a four-point scale was 3.50 with a standard deviation of .68, which was skewed in the direction of a reform-based approach (-1.04). The median response for this item was 4, which was significantly different than the hypothesized median ($p < .001$) suggesting that the study sample was in alignment with reform-based beliefs on this statement.

Summary of MBI Item Analysis

The analysis of items on the *MBI* provided insight into the beliefs of the study sample according to each of the domain questions related to the research question guiding the study. Analyses provided depth to the descriptive analysis conducted on participant scores on the *MBI* by analyzing questions related to study sample participants' beliefs about the nature of mathematics, teaching mathematics, and learning mathematics. The item analysis confirmed descriptive results showing general alignment of the study sample beliefs with reform-based beliefs in the majority of domains addressed within the *MBI*.

Concluding Thoughts

The mixed method design of the current study enabled results from the two phases of the study to be combined in order to elaborate on the findings to enhance their significance. Results from Phase One of the study described in this chapter will be combined with results from Phase Two of the study to better understand the beliefs special educators hold about mathematics, teaching mathematics, and learning mathematics. Mixing the findings from each phase and type of data allows for greater depth in the conclusions that can be drawn from the study and more clarity for the direction of future study.

CHAPTER 5

PHASE TWO RESULTS

Phase Two of the study considered the main research question and domain-related questions of the entire study. The main research question was, “What is the nature of the beliefs and attitudes held by special educators about the discipline of mathematics and the teaching and learning of mathematics?” The research question was explored by answering four domain-related questions: (a) what are the attitudes of special educators about mathematics, (b) what are the beliefs of special educators about the discipline of mathematics, (c) what are the beliefs of special educators about teaching mathematics, and (d) what are the beliefs of special educators about learning mathematics?

Phase Two of the study was primarily concerned with elaborating on the attitudes and beliefs of study participants through analysis of qualitative data collected from a sub-sample of participants from the full-study sample. A sub-sample of the full-study sample was selected to participate in Phase Two by stratifying the participants according to two dimensions, degree of alignment with reform-based mathematics (mathematics beliefs or MB) and level of mathematics anxiety (MA). The sub-sample for Phase Two was selected in order to have representation of participants in each of the cells of the Variable Dimension Matrix (Table V.28), low alignment of mathematics beliefs, high mathematics anxiety (I); high alignment of mathematics beliefs, high mathematics anxiety (II); low alignment of mathematics beliefs, low mathematics anxiety, (III); and high alignment of mathematics beliefs, low mathematics anxiety. The stratification process allowed for greater elucidation of the beliefs of special educators.

Table V.27 Variable Dimension Matrix

Mathematics Anxiety Level		Mathematical Belief Alignment: Low	Mathematical Belief Alignment: High
	High	I	II
	Low	III	IV

Given the MA and MB profiles of the study participants, analysis of participants' attitudes and beliefs among cells was not possible. Instead, analyses consisted of comparing attitudes and beliefs of participants along the dimensions of MA and MB. To understand the attitudes special educators hold about mathematics, data from participants with low MA was compared with participants indicated with high MA (Table IV.28).

Table IV.28 Analysis of Mathematics Attitude

Mathematics Anxiety Level		
	High	High MA: Cells I and II
	Low	Low MA: Cells III and IV

Similarly, to understand the beliefs special educators hold about mathematics, data from participants identified as low alignment of beliefs with reform-based mathematics was compared with participants identified as high alignment of beliefs with reform-based mathematics (Table IV.29).

Table IV.29 Analysis of Mathematics Beliefs Alignment

Low MB: Cells I and III	High MB: Cells II and IV
Low	High

Mathematical Belief Alignment

Phase Two data collection consisted of conducting a semi-structured interview (Appendix E) with the sub-sample selected through the stratification process. The questions within the semi-structured were designed to provide data related to the four domain-related questions. By collecting qualitative data related to the sub-sample beliefs about the discipline of mathematics, teaching mathematics, and learning mathematics, the findings from the Phase One quantitative analysis could be elucidated and enhanced providing greater confidence in the conclusions drawn from the study.

In this chapter, the findings from Phase Two of the study will be described and related to Phase One of the study. First, the mathematics attitudes of the Phase Two participants will be reported. Next, the mathematics beliefs of the Phase Two participants will be described with respect to their beliefs about the nature of mathematics, beliefs about teaching mathematics, and beliefs about learning mathematics. Finally, the influence of teaching mathematics on the mathematics attitudes and beliefs of Phase Two participants will be described.

Phase Two Participants' Attitude about Mathematics

Measurement of participants' attitude toward mathematics for the full study sample was limited to the data from the *MARS-SV*. For Phase Two of the study, measurement of participants' attitudes toward mathematics was enhanced by their responses to questions on the semi-structured interview (Appendix E). The combined results of data collection related to mathematics attitudes of Phase One and Phase Two participants are displayed in Table V.30. Table V.30 shows the *MARS-SV* raw score, the respective normative percentile, and the MA rating for each Phase Two participant. The *MARS-SV* normative percentiles for individual *MARS-SV* scores are not available; instead normative data are available by *MARS-SV* raw score range (see Appendix H). The MA rating column indicates whether the *MARS-SV* score meets the criteria of high or low MA according to Ashcraft and Kirk (2001). Phase Two data included in Table V.29 show the self-rating that the Phase Two participants provided related to their comfort and enjoyment of mathematics.

Table V.30 Mathematics Attitude Summary for Phase Two Participants

Participant	*MARS-SV* Raw Score	*MAR-SV* Normative Percentile	MA Rating*	Self-rating of Mathematics Enjoyment on a Scale of 1 to 10 (1 is low, 10 is high)	Self-rating of Mathematics Anxiety on a Scale of 1 to 10 (1 is low, 10 is high)
Betty	35	<10th	Low	8-9	1
Callie	93	85-90th	High	9	2
Carson	41	<20th	--	10	"On low end"
Natalie	100	90-95th	High	3	7
Sally	41	<20th	--	9	2
Steven	34	5th	Low	5	2-3
Tammy	45	<25th	--	8	3

*Using the Ashcraft and Kirk (2001) definition of high and low MA.

A discussion of the mathematics attitudes of the Phase Two participants must include an acknowledgment of the limitations of data. Only six participants from the full study sample met Ashcraft and Kirk's (2001) criteria of high MA. Having so few participants with high MA complicated selected of extreme cases that also had high and low MB. Of these six potential participants for Phase Two of the study, only one had an *MBI* score indicating high MB. Also, of the two of the high MA participants met criteria of low MB and just one would respond to invitation to participate in Phase Two. As noted in Chapter 3, in order to expand the data set for Phase Two, criteria for high and low MA was broadened to the top and bottom quartile of MA of the *MARS-SV*.

Mathematics Attitudes of Phase Two Participants with High MA

Participants identified with high MA were Natalie and Callie. Natalie's *MARS-SV* score was 100, which was the second highest score in the entire study sample and above the 90th percentile of the normative *MARS-SV* data. Natalie rated her enjoyment of mathematics as a three on a scale of one to ten. Natalie rated her anxiety level as a seven on a scale of one to ten. She indicated that her anxiety has diminished over time.

For Natalie, mathematics was a source of discomfort and painful memories. When asked about her feelings about mathematics, Natalie described the impact of her high school mathematics teacher on her negative attitude toward mathematics:

> I struggle with it. I only got through Algebra I in high school and then I was told by my math teacher that he couldn't teach me anything so I never took another math class until I got to college. I still remember after all of these years. Back in 1975. He told me that right in front of my whole class.

The memory of this experience was clearly fresh in Natalie's mind such that 37 years later she still recalled the episode.

Natalie indicated that her attitude toward mathematics had improved over time, and that she attributed the change to teaching mathematics as the interview transcript portion illustrates:

INTERVIEWER:	In terms of anxiety about math with 1 being really comfortable with it and 10 being very anxious, how would you rate yourself?
NATALIE:	Probably a seven.
INTERVIEWER:	Do you think that's changed over time?
NATALIE:	Yes.
INTERVIEWER:	In the past you might have been higher than that?
NATALIE:	The anxiety would have been a 10. Even when I was in undergraduate school.
INTERVIEWER:	What do you think has helped you overcome a little bit of that anxiety?
NATALIE:	I'm in situations where I have actually have to teach the math.

Going further, Natalie explained,

> As a person it [mathematics] doesn't scare me as much as it used to because I've learned how to do it. I have taught myself or somebody else has taught me how. The feeling it doesn't, doesn't bother me anymore. It used to terrify me.

Despite her improved attitude in mathematics, Natalie continued to hold negative feelings and a high level of anxiety about mathematics.

The other participant identified as high MA was Callie. Callie's *MARS-SV* score was 94, which placed her at within the 85^{th}-90^{th}-percentile range for high MA. Callie's interview data and follow up conversations revealed a more complex attitude than the *MARS-SV* score suggested. In her interview, Callie indicated that she loves mathematics, rating her enjoyment at a nine on a scale of one to ten, ten being mathematics is very enjoyable. Callie rated her anxiety level toward mathematics at a two on a scale of one to ten with ten as very anxious. Her answer to a question about her feelings toward mathematics hinted at the complexity of her attitude:

> How do I feel about mathematics? I don't know, that's sort of broad, let me think. I mean it's sort of evolved. When I was growing up I was always really good at math. My mother, my parents told me you know when I was younger, like in second and third grade, we would take all these kinds of tests and I would always come out really good at math. I think about what does that mean to be really good at math? And for me when I was growing up it sort of felt like rote, didactic instruction and now it has sort of evolved more into—I teach more in a more of a problem solving, problem-task way and I think gosh I really get stumped on a lot of these problems and I'm constantly like maybe I am not that good at math.

Callie indicated that as her beliefs about teaching mathematics shifted from an instrumental to a problem solving approach, she began question her beliefs about what it means to be good at mathematics. In turn, she began to question her own ability in mathematics. As to the accuracy of the *MARS-SV* score, Callie was contacted following the data analysis to ask her perspective on the why her MA level would be considered high on the instrument in contrast to her interview. Callie's response was, "I would not consider myself to have a high anxiety towards mathematics. I don't want to skew your data but that just can't be correct.” Given the questions about the validity of Callie’s MA level, there are limits to the ability of this study to illuminate the beliefs and attitudes of special educators who have high MA.

Mathematics Attitudes of Phase Two Participants with Low MA

Participants identified with low MA were Steven, Tammy, Carson, Sally, and Betty. All participants with low MA had similar self-ratings ranging from 1 to 3. Self-rating on the low MA participants’ level of enjoyment of mathematics varied with the majority of participants expressing high levels of enjoyment; the exception was Steven whose rating of 5 suggested a more ambivalent attitude toward mathematics.

Participants with low MA provided varying amounts of detail related to their attitudes toward mathematics. Steven did not express strong positive or negative emotions related to mathematics, “I have no qualms about math, and I never really did have a high anxiety about math.” Steven’s attitude toward mathematics primarily related to his instructors and their instructional approaches, “I like when math is taught in a logical order. It builds upon itself. You know for me that was the easiest way to learn it. I think it’s the easiest way to teach it.” Steven expressed positive attitudes related to his middle school mathematics teacher and the structured, methodical instructional approach he employed. When asked what he thought of when he heard the word ‘mathematics’, Steven replied,

> I think of my seventh grade math teacher Mr. Conti. He was just a really good teacher, really cool with us. He was one of my favorite teachers in school and so I think of math, I think of him.

The only negative experiences in mathematics Steven could recall related to a personality conflict he experienced with his high school trigonometry teachers and the difficulty he had learning college algebra, which he attributed to the challenge of understanding his instructor's English. Steven's attitude toward mathematics was closely tied to his teachers and the instructional approaches they employed.

Betty expressed a more sustained positive attitude about mathematics throughout the interview than Steven. When asked about her attitude toward mathematics, Betty exclaimed,

> I love mathematics! I mean, I love teaching it and I love seeing kids discover it. I feel like the more I teach it the more deep that I learn the concepts and just see the connections in a different way and I really enjoy it that just on a personal level too.

Betty recalled always being successful in mathematics and could not remember a negative experience related to mathematics.

Similarly, Carson expressed enthusiasm about mathematics recalling that she has always enjoyed mathematics. Carson said,

> I love math! I really have loved math all my life. I was one of those quirky kids. I always played math games, just myself in my head. I still do it to this day. I'll look at the clock and play games with the numbers. My favorite is of course, one, two, three, four, 12:34.

Carson attributed her enjoyment of mathematics to the games she played growing up and her ongoing fascination with mathematics.

Sally provided a positive but nuanced explanation of her high self-rating of mathematics enjoyment and low level of mathematics anxiety. For Sally, mathematics always made sense, and she never experienced difficulty learning mathematics except in college calculus. Her attitude toward mathematics was related to both the level of mathematics and teaching students who struggle. Sally expressed hesitation about mathematics at the calculus level and above, specifically mentioning the course twice: "I don't like to do calculus anymore because I forgot so much," and "Don't ask me to do calculus, by the way." Sally also referenced her students in relation to questions about her attitude toward mathematics, "That's a very complicated question, isn't it? I like math myself, but I think teaching it to a lot of our kids is difficult because they struggle with it so much."

Tammy expressed that mathematics has always been her favorite subject. During the interview, Tammy often made an association with her success in mathematics to her positive attitude toward mathematics. When asked to what she attributed her positive attitude toward mathematics, Tammy said, "I think probably just my success in math growing up." Tammy noted that mathematics "just clicked" for her, a subject that always came naturally to her. In response to a question about what she remembers most about learning mathematics in school, Tammy replied, "I just enjoyed it. I know that it was fun for me, it was enjoyable. I got good grades."

Whereas the teachers with low MA were less apprehensive about mathematics, they had differing affinities toward the subject. Betty, Carson, and Tammy expressed enthusiasm about the subject of mathematics. In contrast, Sally expressed a more muted attitude about mathematics recognizing the struggles her students have with the subject, while Steven expressed a level of indifference about mathematics. For the sub-sample, a low level of MA did not necessarily equate with an affinity for the subject.

Themes Related to Mathematics Attitudes

In addition to responses directly related to the Phase Two participants' attitudes about mathematics, two additional themes emerged across each of the interviews, each involving the role of the mathematics teacher in mathematics attitude formation. First, as participants were asked to describe their worst mathematics experience, the salience of the experience varied across the sub-sample with the most prominent negative memories involving a teacher from participants' K-12 education experiences. Second, Phase Two participants provided responses that demonstrated the role their mathematics teachers had on their own mathematics attitude development or their recognition of their role in influencing the mathematics attitude development of their students.

Salience of worst mathematics experience. The salience of participants' worst mathematics experience varied across the Phase Two participants. For Natalie, the one participant with very high MA, the prominence of the negative feelings she had toward mathematics was evident in her first response in the interview. When asked about her feelings toward mathematics, Natalie immediately shared a humiliating experience in her high school mathematics class:

> I struggle with it. I only got through algebra I in high school and then I was told by my math teacher that he couldn't teach me anything so I never took another math class until I got to college.

For Natalie, the experience was still salient 37 years later.

For the other four participants with lower MA, the prominence of a negative mathematics memory was not as strong. Neither Betty nor Callie could recall any negative mathematics experience. Sally indicated that she struggled with concepts in college calculus but was able to get assistance. Tammy simply indicated that getting a bad grade at some point was probably her most negative experience.

Only Carson and Steven had stronger memories of a negative experience in mathematics, both of which were related to their teachers. Carson (high MB, low MA) discussed the challenges she faced with three-dimensional visualization, and the response of her mathematics teacher:

> School was always very easy for me. And I—my teacher, this illustrious coach, whose son was in my grade and a friend, thank you very much—would try to put up 3-D graphs and we had the x, y, and z axis and he would say, "Now, does everyone see it?" And I would get headaches a lot because I would try so hard to see it. And I would say, "No, no I don't." And I would be honest and I was extremely shy, and I would say, "No, can you try it again? Try another way. Try something. Help me see this." He got to the point where he would say, "Does everyone but Carson see it?"

The experience motivated Carson to respond to struggling students differently,

> Oh, it was humiliating, horrible, terrible, and so that was one of the times I thought, you know what? I want to teach kids and not humiliate them. I am going to build on their strengths and I'm gonna find different ways. Not just one way. I'm gonna help them see math when they have trouble. This might be my problem, but there are other kids who can't see what I can see, and I'm going to help it make sense. This is one of my passions.

Steven's (low MB, low MA) most negative experience in a mathematics class was not related to mathematics but to a personality issue with the teacher.

> Worst experience…probably my high school trigonometry teacher; he was a jerk. He didn't really like me because I was kind of ornery sometimes. We'd get out our homework and he would pick five or six students out of the class. And he would have you go up to the board and you were to write the homework problem out on the board, how you solved the problem whatever. And I didn't always have my homework done, but I didn't have it done sometimes out of spite. And he would put me up on the board and purposely tried to make me look bad. Then he'd be condescending and try to point me out or whoever out in front of the class, you didn't get your work done, you can't do it.

For each of the three participants with a salient negative mathematics experience, the teacher played a prominent role in the experience. Whether it was a teacher who was not compassionate about the struggles the participants faced learning mathematics or a teacher who used public humiliation as a classroom management tool, the role of the teacher in negative experiences for students is notable.

Role of teacher in student mathematics attitude development. Many of the participants acknowledged the role of the mathematics teacher in the development of students' mathematics attitudes. Natalie's negative experience with her Algebra 1 teacher illustrated the long-term impact of a teacher's action on the attitude of students. Natalie also shared the how the middle school mathematics teacher in her school system did not like boys and as a result, boys were coming to her high school class with a poor attitude toward mathematics.

Carson most clearly articulated this theme when she was asked to elaborate on a statement she made relating teacher anxiety to student attitude.

INTERVIEWER: So, you think that there's like a relationship maybe between the anxiety of the teacher and what happens with kids?

CARSON: Huge, huge, even if it is subconscious because of their previous experience and they're trying to cover it up, it comes through. You can totally tell and those teachers don't have enough experience working with it and they feel out of control. They are afraid to let kids experiment or do different kinds of methods because they were only taught, like I said, the old way where there is only one algorithm.

Further, Carson explained how she attends to how she projects her attitudes about mathematics and every subject,

> When I was a classroom teacher, my philosophy was the kids aren't gonna have fun unless you have fun, and I applied that to every subject and it didn't matter if I loved it or not. It didn't matter and at the end of the year, I always did a little survey, and I have the kids write down "What subject do you think Mrs. R. loves the most?" And I always got votes in every single subject.

Carson clearly believed that the attitude of the teacher is relayed to students.

Both Tammy and Sally referred to the role of teachers in support of student attitudes in mathematics. When asked what the most important aspect of teaching mathematics was, Tammy responded, "It's making children feel good about themselves and feel confident and feel successful. I think that's one of the most important things."

Sally commented that patience was most important for teaching mathematics given the frustration that students who struggle experience in mathematics. When asked to elaborate, Sally said,

> I think kids get frustrated very easily, and they get more frustrated if the teacher is frustrated. So if you can be patient and just try to figure out where they're getting hung up and try to teach it to them a different way, give them some different opportunities, think of different ways to approach it, and just be patient. Because most kids eventually, the light bulb comes on, and they're like 'Oh, I get it.'

To Sally, the patience of the teacher allows students to work through their difficulties without the added challenge of teacher frustration.

Callie echoed the importance of teacher focus on student attitude. When asked what was most important about teaching mathematics, she said,

> I think teachers really need to be focusing more on motivation and kids' identity in a math classroom and how they feel about it. Do they feel like they are good at it? Do they feel like they are bad at it? And most kids start to find that they are good at it and it's a motivating environment.

Callie also expressed that in her opinion, a contributor to the negative attitude students have toward mathematics relates to the rote, procedural way mathematics is taught in the United States.

Betty discussed the role of the teacher in developing a positive attitude toward mathematics noting that when the teacher is excited, so are the students:

> I was talking to a third grade teacher this year and we were both just really fired up about math and it was the last hour of the day and it was really hard, but I think

that because we were so excited about it the kids tended to be really excited during that time doing it. I think that came out in the classroom. It was great.

Interview participants provided multiple examples of the relationship between the mathematics attitudes of the mathematics teacher and the mathematics attitude of students. From their own experiences as students or their experiences teaching mathematics, sub-sample participants pointed to the role of the mathematics teacher in students' mathematics attitude development.

The design of the current study limited an exploration of mathematics attitude in Phase One of the study to mathematics anxiety. The qualitative design of Phase Two of the study allowed for greater elaboration on the participants' attitudes about mathematics. The qualitative data provided insight into the causes of positive and negative emotions related to mathematics, often implicating the role the mathematics teachers played in attitude development. Phase Two of the study also included data collection related to the beliefs the participants held about the discipline of mathematics, teaching mathematics, and learning mathematics. Results related to these domains are in the section that follows.

Phase Two Participants' Beliefs about Mathematics

In addition to further exploring participants' attitudes in Phase Two of the study, participants' beliefs about the nature of the discipline of mathematics, teaching mathematics, and learning mathematics were investigated. In Phase One of the study, all participants completed the *MBI*, which included 28 items related to mathematics and mathematics teaching and learning (Appendix D). Item analysis of the *MBI* proved useful in determining trends in participants' beliefs within the Phase Two participants as a whole and among those participants with high MB and those participants with low MB.

The semi-structured interview process enabled participants to provide information beyond the questions within the *MBI* and the interview questions. As such, themes emerged from the interview data that were not included within the *MBI* or semi-structured interview questions. Themes related to each of the belief domains explored in the study emerged through the interview data. Within the nature of mathematics domain, a theme related to participants' distinction between school mathematics and informal mathematics was found. Within the beliefs about teaching domain, two themes became apparent. The first theme involved participants' generalizations of their own mathematics learning preferences to teaching mathematics. The second theme related to participants' common instrumentalist mathematics learning experience. Finally, within the beliefs about learning domain, a theme related to teacher attribution of success factors in mathematics was apparent across the Phase Two interview data.

In the sections that follow, results related to the beliefs of the Phase Two participants will be summarized including analysis of relevant *MBI* items and semi-structured interview data. Data will be presented according to the domains of beliefs about the nature of the discipline of mathematics, beliefs about teaching mathematics, and beliefs about learning mathematics.

Phase Two Participants' Beliefs About the Nature of Mathematics

The summary of beliefs of the participants in this section includes data from both Phase One and Phase Two data collection. Presenting results in this manner allowed for greater elaboration on the beliefs of the Phase Two participants. The analysis in this section first considered the results of an analysis of *MBI* items that pertain to beliefs about the nature of mathematics. Next, results of interview data analysis are presented for participants with high MB and low MB. Included in the results that follow is the emergent theme noted earlier related to beliefs about mathematics as a subject in school contrasted with mathematics as a way of understanding the world.

MBI item analysis of Phase Two participants' responses related to the nature of mathematics. Seven questions on the *MBI* related to beliefs about the nature of mathematics (Table V.30). Analysis of the responses to the items suggests general agreement with reform-based views of the nature of mathematics with a few exceptions. All participants were in consensus and aligned with reform-based beliefs about these statements: (a) mathematics can be thought of as a language that must be meaningful if students are to communicate and apply mathematics productively, (b) a demonstration of good reasoning should be regarded even more than students' ability to find correct answers, and (c) math problems can be done correctly in only one way.

There was near consensus on two *MBI* items in agreement with reform-based beliefs: (a) problem solving should be a separate, distinct part of the mathematics curriculum, and (b) the mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation.

There was a difference in the pattern of responses between participants with high MB and low MB on two statements: (a) everything important about mathematics is already known by mathematicians, and (b) in mathematics something is either right or it is wrong. A display of the responses is found in Table V.31. Participants with high MB were in general alignment with reform-based beliefs on both statements whereas participants with low MB were generally not in alignment.

Table V.31 *MBI* Item Analysis of Phase Two Participants' Responses Related to the Nature of Mathematics

MBI Statement	Reform-based Belief	High MB Participants	Low MB Participants
Mathematics can be thought of as a language that must be meaningful if students are to communicate and apply mathematics productively.	Agree	All agreed	All agreed
A demonstration of good reasoning should be regarded even more than students' ability to find correct answers.	Agree	All agreed	All agreed
Math problems can be done correctly in only one way.	Strongly disagree	All strongly disagreed	All disagreed or disagreed strongly
Problem solving should be a separate, distinct part of the mathematics curriculum.	Disagree	All disagreed but Sally	All disagreed
The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation.	Disagree	All disagreed	All disagreed except for Steven
Everything important about mathematics is already known by mathematicians.	Strongly disagree	All strongly disagreed	One disagreed strongly; one disagreed; one agreed
In mathematics something is either right or it is wrong.	Disagree	All but one disagree	All agree

Analysis of the results of the item analysis of participants' responses to the *MBI* revealed that there was a degree of alignment with reform-based beliefs about the nature of mathematics among the sub-sample. However, there were differences among the responses to questions that relate to fundamental beliefs about the discipline of mathematics: (a) everything important about mathematics is already known by mathematicians, and (b) in mathematics something is either right or it is wrong. The questions relate to whether mathematics is perceived as fixed, predictable, and consisting of rules, facts, and procedures or whether mathematics is perceived as a dynamic, problem-driven, and continually expanding discipline. On these two points the responses differentiated between participants with high MB and low MB. Further differences in beliefs of participants according to high and low MB are explored in the following sections.

Beliefs held by Phase Two participants with high MB. Four of the participants were classified as having high MB: Callie, Carson, Sally, and Betty. In this section, a description of the beliefs that participants with high MB related to the nature of mathematics will be provided.

Participants with high MB tended to consider the nature of mathematics to be dynamic, less as a fixed body of knowledge consistent with a problem solving or discovery perspective. These teachers disagreed with the statement that in mathematics something is either right or it is wrong. Also, the teachers were in strong disagreement with the statement that everything important about mathematics is already known by mathematicians.

All participants with high MB described mathematics as consisting of more than algorithms and rules. Instead they portrayed mathematics as a discipline involving problem solving. For instance, Betty noted, “I think that teachers can get caught up in the procedures and rules piece, but miss the fact that there’s a much deeper understanding for mathematics.” Carson explained,

> Math should be used as a way to problem solve, but not just memorize because if you only memorize an algorithm or a rule and not understand what you’re doing it cannot be used later. So you can’t think of it as a rule or an algorithm if you don’t have the concept.

Sally agreed saying, “the purpose of all the rules and the logic and everything is to figure things out and solve problems.” To Callie, the essence of mathematics is problem solving, a perspective that has changed how she approaches teaching.

Participants with high MB also referenced the role of mathematics in the world. Carson described mathematics as connected to day-to-day activities. Sally described mathematics as deeply embedded in the world. Betty framed mathematics as “a perspective that you might bring to looking at the world” and a way of “connecting with the world through numbers and mathematical concepts.” Statements about the nature of mathematics made by participants with high MB are displayed in Table V.32.

Table V.32 High MB Phase Two Participants' Beliefs Related to the Nature of Mathematics

Participant	Beliefs About the Nature of Mathematics
Betty	Mathematics is about connecting with the world through numbers and mathematical concepts. Mathematics is a way of looking at the world. Mathematics helps people connect and solve problems that they encounter in daily life. Mathematics involves procedures, but it is more than accumulation of procedures.
Callie	Mathematics involves reasoning and problem solving.
Carson	Mathematics is more than algorithms and rules; it is problem solving. Mathematics is fun; play is part of mathematics. Concepts in mathematics have inner relationships and are connected to the activities of adults in the real world. Mathematics can be represented with concrete and visual representations.
Sally	Mathematics is embedded in world. The purpose of mathematics is to solve problems. Mathematics involves both procedures and solving problems.

Participants with high MB tended to view mathematics more in terms of a dynamic, problem-driven, and continually expanding discipline than a static discipline consisting of rules and procedures. Furthermore, participants with high MB were inclined to view mathematics beyond the mathematics classroom and school. Instead, high MB participants described mathematics in terms of a problem solving or discovery view of the discipline where mathematics is considered to be contextually-bound, inextricably interwoven into the fabric of life.

Sally and Betty, both participants with high MB and low MA, illustrated this viewpoint in their interviews. Sally described her upbringing with educators as parents as impacting her perspective of the application of mathematics in life. In her words, “I lived with two people who the world was school and so, everything was a story problem, but you didn’t know that’s what you were doing. So, I got that [application of mathematics] outside of school.” In answer to a question related to why some students struggle, “They don’t have the conceptual understanding because they didn’t count change with their mom when they were little kids and things like that.”

A theme that emerged from Sally’s interview was that learning mathematics should be experiential and that experiential learning involves solving problems related to real life. Relating how she applied this in her instruction, Sally explained,

> I had a geometry class that I taught several years ago that we built houses and so they had to figure out—they had to use math, like money math, to figure out how to pay for things, but they also had to figure out angles and volume and area, or they could do some experiential stuff and then they could go back and look at the problems in the book and go “Oh, that makes so much more sense now.”

Similarly, Betty described mathematics as deeply connected to everyday life and this connection was important for her students to understand. In answer to what she thought of when she heard the word mathematics, Betty responded,

> I then really think that it’s true that math can be so much more than procedures and rules and it really is—it can help individuals find a way to connect and solve problems. There is sort of the conceptual foundation of thinking and working with numbers. But then I think it also sort of—I mean this is quite broad, but like a

> perspective that you might bring to looking at the world also. So I see it definitely beyond just sort of arithmetic and stats piece but more at—when you think about connecting with the world through numbers and mathematical concepts.

Betty noted the motivating and sense making aspect of problem solving distinguishing between school mathematics and real life applications of mathematics:

> Not just problems that sort of teacher created problems, but problems that they may encounter sort of in their daily life and just throughout life, because I think that it's that deeper understanding of math is kind of really the essence of it. And that a lot of times I think that teachers can get caught up in the procedures and rules piece, but miss the fact that there's a much deeper understanding for mathematics. And that's kind of for me where the excitement is and I think where, for a lot of kids where it really makes sense.

The statements from both Sally and Betty portray mathematics as relevant to life outside of school and closely tied to everyday life. In contrast, participants who portrayed mathematics from an instrumentalist perspective tended to describe mathematics almost exclusively in terms of school.

A discovery perspective of mathematics considers the discipline to be a unified body of knowledge existing outside of cultural contexts that people can discover through inquiry. Carson and Callie both participants with high MB, described perspectives of mathematics that could be classified as discovery. While both participants emphasized the importance of problem solving and reasoning in mathematics instruction, neither shared examples of the type of problems students solve in their respective classrooms. That is, it was not evident in their interview data that the problems or tasks in which their students engage involve the real world.

The perspective Callie brought to school mathematics and real life mathematics can only be inferred by her deep emphasis on problem solving and reasoning. To Callie, a mathematics class must involve students in solving engaging problems and deepen students' abilities to reason. Despite a constant reference to problem solving tasks, Callie never provided information related to the nature of the tasks or whether the task related to real applications of mathematics.

Like Callie, Carson portrayed mathematics as consisting of more than algorithms and rules; instead to Carson mathematics is about problem solving. Unlike Callie's general discussion of the importance of problem solving in the abstract, Carson articulated a belief that it is important for students to see how mathematics is useful in solving problems related to life. Discussing this, Carson relayed a conversation she has with her students about the relevance of mathematics:

> Have you ever seen an adult write something? It can be writing out a grocery list. It could be a letter. It can be a note to your teacher. Have you ever seen them write a bill, write a check? That's writing. So have you ever seen an adult do math? Fewer hands go up and I say, "Hmm, all right. Let me ask you some more questions" because they don't get the real life application of math. Have you ever seen an adult buy gas? That's math. Have you ever seen an adult buy groceries? That's math. Have you ever had an adult tell you, "We can't afford that yet this month. You have to wait until I get paid?" Boom, every hand goes up. That's math. It's budgeting. You have to understand money. That's math. Have you ever

had an adult say, “We don’t have time to do that we need to get somewhere?” That’s math.

Thus, both Carson and Callie described mathematics in ways consistent with a discovery perspective yet the role of real life mathematics was less evident than the problem solving perspective.

In summary, participants with high MB see mathematics as a discipline that is dynamic and related to human experiences in the world. Teaching mathematics involves providing students with rich learning experiences and guiding their discovery. Learning mathematics involves engaging in problem solving and discovery activities to make connections between concepts and the world. Such a view of the discipline of mathematics and how it should be taught and learned is in contrast with the perspective provided by the low MB sub-sample participants.

Beliefs held by Phase Two participants with low MB. Three of the Phase Two participants were classified as having low MB: Natalie, Steven, and Tammy. In this section, a description of the beliefs that participants with low MB related to the nature of mathematics will be provided.

Participants with low MB described the discipline of mathematics in ways consistent with an instrumentalist perspective. Participants described mathematics as consisting of immutable rules and procedures. For instance, participants with low MB tended to agree with the *MBI* statement that in mathematics, something is either right or wrong. In line with an instrumentalist perspective, participants with low MB portrayed mathematics as an accumulation of rules and procedures. Steven described mathematics in terms of sequence, steps, and order that relates mainly to school. Tammy conveyed a similar viewpoint indicating that mathematics involves following procedures to solve problems encountered in school. Natalie portrayed mathematics at its essence as procedures and processes that require sequential thinking. Table V.33 displays a summary of the beliefs sub-sample participants with low MB have about the nature of mathematics.

Table V.32 Low MB Phase Two Participants' Beliefs Related to the Nature of Mathematics

Participant	Beliefs About the Nature of Mathematics
Natalie	Mathematics consists of a series of accumulated skills.
	There is an exact way to do mathematics problems.
	Mathematics involves sequential thinking.
Steven	Mathematics is a discipline of sequence, order, logic, and reasoning.
Tammy	Mathematics consists of numbers, calculation, and procedures.
	Problems in mathematics relate to math textbook and the four operations.

An instrumentalist perspective of mathematics depicts the discipline as a body of knowledge consisting of facts, rules, and procedures. Steven and Tammy, both participants with low MA and low MB, portrayed mathematics as related almost exclusively to school with Tammy admitting that sometimes school mathematics does not relate to life.

The perspective Steven portrayed about mathematics was related almost exclusively to school. In response to the question, what do you think of when you hear mathematics, Steven replied, "I guess I just think of school and it just makes me remember my math classes, my math teachers. Honestly when I think of math, I think of my seventh grade math teacher Mr. Conti, that's what I think about". Steven's discussion of mathematics and teaching mathematics centered on mathematics involving sequence and order and quality instruction consisting of following a textbook from beginning to end. When asked which of three perspectives of mathematics reflected his view, Steven acknowledged the problem solving, everyday aspect of mathematics,

> [Math is] problem solving, you know, math is every day, it's all around us all the time and you do have to think logically and, you know, lots of different things that you do, the things you deal with in real life and you know, just you have to do a little problem solve to it.

Despite expressing this sentiment, nowhere in the interview did Steven use examples of applications of mathematics in his own life or instances of problem solving with his students.

The perspective Tammy portrayed was mainly that of mathematics as school oriented, that the mathematics one learns in school does not always apply to life. In response to the question, what do you think of when you hear the word mathematics, Tammy said, "[Math] is like the numbers, multiplication. I just think of numbers and calculations."

Tammy elaborated on this point further illustrating a school-based view of mathematics,

> In mathematics growing up you're solving problems on a day-to-day basis with your math textbook and homework. In life, I guess it's not—I mean it's obviously used like for addition and subtraction and multiplication, division and stuff that you're going to use in your life, but a lot of this stuff you don't use. A lot of this

stuff you just don't use with your life when you get older. It's kind of used for the purpose of solving the problems and getting a good grade in math, unless you choose a career that's in the field of mathematics.

The statements from both Tammy and Steven portray mathematics as almost exclusively related to school.

Natalie related mathematics beyond the school to a much greater degree than her low MB counterparts. In email correspondence to set up her interview, Natalie noted that she had been busy working on her farm. This provided an opportunity for prompting Natalie to discuss how she and her husband use mathematics in their everyday life.

When asked her views on the nature of mathematics, Natalie indicated that she viewed mathematics as a tool for solving problems. A follow up question was asked related to whether she used mathematics in her farming. Natalie responded, "You do all the time. Yeah, we use it. We use it as far as water shares or we use it as rations for cows, how much hay to feed. We use it every day."

Natalie also noted the need to bring in real work applications of mathematics into the classroom. The example of real world mathematics that Natalie provided was packing a box for shipping, an illustration of the mathematical concept of volume. Natalie said,

> But if I have a box and I have to fill it with peanuts and I'm going to put something in there and ship it, I have to be able to figure out how many peanuts to put in there right? But if I am not very good at math I'm not even going to try figure out the volume of that.

Interestingly, the example Natalie provided is not one in which any formal mathematics would be used. That is, it is unlikely that an individual would find the volume of the box nor determine the number of peanuts needed. Instead, it is more likely that informal estimation would be used for the situation Natalie provided.

Except for Natalie, participants with low MB tended to portray mathematics devoid of context and related mainly to school. Such a view of mathematics is consistent with an instrumentalist perspective of the discipline that considers mathematics to be immutable truth played out through rules and procedures devoid of context.

The beliefs that participants expressed related to the discipline of mathematics differed according to their degree of alignment with reform-based approaches to mathematics. Participants with high MB tended to portray the discipline in terms of a discovery or problem solving perspective whereas participants with low MB tended to portray mathematics from an instrumentalist perspective. In the section that follows, the beliefs that the sub-sample participants expressed about teaching mathematics are explored.

Phase Two Participants' Beliefs About Teaching Mathematics

Through Phase One and Phase Two of the current study, data were collected related to beliefs that participants held about teaching mathematics. Questions related to beliefs about teaching mathematics were included on both the *MBI* (Appendix D) and the semi-structured interview (Appendix E). Mixing analysis of data collected from both Phase One and Phase Two of the study allowed for greater elaboration on the beliefs the participants expressed about teaching mathematics. Included in the results is the emergent theme noted earlier related to participants' generalizations of their own mathematics learning preferences to teaching mathematics. In the section that follows, a summary of the Phase Two participants' beliefs related to teaching mathematics will be presented.

MBI item analysis of Phase Two participants' responses related to teaching mathematics. Eight questions on the *MBI* related to beliefs about the nature of mathematics (Table V.34). Analysis of the Phase Two participants' responses to the items suggests agreement with reform-based views of the teaching mathematics with variation across participants with low and high MB. All participants were in consensus or near consensus and alignment with reform-based beliefs about these statements: (a) a major goal of mathematics instruction is to help children develop the belief that they have the power to control their own success in mathematics, (b) good mathematics teachers show students lots of different ways to look at the same question, and (c) children should be encouraged to justify their solutions, thinking, and conjectures in a single way. Interestingly, there was consensus or near consensus among the sub-sample participants contrary to reform-based approaches to teaching mathematics related to these statements: (a) in K-5 mathematics, increased emphasis should be given to reading and writing numbers symbolically, and (b) mathematics should be taught as a collection of concepts, skills, and algorithms. Finally, there were differences in beliefs according to participants who hold high MB and low MB on the following statements: (a) in K-5 mathematics, increased emphasis should be given to use of clue words (key words) to determine which operation to use in problem solving, (b) appropriate calculators should be available to all students at all times, and (c) good math teachers show you the exact way to answer the math question you will be tested on. On these statements, participants with high MB tended to align more closely with reform-based approaches.

Table V.34 *MBI* Item Analysis of Phase Two Participants' Responses Related to Teaching Mathematics

MBI Statement	Reform-based Belief	High MB Participants	Low MB Participants
A major goal of mathematics instruction is to help children develop the belief that they have the power to control their own success in mathematics.	Agree	All agreed	All agreed
Good mathematics teachers show students lots of different ways to look at the same question.	Agree	All agreed	All agreed
Children should be encouraged to justify their solutions, thinking, and conjectures in a single way.	Disagree	Three of four disagreed	All disagreed
In K-5 mathematics, increased emphasis should be given to reading and writing numbers symbolically.	Disagree	All agreed	Two of three agreed
Mathematics should be taught as a collection of concepts, skills, and algorithms.	Disagree	Three of four agreed	All agreed
In K-5 mathematics, increased emphasis should be given to use of clue words (key words) to determine which operation to use in problem solving. given to reading and writing numbers symbolically.	Disagree	Two of four disagreed	All agreed
Appropriate calculators should be available to all students at all times.	Agree	Two of four agreed	All disagreed

Table V.34 (Continued)

Good math teachers show you the exact way to answer the math question you will be tested on.	Disagree	All disagreed	Two of three agreed

Analysis of items on the *MBI* revealed some aspects of alignment of participants' beliefs with reform-based approaches to teaching mathematics related to major goals of mathematics instruction and promotion of multiple approaches to solving problems. Differences existed among the participants about the role of procedures, calculators, clue words, and testing. Further differences in beliefs of the participants according to high and low MB are explored in the following sections.

Beliefs about teaching mathematics held by Phase Two participants with high MB. The perspective participants with high MB expressed about the nature of mathematics relates to beliefs the participants hold about teaching mathematics. Participants with high MB tended to describe the role of the teacher as a facilitator or guide to engage students in understanding math concepts and solving problems. A summary of the beliefs participants with high MB expressed about teaching mathematics is displayed in Table V.35 and described here.

Callie believed the role of the teacher to be establishing the right environment for students, selecting engaging tasks for students to do, and facilitate the learning process. She indicated that teachers should not talk too much nor direct kids to particular problem solving methods. Betty indicated that the role of the teacher is a guide or coach to help students make connections to what they have learned in the past, what they have studied, and what they have explored. Expanding on this, Carson described the role of the teacher as creating learning experiences for students through games or physical representations to help students discover mathematics. Sally described the role of the teacher as helping kids figure out the mathematics. She stressed the importance of the teacher having deep knowledge of mathematics, understanding the needs of students, and presenting material in multiple ways. Among the high MB participants, Sally expressed the most frequent reference to ensuring students could also have procedural fluency with mathematics.

A theme that emerged from the interview data related to beliefs about teaching mathematics was the tension the participants felt in terms of time. In order to provide the type of instruction that the participants believed to be important, three of the four participants with high MB indicated that they felt pressured by time constraints. Betty indicated that she felt there was a tradeoff between the deep learning she seeks to provide and the pressure to cover content. Sally also referenced time as a constraint, while Callie indicated that she struggled with the balance between teaching in a problem solving manner and content coverage concluding that the experience solving problems is much more valuable for students.

Table V.35 High MB Phase Two Participants' Beliefs Related to Teaching Mathematics

Participant	Beliefs About Teaching Mathematics
Betty	The role of the teacher is to be a guide and a coach. Teachers need to help students make connections to what students have learned in the past, what they have studied, and what they have explored. Teachers can lose focus on deeper understanding by focusing too much on procedures and rules. Teachers should ensure students understand the rationale for what they are learning.
Carson	The role of the teacher is to create a safe place for students to solve problems and impart in students that mathematics is fun. The teacher's role is creating learning experiences through games or physical representations that help students see, or discover, mathematics. Teachers should help students acquire strategies for learning not only mathematics but for lifelong learning.
Callie	Teachers need to establish the learning environment for kids, find engaging tasks, and facilitate the learning process. Teachers should not talk too much or direct students to particular problem solving methods.
Sally	An ideal math class involves traditional math and problem solving. The role of the mathematics teacher is to have deep knowledge of mathematics, understand the needs of students, and present material in multiple ways. The mathematics teacher needs to find ways to help kids figure out mathematics.

The beliefs about teaching that the participants with high MB expressed are consistent with the problem solving and discovery views of mathematics which places teachers in the role of guiding student learning and providing rich experiences in which to engage students. The role of the teacher and beliefs about teaching mathematics described by the sub-sample participants with high MB differed from the beliefs expressed by sub-sample participants with low MB.

Beliefs about teaching mathematics held by Phase Two participants with low MB. The perspective Phase Two participants with low MB participants expressed about the nature of mathematics related to their beliefs the participants hold about teaching mathematics. Participants with low MB tended to describe the role of the teacher in terms of sequencing instruction for students to ensure mastery of skill attainment. A summary of the beliefs participants with low MB expressed about teaching mathematics is displayed in Table V.36 and described here.

Steven described good mathematics teaching as explicitly teaching the correct steps to in a sequential order in an engaging way. To Steven, being able to show steps is important to good mathematics teaching. Tammy's description of a mathematics teaching included three phases: (1) review and activation of prior knowledge, (2) teaching the new skill and using guided instruction, and (3) students doing independent practice. Tammy also indicated that repetition was important in teaching mathematics. Natalie emphasized that students learn in different ways so it is the teacher's responsibility to show students different ways of solving problems, such as using both pictures and numbers.

Table V.36 Low MB Phase Two Participants' Beliefs Related to the Teaching Mathematics

Participant	Beliefs About Teaching Mathematics
Natalie	The role of the teacher is to sequence instruction for students.
	Teacher's role is to determine levels of students, instruct, and move forward.
	Teachers need to be able to show students many ways to do mathematics.
Steven	Mathematics should be taught in a sequential order.
	Good mathematics teaching involves explicitly teaching the correct steps in a sequential order in an engaging way.
	Teachers should clearly model how to solve problems. Modeling by the teacher allows students to learn to do mathematics on their own by repeating the model.
	A math class needs to be highly structured.
Tammy	Teaching mathematics involves a sequence of instruction. The first part of the lesson is review and activation of prior knowledge, the second part is teaching the new skill and using guided instruction, and the final part is independent practice by students.
	Repetition is very important to teaching math.
	The teacher's role is to making kids feel confident and successful, to have a positive attitude and provide praise, and to make mathematics fun.
	Teachers need to find out how students learn best and provide the right instruction.

The beliefs about teaching that the participants with low MB expressed are consistent with the instrumentalist perspective of the nature of mathematics. The instrumentalist perspective of teaching mathematics involves conveying rules and demonstrating procedures to students. From an instrumentalist perspective, the role of the teacher is to sequence the presentation of skills and concepts to students through demonstration, explanation, and definitions.

Whereas there were differences in the beliefs the participants expressed related to teaching mathematics, there were similarities among the participants related to the relationship the educators identified about their own experiences learning mathematics to their beliefs about teaching mathematics. This relationship is described in the section that follows.

Relationship of mathematics learning experiences to teaching mathematics. A theme that emerged across the Phase Two interview data was the relationship between the mathematics learning experiences described by the participants to their beliefs about teaching mathematics. First, nearly all of the participants described their mathematics learning experience as instrumentalist. Second, all of the participants generalized their own mathematics learning experiences to their approach to teaching mathematics.

Within the Phase Two participants, six of the seven participants described their mathematics learning experience in school as an instrumentalist approach to mathematics instruction. Callie described her school experience in mathematics as not motivating, sterile, and boring. Her recollections were of a teacher centered and textbook driven class with a focus on repetition that she described as “drill and kill”. Sally recounted her mathematics learning experience centering on memorization and procedures. Sally described this as the “era of flashcards”. Carson described her background as memorizing algorithms and referenced this approach as the “old ways”. Betty’s memories were not as salient as but recalled being taught using a procedural approached that lacked an emphasis on depth of understanding. Finally Steven’s recollections of his mathematics instruction included a heavy emphasis on textbooks, procedures, and sequential, explicit, and direct instruction. Table V.36 displays statements from the participants that illustrate this common mathematical learning experience.

Table V.37 Statements of Phase Two Participants' Mathematics Learning Experiences

Participant	Statements Related to Mathematics Learning Experiences
Betty	I remember a lot of procedures and shortcuts, but didn't know why they worked.
Callie	And for me when I was growing up it sort of felt like real didactic instruction and now it has sort of evolved more into—I teach more in a more of a problem solving, problem-task way. There was always just get like a set of problems, mainly didactic based, drill and kill problems. And I sort of remember having the text book and during the math class my teacher would often place the homework for the week.
Carson	That I've become a better teacher of it, and that my depth of knowledge has increased. How do you help them discover it so that it makes sense to them? Because the ways of—the old ways of teaching, it didn't work. I don't think I was taught successfully. I just think it worked for me because I was strong enough. If I was—it was due to my mom. My mom was a teacher.
Sally	I was raised in the era where you take out your workbook and do the next 40 problems and carry your flashcards, learn your multiplication tables. I don't remember doing story problems probably until high school.
Steven	That's what I remember about math, being really explicitly taught. I really thought I had good teachers especially in middle school. Mr. Conti, our eighth grade teacher; he was good. But just being explicitly taught out of that book, follow lessons that they had so by the end of the year, the end of the book.
Tammy	So the way that I was taught it. I learned stuff the way that is was taught, the old school way. It wasn't anything of the newer math that we're doing nowadays with kids. It was more drill and practice.

To varying degrees, nearly all of the participants characterized their mathematics learning experience in terms of an instrumentalist approach, yet the mathematics teaching beliefs of the educators did not follow the instrumentalist approach they experienced. Of the six teachers who relayed their mathematics learning experience in terms of an instrumentalist approach, four participants expressed views of teaching mathematics characterized as either discovery or problem solving.

Another theme that emerged from the interview data related to the experiences participants had learning mathematics was a phenomenon in which the teachers seemed to generalize their own mathematics learning experience (either positive or negative) to their approach to teaching. Each participant made specific references to an aspect of their learning experiences in mathematics that was relatable to their beliefs and approaches to teaching mathematics.

Natalie described her worst experience in mathematics as being publically humiliated by her teacher who declared that "he couldn't teach me anything so I never took another math class until I got to college". When asked to describe the ideal mathematics classroom, Natalie said, "I think it's a place where kids feel safe, to do well or to make mistakes. They don't have to feel bad about being good at math and they don't have to feel bad about not understanding the math." Also, Natalie shared her difficulties learning mathematics, noting the devastating effect of experience being placed in an Algebra 1 class as a freshman in high school without adequate preparation. Throughout her interview, Natalie emphasized the importance of students being ready (or adequately prepared) for their mathematics class. When asked the most important aspect of teaching mathematics, Natalie responded, "Understanding that kids come to you at different levels of readiness and understanding of math." Natalie also noted her inability to memorize due to a learning disability and later commented,

To memorize numbers or steps or processes, it doesn't happen for me and I see that a lot with kids. If you write down the steps and have them up someplace where they can look back to them and refer they can get it but if they have to remember the steps or sequence of things like with a learning disability can't do that.

Callie described her mathematics learning experience as didactic, de-motivating, and sterile. She contrasted her mathematics learning experience with the interest she took in social studies given the greater amount of discussion and interaction. When describing an ideal mathematics classroom, Callie response was "I think it should be where there's a lot of discussion."

Sally experienced her most valued mathematics learning outside of the classroom through the interactions with her parents and the connections they made between their everyday experiences and mathematics. As she described, "I lived with two people who, you know, the world was school and so, everything was a story problem, but you didn't know that's what you were doing. So, I got that [problem solving] outside of school". The importance of mathematics to solving problems in life was illustrated in Sally's response to a question related to the nature of mathematics, "the purpose of it [mathematics] is to solve problems, to figure out something".

Betty described her mathematics learning experiences as a process of learning isolated "nuggets", a series of disconnected procedures and facts. In teaching mathematics Betty has found the connectedness of these nuggets and "it's really encouraging for me to see it in sort in the broad web or interconnected network that math can represent". Betty related her view of mathematics to the role of a mathematics teacher as "sort of playing with connections that they [students] might not necessarily see whether it's to different things that they've learned in the past or other things that we've studied or that they've explored". Also, Betty shared her memory of being dismayed by the inability of her Algebra teacher to articulate the rationale for why students were learning a particular mathematics skill. Betty shared, "I just remember thinking, 'Well if you don't know, then why are we doing it?' And now I feel like if I was asked that question now I feel like I would know and I could just sort of talk with a student through it".

For Tammy, the relationship between her learning experience and her approach to teaching came through her struggles to learn to read. Tammy related her comfort with mathematics but described the difficulties she had learning the read and the role practice played in her overcoming this challenge. When asked what accounts for differences between good and poor mathematics students, Tammy replied, "I think it has to do with practice. And like I said, I struggled with reading and it is just a matter of practicing you practice, practice, practice and you get better. And the same thing with math you have to practice it to get better."

Carson learned mathematics as play and shared how she uses games as an approach to teaching mathematics. Carson described herself as "one of those, I would say quirky kids, but I always played math games, just myself in my head. I still do it to this day. I'll look at the clock and play games with the numbers". In answer to how she learned mathematics, Carson explained, "So a lot of it is that I—we played games as I grew up—Monopoly, you name it, Holly Hobby. So math has always been enjoyable to me." When she described the instructional approach for a student who was struggling, Carson explained a game called "Trash" and how "We'd play game- game after game after game, until she [the student] could get it [the mathematics concept].”

Steven’s learning experience in mathematics was one in which his teachers presented mathematics in an ordered and sequential way, closely following the textbook. Steven recounted his learning experience,

> That’s what I remember about math, just being really explicitly taught. I really thought I had good teachers especially in middle school. Mr. Conti, our eighth grade teacher, he was good. But just being explicitly taught out of that book, follow, whatever lessons that they had but we went from step one to, so by the end of the year the end of the book.

In describing the ideal classroom, Steven described a teacher and instruction very much like his middle school mathematics teacher,

> An ideal math class, ideal math teacher to me is somebody that really—how am I going to put into words— to really explicitly teach and model the correct steps and the different ways of working through problems in a sequential order and really be able to model that exactly what things look like, how to calculate and do it in some type of an engaging way.

Punctuating the connection between his own experience learning mathematics and his beliefs about teaching mathematics, Steven explained, “I like when math is taught in a logical order, and it goes—it builds upon itself you know. You know for me that was the easiest way to learn it. I think it’s the easiest way to teach it.”

For each teacher in Phase Two, a relationship existed between their experience learning mathematics and their beliefs and approaches to teaching mathematics. Questions evoking learning experiences in mathematics elicited positive and negative memories that could be related to participant beliefs about teaching mathematics. A summary of the relationships found in the interview data is illustrated in Table V.37.

Table V.38 Display of Generalized Mathematics Learning Experiences to Beliefs about Teaching Mathematics

Participant	Mathematics Learning Experiences	Beliefs about Teaching Mathematics
Natalie	Was not prepared for high school mathematics	Frequently noted the need for readiness of her students for success in mathematics
	Was publically humiliated by her high school mathematics teacher for her struggle with mathematics	Emphasized the need for a safe environment in mathematics class
	Could not memorize anything in mathematics	Recognized that some students cannot memorize
Callie	Was bored by instrumentalist approach to learning mathematics and enjoyed the interactions and discussions in social studies classes	Emphasized the importance of an engaging mathematics classroom based on problem solving and interaction
Sally	Was taught by parents to understand the utility and importance of mathematics for solving problems	Described the purpose of mathematics as solving problems Emphasized both problem solving and practice as important in learning mathematics
Betty	Learned mathematics as isolated “nuggets”	Described mathematics as an interconnected web that should be taught as such
	Dismayed by her teacher’s inability to articulate a rationale for learning Algebra	Emphasize how she can always share a rationale for learning with her students

Table V.38 (Continued)

Carson	Enjoyed and learned mathematics by playing games	Emphasized the belief that games and enjoyment are important for learning mathematics
Steven	Appreciated learning mathematics in a sequential manner through explicit instruction	Described an ideal mathematics class as one using explicit instruction

The relationship between participants' experiences learning mathematics and their beliefs about teaching mathematics points to a biographical component to teaching beliefs. The connections the participants drew between their instrumentalist learning experiences and the generalization of their mathematics learning experiences illustrates an almost inseparable relationship between teaching and learning. The next section of this chapter explores the beliefs that the Phase Two participants expressed about learning mathematics.

Phase Two Participants' Beliefs About Learning Mathematics

The summary of participants' beliefs related to learning mathematics in this section includes data from both Phase One and Phase Two data collection. Presenting results in this manner allowed for greater elaboration on the beliefs of the sub-sample participants. The analysis in this section first considered the results of an analysis of *MBI* items that pertain to beliefs about learning mathematics. Next, results of interview data analysis are presented for participants with high MB and low MB. Included in the results is the emergent theme noted earlier related to attribution of student success in mathematics. The theme of attribution emerged from analysis of the quantitative and qualitative data collection related to how the sub-sample participants attributed success in mathematics that is whether success in mathematics can be attributed to internal or external factors.

MBI item analysis of Phase Two responses related to learning mathematics. Twelve questions on the *MBI* related to beliefs about the nature of mathematics (Table V.39). Analysis of the responses to the items suggests some agreement with reform-based views of the learning mathematics with greater variation across participants with low and high MB. There was consensus or near consensus with reform-based beliefs on these questions: (a) students should share their problem solving thinking and approaches with other students, (b) the study of mathematics should include opportunities of using mathematics in other curriculum areas, (c) in K-5 mathematics, skill in computation should precede word problems, and (d) learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement. There was a clear distinction between the beliefs of participants with high MB and low MB on the remaining questions, with the beliefs about learning mathematics of high MB participants more in alignment with reform-based approaches than those of low MB participants.

Table V.39 *MBI* Item Analysis of Phase Two Participants' Responses Related to Learning Mathematics

MBI Statement	Reform-based Belief	High MB Participants	Low MB Participants
Students should share their problem solving thinking and approaches with other students.	Agree	All agreed	All agreed
The study of mathematics should include opportunities of using mathematics in other curriculum areas.	Agree	All agreed	All agreed

Table V.39 (Continued)

In K-5 mathematics, skill in computation should precede word problems.	Disagree	All disagreed	All but one disagreed
Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.	Disagree	All disagreed	All but one disagreed
Children enter kindergarten with considerable mathematical experience, a partial understanding of many mathematical concepts, and some important mathematical skills.	Agree	All agreed	All disagreed
Some people are good at mathematics and some aren't.	Disagree	All disagreed or strongly disagreed	All agreed or strongly agreed
To solve most math problems you have to be taught the correct procedure.	Disagree	All disagreed or strongly disagreed	All agreed or strongly agreed
In mathematics you can be creative and discover things by yourself.	Agree	All agreed or strongly agreed	All but one agreed or strongly agreed
To be good in math you must be able to solve problems quickly.	Disagree	All strongly disagreed	Two of three disagreed; one agreed
Males are better at math than females.	Disagree	All strongly disagreed	Two of three disagreed; one agreed
Some ethnic groups are better at math than others.	Disagree	All strongly disagreed	Two of three disagreed; one agreed
To be good in math you must be able to solve problems quickly.	Disagree	All strongly disagreed	Two of three disagreed; one agreed

Analysis of items on the *MBI* revealed some aspects of alignment of participants' beliefs with reform-based approaches to learning mathematics related to the relevance of mathematics learning across the curriculum, the need for children to share their solution methods, ability of students to engage in problem solving and discovery, and a rejection of a transmission model of learning. Differences existed among the participants related to the mathematics knowledge students bring to kindergarten, and whether mathematics ability is inherent in some people. Further differences in beliefs of participants according to high and low MB are explored in the following sections

Beliefs about learning mathematics held by Phase Two participants with high MB. Participants with high MB described the role of students in learning mathematics in similar ways. High MB participants minimized the role of memorization and learning algorithms as the primary activity of the mathematics classroom. All high MB in strong disagreement with the *MBI* statements (a) to solve most math problems you have to be taught the correct procedure, (b) the best way to do well in math is to memorize all the formulas, and (c) to be good in math you must be able to solve problems quickly.

While there was acknowledgement of the need to know facts and procedures, the high MB participants expressed the belief that student must first have a conceptual understanding of the mathematics and a connection to the real world. Betty indicated that a deep level understanding of concepts helps students to understand why procedures work. Callie explained that students practice and learn much mathematics as they work through problems; they do not need more repetition in order to learn procedures. Sally placed a greater emphasis on procedural knowledge. However, she noted that students should be engaged in problem solving prior to practicing procedures. Sally acknowledged that some students need more practice than others, but this additional practice should relate to real world experience.

The high MB participants expressed a perspective of students as active constructors of their own understanding and described the importance of discovery, making connections between mathematics concepts, and problem solving to support student learning of mathematics. All high MB participants agreed with the *MBI* statement that children enter kindergarten with considerable mathematical experience, a partial understanding of many mathematical concepts, and some important mathematical skills. They also strongly agreed with the statement that in mathematics you can be creative and discover things by yourself. For Betty, the role of the student is to make connections between what they see and learn and what they have experienced. Mathematics concepts can be discovered by students and learning mathematics involves flexibility, play, and experimentation. Callie conveyed that to learn mathematics, students should be engaged with interesting tasks, collaborate and discuss solutions with their peers, and stretch their understanding. For Sally, learning mathematics should be experiential, which involves real uses of mathematics from everyday life. Finally, for Carson, learning math involves discovery and exploration, where games and play have a prominent role in learning. Results are summarized in Table V.40.

Table V.40 High MB Phase Two Participants' Beliefs Related to Learning Mathematics

Participant	Beliefs About Learning Mathematics
Betty	Mathematics can be learned deeply, which involves learning concepts, connections among concepts, and why procedures work.
	Mathematics concepts can be discovered and learned over one's lifetime.
	Learning mathematics involves play and experimentation; students should be engaged in hands-on learning and doing projects.
	Students learn from peers; diversity in the mathematics class is helpful to learning.
	The role of the student is to make connections between what they see and learn and what they have experienced.
Carson	Learning math involves exploration and discovery.
	Students learn math through an instructional sequence that involves concrete materials, representations, and abstraction (CRA sequence). Students struggle in mathematics when they become stuck in a part of the CRA sequence.
	Making mistakes and playing games are part of learning math.
	Memorizing algorithms and rules without meaning does not support number sense.
	Students should develop strategies to become lifelong learners.
Callie	Students learn mathematics as they work through problems; they do not need more repetition.
	Problem solving and reasoning engages students in mathematics.
	Students should be given engaging tasks, time to collaborate and discuss with peers.
	Children learn math through trial and error, reasoning, and discussion with their peers.

Table V.40 (Continued)

Sally	Kids learn math through practice, problem solving, and hands-on approaches. Problem solving should relate to real uses of mathematics; conceptual understanding comes from using math in life. Students should be engaged in problem solving and then have opportunities to practice skills. Some students need more practice; additional practice should relate to real world experience. Learning math should be experiential, involving solving problems related to real life.

The beliefs about learning mathematics that the participants with high MB expressed are consistent with the problem solving and discovery views of mathematics, which suggest that learning mathematics involves developing one's own conceptual understanding of mathematical concepts and relationships, and involves active construction of understanding by the learner through problem solving, inquiry, and discourse.

The beliefs the participants expressed about learning mathematics extended to beliefs about attribution of success in learning mathematics. Participants with high MB specifically attributed success in mathematics to confidence, motivation, and experience. Callie expressed that student confidence and success comes from solving engaging mathematics tasks. Interesting tasks involve students in mathematical thinking, promote perseverance, and spark the natural interest of students. Carson described the importance of confidence for success in mathematics while Betty attributed success to experiential learning. Sally was the only participant with high MB who added a component of innate ability to her response. Sally expressed a strong belief in the importance of real world experience to build mathematics success but recognized that she provided this for both of her children and saw different outcomes. The struggle her daughter has with mathematics despite a similar background as her son caused Sally to believe that innate ability plays a role in mathematics success.

Analysis of *MBI* items supports the contention that participants with high MB attribute success to factors other than those within the learner. No participants with high MB disagreed with statements that would attribute success in learning mathematics to gender, ethnicity, or inherent ability. Furthermore, the participants tended to believe that students with SLD did not possess inherent differences in the ability to learn mathematics. Both Callie and Sally indicated that students with SLD needed more time to learn concepts. Betty expressed the belief that students with SLD need more hands on learning approaches and guidance from peers or the teacher. Carson noted that students with SLD needed to have support building number sense.

The beliefs about teaching mathematics described by the participants with high MB differed from the beliefs expressed by participants with low MB. In the next section, the beliefs of participants with low MB will be described.

Beliefs about learning mathematics held by Phase Two participants with low MB. The beliefs related to the role of the teacher described by the participants with low MB corresponded with the beliefs the participants expressed about learning mathematics. For the participants with low MB, learning mathematics requires instruction in certain processes and procedures. In fact, participants with low MB disagreed with the *MBI* statement that children enter kindergarten with considerable mathematical experience, a partial understanding of many mathematical concepts, and some important mathematical skills. Also for participants with low MB, there was common agreement that students need to follow the correct steps to solve problems and learning these steps requires repetition. All participants with low MB agreed with the *MBI* statement that to solve most math problems you have to be taught the correct procedure.

For participants with low MB, the role of the student is to pay attention and practice. To Tammy, learning mathematics involves practice, review, and repetition, and students who struggle in mathematics do so because they have not practiced enough. Similarly, Steven expressed that students need to pay attention, follow the steps the teacher demonstrates, and work hard. The basic understanding students gain from learning procedures is necessary for students learn higher-level mathematics and problem solving. Natalie also expressed the belief that students need to learn certain processes and have “the basics” before being able to be successful. She also recognized a role for experiential learning in order to understand mathematics.

In summary, participants with low MB depicted mathematics in ways consistent with an instrumentalist perspective, consisting of an accumulation of procedures, rules, and facts. The participants described the role of the teacher as providing direct, sequential instruction for students in the procedures and rules of mathematics and the role of students to follow and practice the procedures modeled by the teacher. An overview of the beliefs Phase Two participants with low MB expressed about learning mathematics can be found in Table V.41.

Table V.40 Low MB Phase Two Participants' Beliefs Related to the Learning Mathematics

Participant	Beliefs About Learning Mathematics
Natalie	Children need to be taught certain processes in math.
	Students need to have "the basics" before being able to be successful.
	Some students are wired to understand mathematics; they think sequentially.
	Kids understand mathematics in different ways. Drill, practice, and memorization do not work for all kids.
	Kids need to feel safe in the mathematics class.
	The students' role is to have a positive attitude, try, and practice.
Steven	Student practice follows explicit instruction from the teacher. Practice involves doing examples and is part of learning math.
	The role of the student is to listen and practice.
	Learning math involves repetition, effort, working through problems, referring back to notes, and studying.
	Students need to know basics of mathematics before they can access higher-level math or problem solving.
	Order and organization are important to learning mathematics. There are correct ways to hold pencils, sit in desks, and organize papers.
Tammy	Learning mathematics involves repetition, practice, review, and a positive attitude.
	Memorization of steps is a precondition for getting correct answers.
	Practice is essential to success in mathematics; students who struggle have not had sufficient practice.
	Children learn math differently. Some need to use manipulatives, physical models, or songs in order to learn to do procedures correctly.

The beliefs about learning mathematics that the participants with low MB expressed are consistent with the instrumentalist view of mathematics, which suggests that learning mathematics involves acquisition of rules and procedures through demonstration and practice.

The beliefs the participants expressed about learning mathematics extended to beliefs about attribution of success in learning mathematics. Participants with low MB tended to attribute success in mathematics largely to innate ability bolstered by practice and effort. For Natalie, mathematics ability is hard wired; people are either sequential thinkers or they are not. Tammy also expressed the belief the brains of some people are just different, and they need more repetition to learn mathematics. Steven agreed that mathematics just comes easier to some people but also recognized a role for effort on the part of the student. Furthermore, the participants tended to believe that students with SLD possessed inherent differences in the ability to learn mathematics. Both Natalie and Tammy expressed that students with SLD have differences in learning that are hard-wired. Steven's beliefs about students with SLD were more aligned with sub-sample participants with low MB. He expressed that students with SLD had difficulty with organizing steps and needed a slower pace in order to learn.

While all participants in Phase Two agreed with the *MBI* statement that a goal of math is to develop the belief that students have the power to control their success, there was a contrast between the high and low MB participants in relation to the *MBI* statement whether some people are just good at math. The participants with low MB tended to believe that mathematics ability is innate whereas participants with high MB tended to believe that mathematics success is determined through external factors such as motivation, effort, and hard work.

Summary of Beliefs Held by Phase Two Participants

The mathematics beliefs expressed by the Phase Two participants illustrated a range of beliefs about domains of the nature of mathematics, teaching mathematics, and learning mathematics. Beliefs could be categorized according to the perspectives within the conceptual framework, with high MB participants expressing beliefs categorized as discovery or problem solving and low MB participants expressing beliefs categorized as instrumentalist.

Summary of beliefs held by Phase Two participants with high MB.

Participants rated as holding high MB expressed views of mathematics consistent with either discovery or problem solving views. From a discovery perspective, mathematics is dynamic discipline that exists external to human beings and can be discovered. Teaching mathematics involves guiding learners to discover mathematical concepts emphasizing why mathematical relationships exists. Learning mathematics involves developing one's own conceptual understanding of mathematical concepts and relationships. Table V.42 displays the themes from the participant interview.

Carson was the only one participant in Phase Two who provided responses that would indicate a discovery perspective. Carson described mathematics as concepts that can be represented with concrete and visual representations. These concepts are inter-related and are connected to the activities of adults in the real world. According to Carson, the teacher's role is to create learning experiences through games or physical representations that help students see, or discover, mathematics. Thus, learning math involves exploration and discovery. From Carson's perspective, students learn math through an instructional sequence that involves concrete materials, representations, and abstraction.

Table V.42 Summary of the Discovery Perspective of Mathematics with Related Participant's Responses

	Nature of mathematics	Perspective of teaching mathematics	Perspective of learning mathematics
Summary of Discovery Perspective	Mathematics is dynamic discipline that exists external to human beings and can be discovered.	Teaching mathematics involves guiding learners to discover mathematical concepts emphasizing why mathematical relationships exist.	Learning mathematics involves developing one's own conceptual understanding of mathematical concepts and relationships.
Summary of Carson's Perspective	Mathematics is more than algorithms and rules; it is problem solving. Mathematics is fun; play is part of mathematics. Concepts in mathematics have inner relationships and are connected to the activities of adults in the real world. Mathematics can be represented with concrete and visual representations.	The role of the teacher is to create a safe place for students to solve problems and impart in students that mathematics is fun. The teacher's role is creating learning experiences through games or physical representations that help students see, or discover, mathematics. Teachers should help students acquire strategies for learning not only mathematics but for lifelong learning.	Learning math involves exploration and discovery. Students learn math through an instruction that involves concrete materials, representations, and abstraction (CRA). Students struggle in mathematics when they become stuck in a part of the CRA sequence. Making mistakes and playing games are part of learning math. Memorizing algorithms and rules without meaning does not support number sense. Students should develop strategies to become lifelong learners.

From a problem solving perspective, mathematics is a dynamic discipline that is contextually bound, and mathematics is a way of thinking, a discipline of inquiry. Teaching mathematics involves understanding student conceptions of mathematics and facilitating modifications of student conceptions through problem posing and discourse. Learning mathematics involves active construction of understanding by the learner through problem solving, inquiry, and discourse. Three participants in Phase Two of the study expressed ideas about mathematics that could be considered problem solving viewpoints, Betty, Callie, and Sally. A summary display of the sub-sample participants' beliefs can be found in Table V. 43.

All of these participants expressed views of mathematics consistent with a problem solving perspective. Betty described mathematics as a way of looking at and connecting with the world through numbers and mathematical concepts. For Betty, mathematics helps people connect and solve problems that they encounter in daily life. To Callie, mathematics involves reasoning and problem solving. From Sally's perspective, mathematics is embedded in world and its primary purpose is to solve problems.

The perspective of teaching and learning mathematics the three participants shared exemplifies the problem solving perspective. For Betty, the role of the teacher is to be a guide and a coach for students. Teachers need to help students make connections to what students have learned in the past, what they have studied, and what they have explored. The role of the student is to make connections between what they see and learn and what they have experienced. Learning mathematics involves deeply understanding concepts, connections among concepts, and why procedures work. To Betty, play, experimentation, hands-on learning and projects are also important to learning mathematics.

For Callie, the role of the teacher is to establish the learning environment for kids, find engaging tasks, and facilitate the learning process. Teachers should not talk too much or direct students to particular problem solving methods. Students learn mathematics as they work through problems through trial and error, reasoning, and discussion with their peers. To facilitate learning, students should be given engaging tasks, time to collaborate and discuss with peers.

Finally, to Sally, an ideal math class involves traditional math and problem solving. The role of the mathematics teacher is to have deep knowledge of mathematics, understand the needs of students, and present material in multiple ways. From Sally's perspective, children learn math through practice, problem solving, and hands-on approaches. Conceptual understanding of mathematics comes from using math in life, and practice in mathematics should involve connections to the real world.

Table V.43 Summary of Problem Solving Perspective of Mathematics with Related Participants' Responses

	Nature of mathematics	Perspective of teaching mathematics	Perspective of learning mathematics
Summary of Problem solving Perspective	Mathematics is a dynamic discipline that is contextually bound. Mathematics is a way of thinking, a discipline of inquiry.	Teaching mathematics involves understanding student conceptions of mathematics and facilitating modifications of student conceptions through problem posing and discourse.	Learning mathematics involves active construction of understanding by the learner through problem solving, inquiry, and discourse.
Summary of Betty's Perspective	Mathematics is about connecting with the world through numbers and mathematical concepts. Mathematics is a way of looking at the world. Mathematics helps people connect and solve problems that they encounter in daily life. Mathematics involves procedures, but it is more than accumulation of procedures.	The role of the teacher is to be a guide and a coach. Teachers need to help students make connections to what students have learned in the past, what they have studied, and what they have explored. Teachers can lose focus on deeper understanding by focusing too much on procedures and rules. Teachers should ensure students understand the rationale for what they are learning.	Mathematics can be learned deeply, which involves learning concepts, connections among concepts, and why procedures work. Mathematics concepts can be discovered and learned a lifetime. Learning mathematics involves experimentation; students should be engaged in hands-on learning and doing projects. The role of the student is to make connections between what they learn and their experiences.

Table V.43 (Continued)

Summary of Callie's Perspective	Mathematics involves reasoning and problem solving.	Teachers need to establish the learning environment for kids, find engaging tasks, and facilitate the learning process. Teachers should not talk too much or direct students to particular problem solving methods.	Students learn mathematics as they work through problems; they do not need more repetition. Problem solving and reasoning engages students in mathematics. Students should be given engaging tasks, time to collaborate and discuss with peers. Children learn math through trial and error, reasoning, and discussion with their peers.
Summary of Sally's Perspective	Mathematics is embedded in world. The purpose of mathematics is to solve problems. Mathematics involves both procedures and solving problems.	An ideal math class involves traditional math and problem solving. The role of the mathematics teacher is to have deep knowledge of mathematics, understand the needs of students, and present material in multiple ways. The teacher needs to find ways to help kids figure out mathematics.	Kids learn math through practice, problem solving, and hands-on approaches. Problem solving relates to real uses of mathematics; conceptual understanding comes from using math in life. Students should be engaged in problem solving and then have opportunities to practice skills.

The perspective of mathematics, mathematics teaching, and mathematics described by Betty, Callie, and Sally was consistent with the problem solving perspective of mathematics. Like the other participants in Phase Two of the study, the views of these teachers could be categorized using the conceptual framework guiding the study.

Summary of beliefs held by Phase Two participants with low MB. Participants rated as low MB expressed a view of mathematics consistent with an instrumentalist perspective. From an instrumentalist perspective, mathematics is a body of knowledge consisting of facts, rules, and procedures; teaching mathematics involves conveying rules and demonstrating procedures to students; and learning mathematics involves acquisition of rules and procedures through demonstration and practice. Three participants in Phase Two of the study, Natalie, Steven, and Tammy, expressed perspectives of mathematics consistent with an instrumentalist view. Table V.44 displays the themes from the participant interviews.

The view of the discipline of mathematics expressed by the three participants reflected an instrumentalist perspective. Natalie portrayed mathematics a series of accumulated skills and relies on sequential thinking. To Tammy, mathematics consisted of numbers, calculation, and procedures. For Steven, mathematics was described as a discipline of sequence, order, logic, and reasoning.

The roles of the teacher and students the three teachers described were also consistent with the instrumentalist perspective. Natalie described the role of the teacher as determining the levels of students and sequencing instruction in order to move students forward. Natalie described the need for students to know specific procedures and basic skills in mathematics in order to be successful. According to Steven, teachers should clearly model for students how to solve problems. Modeling by the teacher allows students to learn to do mathematics on their own by repeating the model. Tammy described a specific sequence of instruction for mathematics lessons: the first part of the lesson is review and activation of prior knowledge, the second part is teaching the new skill and using guided instruction, and the final part is independent practice by students. For Tammy, learning mathematics involves repetition, practice, review, and a positive attitude.

The perspective of mathematics described by Natalie, Steven, and Tammy was consistent with the instrumentalist perspective of mathematics although there was variation related to MA level. Steven and Tammy were classified as low MA whereas Natalie was classified as high MA. Steven and Tammy's responses were most consistent with an instrumentalist approach whereas Natalie acknowledged that memorization and repetition does not necessarily work for all students.

Table V.44 Summary of the Instrumentalist Perspective of Mathematics with Related Participants' Responses

	Nature of mathematics	Perspective of teaching mathematics	Perspective of learning mathematics
Summary of Instrumentalist Perspective	Mathematics is a body of knowledge consisting of facts, rules, and procedures.	Mathematics teaching involves conveying rules and demonstrating procedures to students.	Mathematics learning involves acquisition of rules and procedures through demonstration and practice.
Summary of Natalie's Perspective	Mathematics consists of a series of accumulated skills. There is an exact way to do mathematics problems. Mathematics involves sequential thinking.	The role of the teacher is to sequence instruction for students. Teacher's role is to determine levels of students, instruct, and move forward. Teachers need to be able to show students many ways to do mathematics.	Children need to be taught certain processes in math. Students need to have "the basics" before being able to be successful. Some students are wired to understand mathematics; they think sequentially. Kids understand mathematics in different ways. Drill, practice, and memorization does not work for all kids. Kids need to feel safe in the mathematics class. The students' role is to have a positive attitude, try, and practice.

Table V.44 (Continued)

Summary of Steven's Perspective	Mathematics is a discipline of sequence, order, logic, and reasoning.	Mathematics should be taught in a sequential order. Good mathematics teaching involves explicitly teaching the correct steps in a sequential order in an engaging way. Teachers should clearly model how to solve problems. Modeling by the teacher allows students to learn to do mathematics on their own by repeating the model. A math class needs to be highly structured.	Student practice follows explicit instruction from the teacher. Practice involves doing examples and is part of learning math. The role of the student is to listen and practice. Learning math involves repetition, effort, working through problems, referring back to notes, and studying. Students need to know basics of mathematics before they can access higher-level math or problem solving. Order and organization are important to learning mathematics. There are correct ways to hold pencils, sit in desks, and organize papers.

Table V.44 (Continued)

Summary of Tammy's Perspective	Mathematics consists of numbers, calculation, and procedures. Problems in mathematics relate to math textbook and the four operations.	Teaching mathematics involves a sequence of instruction. The first part of the lesson is review and activation of prior knowledge, the second part is teaching the new skill and using guided instruction, and the final part is independent practice by students. Repetition is very important to teaching math. The teacher's role is to making kids feel confident and successful, to have a positive attitude and provide praise, and to make mathematics fun. Teachers need to find out how students learn best and provide the right instruction.	Learning mathematics involves repetition, practice, review, and a positive attitude. Memorization of steps is a precondition for getting correct answers. Practice is essential to success in mathematics; students who struggle have not had sufficient practice. Children learn math differently. Some need to use manipulatives, physical models, or songs in order to learn to do procedures correctly.

Although there were areas where the Phase Two participants expressed common beliefs about mathematics, teaching mathematics, and learning mathematics, there were clear distinctions along the domains between participants with high MB and low MB. The beliefs of participants were readily described by the conceptual framework guiding the study, a conceptual framework developed to describe the beliefs of mathematics teachers, not special education teachers.

In addition to the themes related to the main research question and domain questions, a theme emerged across the Phase Two participants related to the influence of teaching mathematics on the mathematics beliefs and attitudes of the participants. In the final section of this chapter, the theme is described and related to the conceptual framework that guided the study.

Influence of Teaching Mathematics on Beliefs and Attitudes

A final theme that emerged across almost all of the participants in Phase Two of the study was that of the impact of teaching mathematics on either teacher beliefs or attitudes. All participants but Tammy expressed to some degree the impact that teaching mathematics has had on their view of mathematics, mathematics teaching, and/or mathematics learning.

Natalie, a participant with low MB and high MA, began her interview by answering a question about her feelings about mathematics by relating an extremely negative and deeply impacting incident:

> I struggle with it [mathematics]. I only got through Algebra One in high school and then I was told by my math teacher that he couldn't teach me anything so I never took another math class until I got to college. I still remember after all of these years. Back in 1975. He told me that right in front of my whole class.

Natalie had a very negative perception of mathematics and her ability to do mathematics.

Teaching mathematics has helped with her attitude but only to a small degree. Natalie shared,

> As a teacher I feel more comfortable with general, you know like life skills math. As a person it doesn't scare me as much as it used to because I've learned how to do it. I have taught myself or somebody else has taught me how.

In contrast to Tammy and Natalie, the other participants expressed a greater impact of teaching on their attitudes and/or beliefs.

Callie reported an interesting effect of teaching mathematics on her comfort with mathematics. When asked how she feels about mathematics, Callie reported, "When I was growing up I was always really good at math. We would take all these kinds of tests and I would always come out really good at math." Yet as the focus of her mathematics instruction has evolved from instrumentalist to problem solving, Callie expressed uncertainty about what it means to be good at mathematics, "I think, gosh, I really get stumped on a lot of these problems and I'm constantly like maybe I am not that good at math." Teaching mathematics from a problem solving perspective has impacted Callie's beliefs about her efficacy as a student of the discipline.

Betty's experience teaching mathematics has made a significant impact on her view of the discipline. When asked whether teaching mathematics has influenced her beliefs, Betty responded,

> In a huge way. I mean I really think that before I started teaching math I really just saw it in the way that I learned it, which was more a set of procedures and rules. And I feel like I—there's so much more depth to my understanding now and that I really see it as—I mean I'm still making connections all the time and I get really excited about it. It's really encouraging for me to see it in sort in the broad web or interconnected network that math can represent and I think that has

> come mostly through going back to a lot of fundamental concepts and really working through them as a teacher.

Betty's remarks illustrate that her perspective about the discipline of mathematics shifted from an instrumentalist view to a problem solving view.

In contrast to changing one's perspective from an instrumentalist to a problem solving perspective, Steven's experience teaching mathematics has intensified his instrumentalist perspective. This is illustrated by his comments about the instructional approach his former school employed that he termed standards-based. The approach Steven described was apparently random and directed at covering material for the state summative assessment:

> It's just they were trying to highlight all stuff that was coming on the test what's going to be here in March. And it never was in sequential order. So for example that you know, we'd be working for a week on line plots. So that was on this day and then the next week we're doing how to solve the area of circle. You know, it just wasn't—it just didn't flow to me. I like when math is taught in a logical order. It builds upon itself you know. You know for me that was the easiest way to learn it. I think it's the easiest way to teach it.

To Steven, the approach only served to bolster his belief that mathematics should be taught and learned sequentially.

The factors that Sally considered to have influenced her beliefs about mathematics teaching were the special education students and teachers with whom she has worked. She attributed the impact of her special education colleagues to the experiential approach to learning that they promoted. She explained that this approach was in contrast with the mathematics educators with whom she worked as illustrated in this exchange during the interview:

INTERVIEWER: Do you think you learned different things from special education colleagues than from your math educator colleagues?

SALLY: Definitely. Sometimes I think we have a slightly different view of how math should work. There still seems to be—the math, the general math classes are getting more experiential, but there's still a lot of drill and kill. Where in special education, I think we try very hard to make it more experiential, make it more real.

In Sally's experience, her special education colleagues have helped her to see mathematics as more experiential and real than the instrumentalist approach advocated by her general education colleagues.

Carson described the impact of "teaching" her peers had on her perspective of teaching mathematics. Carson spent time in her mother's classroom while she was growing up and experienced a more discovery based approach to learning mathematics:

> I have to think so or maybe it was the way—see, my mom started teaching me when I was 10 and she taught in a dual-language school. So she had to teach kids who did not speak English and that means that you have to teach with manipulatives and concretely. That may have been when I started trying to teach in a different way because those kids have to learn in a different way.

Reflecting on the impact of teaching on her view of mathematics, Carson said "Being able to teach always deepens your understanding. Teaching always increases your depth of knowledge on any subject. So I am sure it did."

The impact of teaching on the attitudes and beliefs of the Phase Two participants was noted across almost all of the teachers. The impact related to teacher beliefs about teaching and learning mathematics (as in the cases of Steven, Carson, Sally, and Natalie) or about the nature of mathematics itself as in the case of Betty and Callie.

Concluding Thoughts

The mixed method design of the current study enabled results from the two phases of the study to be combined in order to elaborate on the findings to enhance their significance. Themes from the Phase Two semi-structured interviews amplified the findings from the statistical analysis of the *MBI* detailed in Chapter 4. Combining the findings from each phase and type of data allowed for greater depth in the conclusions that can be drawn from the study and more clarity for the direction of future study.

CHAPTER 6

DISCUSSION

The goal of the present study was to characterize the complex phenomena of the beliefs and attitudes that special education teachers hold about the discipline of mathematics, teaching mathematics, and learning mathematics using a mixed method approach. The research objective was exploration (Johnson & Christensen, 2008). Exploratory research entails "generat[ing] information about unknown aspects of a phenomenon" (Teddlie & Tashakkori, 2009, p. 25) as opposed to explanatory research which seeks to test hypotheses and theories (Johnson & Christensen, 2008). As demonstrated in Chapter 2, the research base related to the mathematics attitudes and beliefs of special educators is limited; therefore, this exploratory study has the potential to generate information upon which future studies may build. The rationale for employing mixed research methods in the study was significance enhancement, which permits the researcher to expand the interpretation of findings from qualitative and quantitative strands of a study to enhance, compare, and clarify across methods (Collins, et al., 2006). Mixing qualitative and quantitative methods promotes complementarity allowing for study of different components of the phenomena (Greene, et al., 1989). In the case of the present study, quantitative methods were utilized in Phase One of the study to measure the mathematics anxiety and beliefs of the full study sample and qualitative methods were employed in Phase Two of the study to enhance and clarify results from Phase One.

The research question guiding the study was "What is the nature of the beliefs and attitudes held by special educators about the discipline of mathematics and the teaching and learning of mathematics?" The primary research question was explored through four domain-related questions: (a) what are the attitudes of special educators about mathematics, (b) what are the beliefs of special educators about the discipline of mathematics, (c) what are the beliefs of special educators about teaching mathematics, and (d) what are the beliefs of special educators about learning mathematics? The questions guiding the study are important because of the hypothesized link between teacher beliefs and attitude and students' experiences and attitude in mathematics.

The discussion of the results of both phases of the study is presented according to the domains of the primary research question, (a) the attitudes of special educators about mathematics, (b) the beliefs special educators hold about the discipline of mathematics, (c) the beliefs special educators hold about teaching mathematics, and (d) the beliefs special educators hold about learning mathematics. Implications of the findings of the study and suggestions for future research are also provided.

The Attitudes of Special Educations about Mathematics

In the literature, the affective domain is generally considered to include emotions, attitude, beliefs, and values (Goldin, 2002; Leder & Forgasz, 2002; McLeod, 1988). Within the affective domain, attitudes are described as less intense but having a longer duration than emotions but more malleable than beliefs and values. Statt (1998) defined an attitude as "a stable, long-lasting, learned predisposition to respond to certain things in a certain way" (p. 10). In the present study, mathematics attitude was defined in terms of anxiety toward mathematics. Phase One of the study involved use of the *Math Anxiety*

Rating Scale, Short Version (*MARS-SV*) to measure the mathematics anxiety of the full study sample. Phase Two of the study involved semi-structured interviews of extreme cases selected from Phase One to further explicate the attitudes of special educators. The section that follows includes a discussion of the results related to special educator mathematics attitudes from both phases of the study in order to answer the research domain question, what are the attitudes of special educators about the discipline of mathematics?

Full Study Sample Attitudes About Mathematics

The mathematics attitude of the full study sample was assessed using the *Math Anxiety Rating Scale, Short Version* (*MARS-SV*). The *MARS-SV* is a 30-item instrument consisting of questions about activities that involve mathematics, such as performing calculations, experiences in mathematics classes, and using mathematics in everyday life (Appendix C). Respondents rate their anxiety for each item on a five-point scale with descriptors of: (a) not at all, (b) a little, (c) a fair amount, (d) much, or (e) very much.

Results from the *MARS-SV* analyses of the full study sample indicate that the MA level of the full study sample was normally distributed. This finding is interesting given the literature that exists related to MA in educators. The majority of studies on MA in educators involve prospective or practicing elementary teachers and point to a prevalence of high MA among these educators (Ball, 1990; DiMartino & Sabena, 2010; Ellsworth & Buss, 2000; Hembree, 1990). No such studies have been conducted on special education teachers. However, given the lack of content knowledge focus of special education teacher preparation programs (Brownell, Ross, Colon, & McCallum, 2005; Maccini & Gagnon, 2002), special education programs may attract a similar demographic of educators as elementary education programs. Also, the coursework of pre-service secondary special education teachers contains significantly less mathematics content than that of secondary general education mathematics teachers (Maccini & Gagnon, 2002). Thus, the normal distribution of MA among the study sample was unexpected.

In fact, the level of MA found among the study sample is counter to the experience of the researcher. As a district level mathematics coordinator, the researcher frequently interacted with special education teachers who provided mathematics instruction or support to students with SLD. The special educators often expressed hesitancy with or fear of mathematics in professional development settings. Thus, based on the literature and past experience, the researcher expected a high level of MA among the study sample.

The lower level of MA than expected among the study sample could be attributed to the study design, professional development initiatives in mathematics, or other factors. First of all, potential volunteers for the study with higher MA may have avoided participation in the study knowing that the topic of was mathematics. Thus, the study topic may have attracted participants with a greater affinity toward mathematics. Alternately, the lower than expected MA might be attributed to increased professional development in mathematics associated with reform-based mathematics. Throughout the 2000s, the National Science Foundation (NSF) funded systemic change initiatives focused on enhancing reform-based mathematics practices that involved over 70,000 teachers nation wide (Banilower, Boyd, Pasley, & Weiss, 2006). The lower than expected MA may be the result of the involvement of Colorado educators in the NSF projects or other local mathematics initiatives. However, without further study, conclusions about the reasons for lower than expected MA in the study sample cannot be determined.

Phase Two Participants' Attitudes About Mathematics

Phase Two of the study involved semi-structured interviews with a sub-sample of participants from Phase One of the study. Phase Two participants were selected by stratifying Phase One participants along two dimensions, level of MA and level of alignment of mathematics beliefs with reform-based mathematics. In order to better understand the range of attitudes special educators hold about mathematics, the profiles of Phase Two participants included: (a) low alignment of mathematics beliefs, high mathematics anxiety, (b) high alignment of mathematics beliefs, high mathematics anxiety, (c) low alignment of mathematics beliefs, low mathematics anxiety, and (d) high alignment of mathematics beliefs, low mathematics anxiety. In the original design of the study, two participants from each category (a) – (d) were to be included in Phase Two of the study. Limitations inherent in the full study sample and difficulties with recruitment for Phase Two narrowed the range of attitudes represented in the results such that high MA participants were underrepresented. As a result, analysis of the Phase Two data was adjusted to compare results according to only two dimensions: degree of math anxiety and degree of alignment of mathematics beliefs with reform-based mathematics.

As with the full study sample, the MA of the Phase Two participants was lower than expected. In fact, it was difficult to select participants who met the criteria Ashcraft and Kirk's (2001) set for high MA. Six participants met the criteria for high MA in the full study sample; however, among those participants there were not a sufficient number of participants with extremes in MB to fill the cells in the variable dimension matrix.

Thus, given the extremely small sample of participants with high MA, there are limits to the ability of this study to illuminate the beliefs and attitudes of special educators who have high MA.

Mathematics attitude of Phase Two participants with high MA. Two participants in the study were indicated as having high MA, Natalie and Callie; however, only Natalie expressed emotions common to people with MA. Natalie's responses illustrate the way MA is described in the literature as she used the terms "scared" and "terrified" to describe her attitude toward mathematics (Aiken & Dreger, 1961; Ashcraft, 2002; Ashcraft & Ridley, 2005; Gresham, 2007). Natalie's expressions of fear and dread related to mathematics demonstrate Ashcraft's (2002) definition of MA as "tension, apprehension, or fear" (p. 181). Natalie's frankness about her fear of mathematics and readiness to share her experiences were remarkable. Within moments of beginning the semi-structured interview, Natalie readily discussed her personal painful stories about how her fear of mathematics developed and was exacerbated by her mathematics teacher. Her forthrightness highlighted the devastating and sometimes lifelong impact of teacher behavior on student attitude. Interestingly, Natalie conveyed that her attitude toward mathematics had improved over the years, a change she attributed to teaching mathematics. Having to learn the mathematics she needed to teach ameliorated her negative emotions toward the subject.

Although data from Callie's semi-structured interview called into question the validity of her *MARS-SV* score, her reflection about the change in her perspective of mathematics was intriguing. Callie noted that her conception of what it means to be competent in mathematics shifted as her perspective of mathematics evolved from an instrumentalist to a problem solving point of view. Callie began to question her own ability in mathematics as her instructional approach changed from teaching procedures to facilitating problem solving.

Callie's responses were consistent with a pilot study conducted by the researcher relating perspective of mathematics with MA. In the unpublished study (Colsman, 2011), participants expressing an instrumentalist perception of mathematics tended to have lower mathematics anxiety than participants expressing a discovery or problem solving perception of mathematics. The study had considerable limitations related to how participants' perspective of mathematics was determined; however, the results were interesting when put in the context of the present study.

The literature provides some insight into the phenomenon experienced by Callie. Clute (1984) conducted a study in order to determine the relationship between MA and the instructional method utilized in mathematics instruction. The study found that students with high levels of MA had higher achievement when taught with an instrumental instructional method. Clute concluded that "instead of trusting his or her own methods of mastering the material, the highly anxious student needs to rely heavily on a well-structured, controlled plan for learning" (pp. 56-57). It may be that an instrumentalist perspective of mathematics conveys the discipline as bound by specific rules which brings a level of comfort to individuals. Further, as individuals begin to move from an instrumentalist perspective of mathematics to a more problem solving view of the discipline, they may begin to question their ability to do mathematics.

If an instrumentalist approach is linked to lower MA, should instrumentalist approaches to teaching mathematics be employed to minimize math anxiety? The answer to this question lies in whether the goal of mathematics education is to lower MA or increase mathematical competence in students. Richland, Stigler, and Holyoak (2012) provided a look at the results of instrumentalist mathematics instruction in their study of students in remedial mathematics classes at the community college level. Richland, Stigler, and Holyoak (2012) found that the students in remedial classes (a) viewed mathematics as a collection of procedures to be memorized, (b) lacked fundamental concepts needed to reason mathematics, and (c) applied mathematics procedures regardless of whether they were needed or made sense. Richland, Stigler, and Holyoak (2012) concluded, “By asking students to remember procedures but not to understand when or why to use them or link them to core mathematical concepts, we may be leading our students away from the ability to use mathematics in future careers” (p. 190).

In summary, no conclusive generalization can be made about attitudes of the sub-sample participants with high MA. However, the apparent ameliorating effect of teaching mathematics on Natalie’s MA and Callie’s questioning of her mathematics ability prompts the question, “what impact does teaching mathematics have on the mathematics attitudes of teachers?” Exploring the influence of teaching mathematics on educators’ mathematics attitudes could prove beneficial to educator preparation programs perhaps indicating a need for increased hours in mathematics practice teaching situations. Such a study would entail measurement of teachers’ MA over time as they gain experience teaching mathematics.

Mathematics attitude of Phase Two participants with low MA. Of the seven participants in Phase Two of the study, five participants were indicated to have low MA. Analysis of the mathematics attitudes of the participants with low MA indicated that low

MA was not necessarily associated with enjoyment of mathematics. Interestingly, only the participants with low MA and a high alignment of reform-based beliefs about mathematics expressed enthusiasm for the subject. Conversely, the participants with low MA and low alignment of reform-based beliefs, Tammy and Steven, expressed more ambivalent attitudes about mathematics.

The limited data from the present study illustrate the complexity of measuring attitudes. Related to the study at hand, the lack of anxiety toward mathematics does not necessarily equate to enjoyment of mathematics. An attitude is more than simply having a positive or negative response. As noted in Chapter Two, McLeod (1988) described dimensions of the affective domain including: (a) the magnitude or intensity of response experienced by the individual, (b) the level of control one has over one's responses, (c) level of consciousness the individual experiences, and (d) the duration of the response (1988, pp. 136-137). The data collected in the present study indicate that mathematics attitude should be more broadly defined than anxiety. Defining mathematics attitudes simply in terms of fear or comfort with mathematics provides only one dimension of the range attitudes possible toward the discipline. Despite the inability to fully answer this aspect of the research question, two themes from the study related to the role of the mathematics learning experience in mathematics attitude formation are worthy of discussion and further study.

Themes related to mathematics attitudes. Interview data pointed to two factors in mathematics attitude development of the study participants: (a) the participants' mathematics learning experiences, and (b) the participants' mathematics teachers. The responses provided by the study participants illustrated the link between their prior mathematics learning experiences and their current attitudes toward mathematics. Specifically, for some participants, negative mathematics learning experiences imprinted long lasting memories. Even when the participants did not express negative experiences, the prominent role of mathematics teachers in the development of the mathematics attitudes of students was apparent.

Salience of worst mathematics experience. Three participants could recall a worst mathematics learning experience, Carson, Natalie, and Steven. All attributed their negative experiences to their mathematics teachers; however, the participants with low MA were more resilient to the negative experiences. For example, both Natalie (high MA) and Carson (low MA) recounted experiences in mathematics classes involving public humiliation by their mathematics teachers. Although they had similar experiences, Natalie and Carson responded quite differently. Carson, who had low MA, continued to love mathematics and desired to become a teacher, vowing never to treat her students as she was treated. In contrast, Natalie avoided mathematics as much as possible as she progressed through school. Unlike Natalie and Carson, Steven's (low MA) negative experience had less to do with mathematics and more to do with a power struggle with his trigonometry teacher. Steven did not express any long-term impact of the experience.

The differential responses to prior negative mathematics classroom experiences described by Carson, Natalie, and Steven appears to be an illustration of a phenomenon noted by Ball's (1988) research with pre-service and novice teachers:

> Whatever their particular experiences, budding teachers develop ideas about how to teach mathematics and about what the roles of students and teacher in a mathematics classroom are. If they were successful in mathematics, prospective

> teachers are likely to approve of the patterns they saw, and thus be uninterested in alternative ways of teaching. If they struggled, they may aspire to teach differently. But even if they are critical of their own past teachers for teaching badly and for making them feel stupid, they may lack alternative models (p. 45).

Carson aspired to teach differently than her mathematics teacher whereas Natalie seemed to lack an alternative to the type of instruction she experienced. Similarly Steven did not consider his experience to be detrimental in the long term and did not question the approaches his mathematics teachers employed.

The differential responses of Natalie, Carson, and Steven to negative mathematics experiences are quite interesting. Researchers (Ashcraft & Krause, 2007; Bekdemir, 2010; Trujillo & Hadfield, 1999) have found a relationship between mathematics anxiety and stressful experiences in the classroom or the hostile behavior of teachers. Caron and Steven exhibited a greater degree of resilience related to the negative experiences and did not develop MA. Natalie, on the other had, suffered from MA, an effect confirmed in the literature. The difference in responses to negative mathematics learning experiences is a potential area further study. Specifically, future research might explore factors associated with resilience in relation to negative mathematics learning experiences. Identification of factors that support resilience to MA could inform interventions to prevent MA or alleviate the effects of MA.

Mathematics teachers played a role in the negative mathematics learning experiences of Natalie, Carson, and Steven. This may be just one aspect of overall role mathematics teachers play in student mathematics attitude development. The next section summarizes how the study participants conceptualized the role of mathematics teachers in student attitude development.

Role of teacher in students' mathematics attitude development. Either from their own experiences as students or their experiences teaching mathematics, participants pointed to the important role mathematics teachers play the mathematics attitude development of students. This theme was also noted by Ellsworth and Buss (2000) who found five factors that influenced the mathematics attitudes of pre-service teachers: (a) the negative and positive experiences related to teachers, (b) family members, (c) the importance mathematics content being relevant to real-life, (d) the tension between conceptual understanding of mathematics and coverage of content, and (e) classroom emphasis on skills and memorization.

Mathematics teacher's attitudes influences more than the mathematics learning environment. Attitudes of mathematics teachers may relate to student achievement in mathematics. Schofield (1981) demonstrated that teachers' positive attitude toward mathematics correlated to student achievement in mathematics. In fact, the hypothesized relationship between teacher and student mathematics attitude led Geist (2010) to contend that "many teachers who have math anxiety themselves inadvertently pass it on to their students" (p. 29). Stopping the cycle of negative mathematics attitudes between teachers and students may support greater student achievement in mathematics. However, further study would be needed to determine the degree to which teacher mathematics attitudes contribute to the mathematics attitudes and achievement of their students before any recommendations for changes in educational practice would be merited.

Conclusions about the Mathematics Attitudes of Study Participants

The results of the present study revealed that the study sample had lower MA than research might suggest. Given the limitations of the presented by the low number of study participants with high MA, no specific conclusions about the attitudes of special education teachers can be made. Instead, further study into the mathematics anxiety and overall attitude of special educators related to mathematics is needed. Regardless, with the role that special education teachers play in the mathematics instruction of students with SLD, whether as students' sole provider of mathematics instruction or as provider of supplementary instruction, special educators must understand the importance of a positive attitude toward mathematics. Given that students with SLD have been shown to have increased MA as they grow older (Lebens, et al., 2011), it is critical that special educators are aware of the influence their mathematics attitudes have on their students.

The Beliefs of Special Education Teachers about Mathematics

The results related to the mathematics beliefs of the special education teachers who participated in the study are quite interesting in relation to the literature. Whereas the participants in Phase Two of the study represented the range of beliefs from instrumentalist (or traditional) to discovery and problem solving (or reform-based), the sample as a whole was more aligned with a reform-based perspective of mathematics than not. This finding is in contrast to what the literature seems to suggest about the beliefs of special educators. A discussion of the beliefs of the full study sample and Phase Two participants is provided here.

Full Study Sample Beliefs About Mathematics

The mathematics beliefs of the full study sample were measured using the *Mathematics Beliefs Instrument* (*MBI*). The *MBI* provided data relevant for answering the domain questions (b) what are the beliefs of special educators about the discipline of mathematics, (c) what are the beliefs of special educators about teaching mathematics, and (d) what are the beliefs of special educators about learning mathematics?

Analysis of participant responses to the *MBI* items showed that for 17 of the 28 items, the study sample was significantly aligned with reformed-based views on mathematics compared with 4 of 28 items where the study sample was not in alignment with reform-based views. These results are interesting in relation to the literature that would suggest that special educators tend to reject the reforms promoted by the NCTM (Hofmeister, 1993; Rivera, 1997; Simon & Rivera, 2007). The results were unexpected given the literature and the experience of the researcher. In the experience of the researcher, special education teachers have been critical of instructional approaches promoted by the NCTM *Standards* and mathematics instructional materials that were designed to align with the *Standards*. A definitive explanation for the difference between the expected and actual mathematics beliefs of special educators who participated in the study is not possible. Possible explanations for these results are elucidated in the sections that follow. However, these results may be related to the lower than expected MA level of the study participants and the voluntary nature of study participation. The study sample may have included special education teachers more inclined to enjoy mathematics and be involved in professional development activities that promote reform-based mathematics.

Phase Two Participants' Beliefs About Mathematics

Qualitative data collected during Phase Two of the study allowed for exploration of the beliefs special educators hold about the discipline of mathematics, teaching mathematics, and learning mathematics. The discussion of the Phase Two participants' beliefs about mathematics, teaching mathematics, and learning mathematics includes four main points: (a) the permeation of reform-based mathematics beliefs within the study sample, (b) the role of teacher biography in mathematics belief formation, (c) beliefs about innate ability in relation to student mathematics success, and (d) the utility of the conceptual framework for describing the mathematics beliefs of special educators. In the sections that follow, the nuances of study participants' mathematics beliefs are explicated.

Permeation of reform-based mathematics beliefs. A striking finding from the present study is the degree of alignment of study participants with reform-based approaches. The finding is in contrast with much of the literature and may suggest that mathematics reform efforts have permeated the education system beyond mathematics teachers. The literature would suggest that special educators lack knowledge of reform-based mathematics and are skeptical of the merits of reform-based approaches, especially for students with SLD. For example, in their study of general and special educators, Maccini and Gagnon (2002) found that whereas 95% of general mathematics educators surveyed were familiar with the goals of the NCTM *Standards*, only 55% of special educators reported familiarity. Also, studies in special education literature tend to focus on instrumental aspects of mathematics such computational fluency and solving routine problems (Bryant, et al., 2000; Calhoon, et al., 2007; Fuchs, et al., 2005; Geary, et al., 1991; Simon & Hanrahan, 2004; Woodward, 2006). Furthermore, the NCTM *Standards* have received significant criticism from special educators (Hofmeister, 1993; Rivera, 1997; Simon & Rivera, 2007).

The results of the present study suggest that influence of the NCTM *Standards* on the beliefs and attitudes of special education teachers may be greater than what has been found in the literature. In earlier studies of special educators' mathematical beliefs (Gagnon & Maccini, 2007; Ginsburg-Block & Fantuzzo, 1998; Grobecker, 1999; Maccini & Gagnon, 2002), questions pertained to special educator knowledge of the NCTM *Standards*. The present study did not reference knowledge of the NCTM *Standards* but instead asked participants to react to statements based on the perspective of teaching and learning mathematics promoted by the NCTM *Standards*. The high level of participant familiarity and general agreement with the view of mathematics promoted by the NCTM may be due to the systemic efforts to train educators in reform-based approaches supported by the National Science Foundation (NSF) in the late 1990s and 2000s. The NSF supported the development of mathematics instructional programs that align with reform-based mathematics approaches (Reys & Reys, 2007). To further promote reform in mathematics instruction, the NSF funded local systemic change professional development initiatives designed to improve mathematics and science instruction consistent with reform-based approaches (Banilower, Boyd, Pasley, & Weiss, 2006). The project impacted 4000 schools in 467 districts across the U. S. involving approximately 70,000 teachers of approximately 2,142,000 students (Banilower, et al., 2006). Colorado was involved in a local systemic change initiative from 1999 to 2003. The project, called the Colorado Mathematics Middle School Teacher Enhancement Project (COMMSTEP), involved 330 teachers in 40 schools across the state ("LSC Project Info").

The degree of alignment with reform-based approaches could also be reflective of policies toward inclusion and collaboration within the Individuals with Disabilities Act (IDEA). As noted in Chapter One, when IDEA was reauthorized in 2004, it allowed for the use of an instructional model called responsive to intervention (RtI) as a method for identifying students for SLD. The RtI framework calls for a comprehensive approach for all instruction and intervention and implies changes in the roles and responsibilities of both special and general educators. Within an RtI framework, special and general educators are called upon to collaborate to best serve students, ideally resulting in higher academic achievement (Hoover & Patton, 2008). Increased collaboration among special and general education teachers may be resulting in broader sharing of instructional approaches.

The permeation of reform-based approach advanced by the NCTM into the beliefs and attitudes of special educators should not go unnoticed by policy makers and education leaders who are working to support implementation of the Common Core State Standards (CCSS) in mathematics. The new standards, adopted by 46 states, will be the basis of instruction for the vast majority of U. S. students in the coming decade. The CCSS build on the NCTM *Standards* and emphasize Standards for Mathematical Practice, which include practices related to a problem solving approach to mathematics. The mathematical practices call for students to: (a) make sense of problems and persevere in solving them, (b) reason abstractly and quantitatively, (c) construct viable arguments and critique the reasoning of others, (d) model with mathematics, (e) use appropriate tools strategically, (f) attend to precision, (g) look for and make use of structure, and (h) look for and express regularity in repeated reasoning (Common Core State Standards for Mathematics, 2010).

The permeation of mathematics reform beliefs to special educators implied by the present study suggests that the systemic approach to supporting mathematics reform in the 1990s and 2000s should inform the implementation of the Common Core State Standards. Applying the learning from the NCTM *Standards* to the new CCSS has the potential to better support all teachers who influence student learning, including general education teachers, special education teachers, intervention specialists, and administrators.

Policy oriented research should be undertaken to learn from past education reform implementation efforts and guide policy makers and education leaders who have an interest in the success of the CCSS. Future research should be three-fold: (a) reviewing past systemic implementation initiatives to ascertain factors that contributed to successful education reforms, (b) using the identified factors as indicators of success for CCSS implementation, and (c) documenting progress along the indicators across the education system.

In addition to important finding related to the permeation of reform-based mathematics beliefs within the study sample, the role the sub-sample participants' biography played in their belief formation was notable. This theme is discussed in the next section.

Role of teacher biography in mathematics belief formation. Another interesting finding from the present study was the role teachers' biographies played in the development of their beliefs about mathematics teaching and learning. Two dimensions of teacher biography emerged from interviews with the sub-sample participants, *teacher socialization* and *generalization of mathematics learning experiences*.

First, teacher socialization relates to what Lortie (1975) described as the "apprenticeship of observation". The term describes the phenomenon children experience as they go through their schooling, observing the practices of teachers and forming notions of what teaching, learning, and school mean. The thirteen years of school experience serves as an apprenticeship of sorts for aspiring teachers. Britzman (1986) described the phenomenon:

> The student teacher enters the apprenticeship classroom armed with a lifetime of student experience. This institutional biography tells the student teacher how to navigate through the school structure and provides a foundation for the stock responses necessary to maintain it. Additionally, implicit in these stock responses are particular images of the teacher, mythic images which tend to sustain and cloak the very structure which produces them. (p. 448)

Studies of mathematics instruction in Germany, Japan, and the U. S. (Hiebert et al., 2005; Stigler & Hiebert, 1999) document the apprenticeship process for mathematics instruction in the U. S. resulting in what Stigler and Hiebert (1999) call a *script* for teaching mathematics. The U. S. mathematics teaching script described by Stigler and Hiebert (1999) describes an instrumentalist classroom with instruction consisting of a warm up or review of the previous lesson, checking homework, presentation of the new lesson with checks for understanding, and seatwork.

Nearly all of the Phase Two participants described learning mathematics from an instrumentalist perspective like the one described by Stigler and Hiebert (1999). Despite this common “apprenticeship” in instrumentalist instruction, the study sample participants expressed relative strong alignment with reform-based approaches to teaching mathematics. In fact, all of the participants with high MB described a reform-based approach to teaching mathematics. That the study sample expressed general alignment of beliefs with reform-based mathematics is interesting on its own. Why study participants either adopted reform-based beliefs or held onto more traditional, instrumentalist beliefs is even more intriguing but not entirely clear.

Ball (1988) examined how prospective and novice teachers approach learning to teach mathematics in relation to their past experiences. She noted,

> Whatever their particular experiences, budding teachers develop ideas about how to teach mathematics and about what the roles of students and teacher in a mathematics classroom are. If they were successful in mathematics, prospective teachers are likely to approve of the patterns they saw, and thus be uninterested in alternative ways of teaching. If they struggled, they may aspire to teach differently. But even if they are critical of their own past teachers for teaching badly and for making them feel stupid, they may lack alternative models. (p. 45)

Ball’s hypothesis may explain the why some study participants abandoned the model of mathematics teaching they experienced as students and why other participants maintained an instrumentalist perspective as teachers.

For example, whereas Natalie, Steven, and Tammy all expressed instrumentalist views of mathematics, only Steven and Tammy expressed comfort with mathematics and described their mathematics experiences as successful. Following Ball’s (1988) reasoning, neither Steven and Tammy found a reason to have their perspective of mathematics teaching and learning challenged. On the other hand, Natalie struggled in mathematics and expressed a high level of MA. Natalie recognized that the way she learned mathematics did not work for her yet she continued to hold an instrumentalist view of mathematics, perhaps because she lacked an alternative model.

In contrast to the participants who continued to embrace the instrumentalist view they experienced as students, four of the participants in the sub-sample did not adopt an instrumentalist view of mathematics teaching and learning. Callie's perspective of mathematics teaching and learning evolved through the early part of her teaching career. Callie attributed the change in her perspective to working with students who struggle and the need to find more ways to engage students in mathematics. She found that by using problem solving tasks as the basis of her classes she was better able to motivate students and contribute to their mathematics learning. Betty described being very good at memorization as a student and attributed her success in mathematics to this ability. Her realization about the limitations of an instrumentalist approach came with the recognition that her mathematics understanding consisted of what she termed isolated nuggets, disconnected rules, procedures, and formulas that did not make sense. Sally's experience learning school mathematics was augmented by out-of-school learning experiences with her parents. Sally's parents demonstrated how mathematics was part of everyday life providing her with a problem solving perspective that countered the instrumentalist perspective presented in school. To Sally, it is essential to connect all school mathematics to real world experiences especially for students with SLD. Finally, Carson also emerged from an instrumentalist school learning experience to approach mathematics teaching and learning differently. Carson described the influence of tutoring her peers and sibling in mathematics as she grew up. Both experiences challenged her to learn to teach mathematics differently than what she experienced in her schooling. For the teachers with high MB, the instrumentalist approach they experienced in their own school was not an approach they have adopted for their own teaching. Each was able to transcend the apprenticeship of observation and create a new image of mathematics teaching and learning.

In addition to this so-called apprenticeship of experience, teachers bring their personal biography to the classroom and may generalize their own learning experiences to teaching and learning in general. Kagan (1992) described this phenomenon in prospective and novice teachers, "Candidates often extrapolate from their own experiences as learners, assuming that the pupils they will teach will possess aptitudes, problems, and learning styles similar to their own" (p. 154). Calderhead and Robson (1991) described a similar phenomenon with pre-service teachers. They found that that pre-service teachers held strong images of teaching based on their own experiences in school. These images served either as models to emulate or as motivation to promote an opposite image. The tendency to generalize one's learning experiences noted by Kagan (1992) and Calderhead and Robson (1991) was a salient feature of the sub-sample participants. During the interview process, participants reported personal stories about their mathematics learning experiences that could be correlated with beliefs they expressed about teaching and learning mathematics. For Steven and Tammy, their experience of mathematics was effective for their learning; thus, there was not the motivation to alter their perspective. For the Betty, Callie, Carson, and Sally, their experiences in and out of school provided the motivation to consider alternatives to the perspective of mathematics teaching and learning modeled in their school. The mathematics learning experiences the participants reported has resulted in the perspective they have constructed about teaching and learning mathematics.

The importance of the participants' personal biographies in relation to their beliefs about teaching and learning mathematics may suggest an addition to Ernest's framework, which will be addressed in a subsequent section of this chapter. Additionally, gaining a better understanding of the underlying reasons for teacher transformation in beliefs may inform teacher education and professional development. One factor that might be considered in future research is the degree to which teachers hold their beliefs about teaching and learning mathematics related to the success of their students. The responses of the study participants suggested that teachers are motivated to adjust their instructional practice and alter their beliefs based on the success of their students. Encouraging teachers to reflect on their practices in relation to student learning could prove instrumental in inspiring teacher change.

Throughout the presentation and discussion of the results for the present study, the beliefs of the study participants have been described using a conceptual framework based on Ernest (1989). In the next section, the importance of the utility of the conceptual framework with special educators is presented.

Relevance of conceptual framework. The conceptual framework that guided the study was one proposed Ernest (1989), which illustrated the hypothesized relationship between mathematics teacher beliefs and instructional practices. Ernest described three views of mathematics, instrumentalist, discovery, and problem solving view. An instrumentalist perspective considers mathematics to be a body of knowledge consisting of facts, rules, and procedures. A discovery perspective of mathematics considers mathematics to be a dynamic discipline that exists external to human beings that can be discovered. Finally, a problem solving view of mathematics considers it to be a dynamic discipline of inquiry that is contextually bound, and a way of thinking about the world. The analysis that has been discussed thus far illustrates the relevance of Ernest's (1985, 1988) framework to describe special educators' beliefs about mathematics, teaching mathematics, and learning mathematics.

Ernest's (1985, 1988) framework was conceived as lens for understanding how mathematics teachers view their discipline. Yet, the framework proved to be a useful tool for describing the mathematics views of the special educators. Specifically, the sub-sample participants described their respective viewpoints about mathematics, teaching mathematics, and learning mathematics in terms of the categories set forth within Ernest's framework. While not representative of special educators at large, that the beliefs of the study participants could be so easily be described using a conceptual framework developed for mathematics teachers suggests that the views of special educators are within the same continuum of beliefs. These findings give credence to the study by Gagnon and Maccini (2007) which found no significant difference between these teachers' perceptions of mathematics between the beliefs held by mathematics and special education teachers. Future exploration into the degree of alignment of the beliefs between general education teachers and special education teachers may be warranted.

According to the Ernest (1989) framework, teaching mathematics is deeply connected to beliefs. Thus, any efforts to change instructional practices of educators may cause resistance if the intended practices are not aligned with educators' beliefs. Mapping the domain of beliefs allows the invisible (beliefs) to become visible and can assist education leaders in designing effective supports for educators. The present study provides some indication that there are commonly held beliefs about mathematics across both general education mathematics teachers and special education.

The beliefs that were the focus of the present study related to mathematics, teaching mathematics, and learning mathematics. An unexpected finding from the study related specifically to beliefs the sub-sample participants held about students' ability to learn mathematics. This final point is presented next.

Beliefs about innate ability in relation to student mathematics success. Another notable theme from study was the difference of beliefs Phase two participants held about the role of innate ability to success in mathematics. All of the Phase Two participants with low MB attributed success in mathematics to innate ability, whereas the Phase Two participants with high MB attributed success in mathematics to factors such as motivation, real world experience, confidence, parent support, and experience.

The influence of student attribution of their success in mathematics is well documented. In their review of the literature related to attribution theory as it applies in mathematics, Middleton and Spanias (1999) found that young children tend to have positive attitudes toward mathematics and their ability to learn mathematics. However, "by the middle grades, many students begin to perceive mathematics to be a special domain in which smart students succeed and other students merely 'get by' or fail" (Middleton & Spanias, 1999, p. 60). Similarly, in her review of beliefs in mathematics, Muis (2004) reported how students tend to believe that "those who are capable of doing mathematics were born with a 'mathematics gene' " (p. 330). Middleton and Spanias

(1999) concluded that, “When students attribute their successes to ability, they tend to succeed; when they attribute their failures to lack of ability, they tend to fail” (p. 70).

Dweck has long studied the theory of attribution as it relates to motivation. Dweck (1975) asserted that,

> If a child believes failure to be a result of his lack of ability or a result of external factors beyond his control, he is unlikely to persist in his efforts. On the other hand, if a child believes failure to be a result of his lack of motivation, he is likely to escalate his effort in an attempt to obtain the goal. (pp. 682-683)

Graham (1991) described attribution along three dimensions: (a) locus, either internal or external, (b) stability, either stable or unstable, and (c) controllability, either controllable or uncontrollable. Described using these dimensions, ability has an internal locus, is very stable, and uncontrollable. Effort described along these dimension also has an internal locus, is unstable as it can vary widely, and is controllable.

How long a child persists when facing a challenge relates to a child’s sense of control. Lacking an internal locus of control can lead to a feeling of learned helplessness, a term coined by Seligman and Maier (1967). Through behavioral experiments, they demonstrated that animals eventually lose their motivation to escape pain when their efforts continue to be met with failure. The phenomenon has been applied in education settings to understand motivation for learning (Dweck, 1975; Graham, 1991; Weiner, 1972). Dweck and Goetz (1978) asserted that learned helpless in students occurs when students perceive their lack of success to factors that they cannot control, such as lack of ability.

The influence of teacher attribution of success may also be impactful for students.

Graham (1991) contended that teachers send indirect cues about their own attribution of student failure and that "failing students can gain information about the causes of their achievement outcomes based on the affective displays of teachers" (p. 9). For instance, teachers may express pity for students they perceive as not having ability or anger for students whose failure the teacher ascribes to lack of effort. Although subtle, Graham suggested that the behaviors of teachers send messages to students conveying teacher perception of student ability.

In contrast to the potentially limiting effect of attributing success to the fixed factor of success, Dweck and Goetz (1978) found that persistence is associated with a mastery orientation, that is, the perspective that success and failure is due to effort. Applied to the mathematics classroom, Ames and Archer (1988) found that the orientation for learning in the classroom impacted student motivation and attribution for success or failure. Students tended to exhibit greater motivation in classes where success was defined in terms of improvement and value was placed on effort compared classes where success was defined in terms of grades and value was on ability.

The literature suggests that the perception of teachers and orientation of a classroom can influence student beliefs and achievement in mathematics. Thus, the beliefs expressed by the Phase Two participants about attribution of mathematics success or failure could be quite important. The importance of the difference in attribution of mathematics success between participants with low MB and high MB cannot be distinguished from the data collected for the study. Whether a relationship exists reform-based beliefs about mathematics and attribution of student factors for success merits further study.

The findings of the present study were both surprising and intriguing and offered the researcher opportunities to reflect on her assumptions about the beliefs of special educators. The results based on the study sample point to special educators having lower MA and holding more reform-based beliefs than the researcher hypothesized. Beyond the interest of the researcher, the study may have implications for others to consider. Implications and suggested areas for future research are presented next.

Implications

The findings discussed in this chapter serve as a potential building block to the literature by illuminating the mathematics attitudes and beliefs of special educators, an area not yet fully explored. The findings herein may ultimately have implications for better supporting the mathematics achievement for students with SLD. The implications of the findings of the present study are three-fold. First, the applicability of the conceptual framework for describing the mathematics beliefs of special educators may imply that the beliefs and attitudes of special educators are not qualitatively different from those of mathematics educators. Second, the permeation of reform-based notions of mathematics, mathematics teaching, and mathematics learning into the beliefs and attitudes of special educators may prove helpful for implementation of future mathematics reform initiatives. Third, the influence of teachers' biographies in their development of beliefs about teaching and learning informs changes to the study's conceptual framework. Fourth, the beliefs teachers hold about the role of innate mathematics ability has the potential to either support or inhibit the mathematics achievement of students. Ultimately, the intention of these findings is to inform better mathematics outcomes for students with SLD. Each point is further examined next.

To begin with, the conceptual framework for the study proved applicable in describing the mathematics beliefs of special educators suggesting that the beliefs of special educators are not qualitatively different from those of mathematics educators. If this assumption is true, then the research related to the beliefs of mathematics teachers may be applicable to special education teachers. This would be important because the differences between educators may not be according to their focus (special or general education) but according to commonly held beliefs articulated through the Ernest (1989) framework.

In fact, the permeation of reform-based beliefs about mathematics into the beliefs of the special educators involved in the present study may suggest that this has already occurred. That the beliefs about the nature of mathematics, teaching mathematics, and learning mathematics of the special educators involved in the present study were relatively aligned with reform-based approaches implies success of past systemic mathematics reform initiatives. As policy makers and educational leaders consider the best approaches to implementing the new Common Core State Standards, the strategies employed by the NSF to influence change in mathematics teachers' beliefs and practices should be carefully studied and employed. Additionally, the Ernest (1989) framework should inform the design of training for special education teachers. The bi-directional arrows within Ernest's (1989) framework imply that changes in teachers' beliefs can be influenced by changes in any level of the framework. That is, a change in a teacher's enacted model of learning mathematics implies a change in each of the other components of the framework. Thus, professional development experiences that intend to change teachers' enacted model of teaching mathematics will, according to the Ernest (1989) framework, inevitably result in examination of teachers' beliefs about learning mathematics and ultimately to their beliefs about the discipline of mathematics. Professional development intended to produce changes in mathematics instructional practices must therefore take into account the impact of beliefs in order to be successful. The role of teacher beliefs and biography may also prove useful in the implementation process as suggested by another finding in the study.

The influence of teacher biography in the development of beliefs about teaching and learning mathematics found in the present study has implications for the study's conceptual framework base don Ernest's (1989). The salience of study participant personal biography in relation to their beliefs about teaching and learning may suggest teacher biography as an additional component for the framework (Figure VI.22).

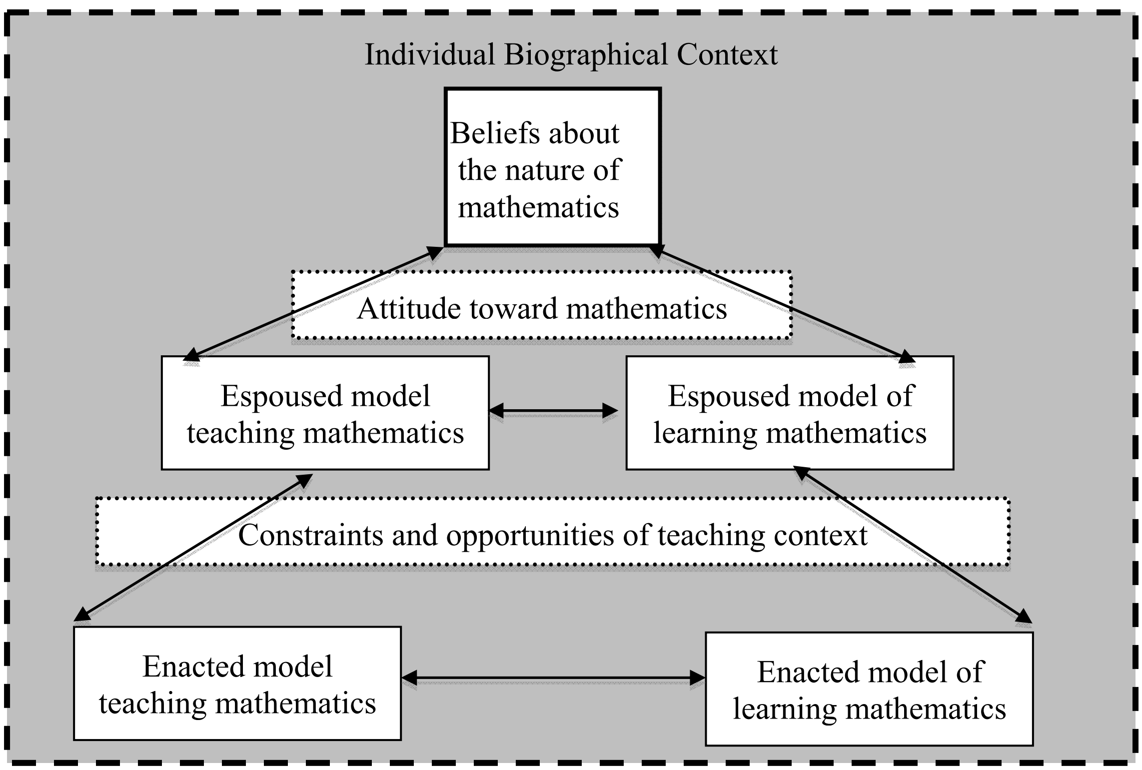

Figure VI.22. Revised Conceptual Framework Relating Mathematics' Teacher Beliefs to Teaching Practices with Attitude toward Mathematics with Teacher Biography as the Context (adapted from Ernest, 1989).

The context provided by each teacher's biography and their own experiences learning mathematics permeated their beliefs about the nature of mathematics, teaching mathematics, and learning mathematics. To explain this phenomenon, teacher biography is added to the conceptual framework as the context in which beliefs and attitudes are developed. The implications summarized herein would need further study to be useful for improving mathematics outcomes for students with SLD. Areas for future research are addressed next.

Recommendations for Future Research

The results from the present may serve as the building blocks for future research. First, in order to verify that the conceptual framework that guided the present is applicable beyond the special educators involved in the present study, a confirmatory study should be conducted. A study design utilizing random sampling would be beneficial in determining whether the permeation of reform-based beliefs indicated within the study sample was due to sampling issues.

Furthermore, the biographical events that influence teacher beliefs either toward or away from the instructional approaches they experienced should be further explored. Understanding how teachers conceptualize their mathematics learning experiences and generalize to beliefs about teaching and learning mathematics may contribute to improvements in pre-service preparation and professional development for practicing teachers. Tapping into the deeply held attitudes and beliefs educators hold about teaching and learning in relation to their own learning may prove fertile ground for precipitating teacher change.

Finally, a deeper exploration into the beliefs teachers hold about the role of innate ability in mathematics achievement is warranted. The role of teachers in the attitude formation of students has the potential to positively or negatively impact student motivation and ultimately achievement. Whether there is a relationship with between low and high MB and beliefs about innate mathematics ability, the potential for either introducing or perpetuating student attribution of success to innate ability is important to understand.

Final Thoughts

The study began with a discussion of the urgency of addressing the mathematics achievement of students with SLD. The problem can be summarized by quote from Olson (2004) in an Education Week special issue on the state of special education in the U. S.:

> Although enormous strides have been made in special education over the past three decades, enormous gaps remain: in the performance of special education students compared with their peers', in understanding how best to assess what students with disabilities know and can do, and in the preparation of special and general education teachers to provide such students with full access to the general education curriculum. (p. 10)

Students with SLD are at risk for failure in mathematics. Students with SLD have lower achievement in mathematics and tend to take fewer mathematics courses than their peers. Yet, mathematics is needed for success in an increasingly competitive global economy. Special educators are uniquely positioned to support the mathematics learning of students with SLD. The present study illustrated the beliefs and attitudes that special education teachers hold about the discipline of mathematics, teaching mathematics, and learning mathematics. Given the influence of teacher beliefs and attitudes on instruction and ultimately on student learning outcomes, consideration should be given to leveraging beliefs and attitudes of special educators during pre-service training and through professional development for practicing teachers. By taking teacher beliefs into account through preparation and professional development, Ernest's (1989) framework would imply changes in teacher practice. So, to affect teacher practice and ultimately the achievement of students with SLD, addressing and challenging teacher beliefs related to mathematics, teaching mathematics, and learning mathematics may prove to be an effective lever for instructional improvement.

Ultimately teacher beliefs influence the mathematics instruction children receive. How do we want students to look back on their mathematics learning experiences? Clearly, the experience should not be that which Schuck (1996) described by undergraduates:

> They speak of their past experiences without enthusiasm, but are quite accepting of the fact that this is how mathematics has to be. Their perception is that mathematics *is* the learning of rules and formulas and the execution of a profusion of decontextualised exercises. These exercises provide, to their eyes, the unpleasant but necessary drill and practice that leads to success in mathematics. (p. 126)

Conversely, the attitude and motivation of students in mathematics can and should be positively influenced by the perspective of mathematics and the type of learning experiences provided by teachers, whether general or special education. As Middleton and Spanias (Middleton & Spanias, 1999) noted,

> Achievement motivation in mathematics is highly influenced by instructional practices, and if appropriate practices are consistent over a long period of time, children can and do learn to enjoy and value mathematics. (p. 82)

It is a national imperative that all students to have the mathematics skills and knowledge they need for success in life and the belief they have the capacity to succeed. It is incumbent on educators and policy makers to ensure this.

APPENDIX A

Recruitment Correspondence

Interested Participant Email Text

Dear [insert name],

Thank you for your interest to participate in the dissertation research study about the attitudes and beliefs that special educators hold about the discipline of mathematics. This research study will add to the literature in an area that has largely been ignored.

Participants who consent to join the research study will be asked to participate in phase one of the research study and may be invited to participate in phase two of the research study. Phase one consists of completing an online survey consisting of questions related to educational and professional teaching background, beliefs about mathematics and teaching mathematics, and attitudes about activities related to doing mathematics. The anticipated time required to complete the online survey is one-hour. A sub-sample of participants will be invited to participate in phase two of the research study, which will consist of a recorded structured interview. The structured interview may take place in person or over the telephone and is anticipated to take one-hour.

Following this email message, I will be sending a separate email which contains a link to the online survey.

Information about your participation is attached to this email. Please review the information and contact me with any questions or concerns at mlcolsman@gmail.com or 303-204-6263. Participants can choose not to continue participation at any time.

Again, thank you, and please let me know what questions I can answer.

Kind regards,

Melissa Colsman
Ph.D. Candidate, University of Colorado Denver
mcolsman@gmail.com
303-204-6263

Phase One Participant Email Text

Dear [insert name here],

Thank you for completing and submitting your consent form to participate in my dissertation research study. As I noted in my previous message, the research study consists of two parts. All subjects are asked to participate in phase one of the research study and may be invited to participate in phase two of the research study. Phase one consists of completing an online survey consisting of questions related to educational and professional teaching background, beliefs about mathematics and teaching mathematics, and attitudes about activities related to doing mathematics. The anticipated time required to complete the online survey is one-hour. A sub-sample of participants will be invited to participate in phase two of the research study, which will consist of a recorded structured interview. The structured interview may take place in person or over the telephone and is anticipated to take one-hour.

The link to the online survey is: [insert link here]. Please allow up to one-hour to complete the survey; however, it is anticipated that the actual time to complete will be much shorter. Please note that you can choose not to continue participation at any time.

Once analysis of the online survey data is complete, a sub-sample of the research study participants will be selected for invitations to participate in the second phase of the research study, which is anticipated to take place during the April to June 2012 timeframe. You will be contacted via email regarding the status of an invitation during this timeframe. Your participation in the second phase is entirely voluntary and you can choose to discontinue participation at any time.

Again, thank you, and please let me know what questions I can answer.

Kind regards,

Melissa Colsman
Ph.D. Candidate, University of Colorado Denver
mcolsman@gmail.com
303-204-6263

Phase Two Notification Email Text: Invitation

Dear [insert name here],

Thank you for participation in phase one of my dissertation research study. Your time completing the online survey is greatly appreciated.

As I noted in my previous messages, the research study consists of two parts. All subjects were asked to participate in phase one and a sub-sample of participants are invited to participate in phase two of the research study, which consists of a recorded structured interview.
The structured interview may take place in person or over the telephone and is anticipated to take one-hour.

Can you please reply with your willingness to participate in phase two of the research study and indicate the best way to reach you to schedule the structured interview?

Again, thank you, and please let me know what questions I can answer.

Kind regards,

Melissa Colsman
Ph.D. Candidate, University of Colorado Denver
mcolsman@gmail.com
303-204-6263

Phase Two Notification Email Text: Non-Invitation

Dear [insert name here],

Thank you for participation in phase one of my dissertation research study. Your time completing the online survey is greatly appreciated.

As I noted in my previous messages, the research study consists of two parts. All subjects were asked to participate in phase one and a sub-sample of participants are invited to participate in phase two of the research study, which consists of a recorded structured interview.

At this time, I do not plan to request your participation in phase two of the research study. I wish to thank you for your time and assistance in this research study.

Kind regards,

Melissa Colsman
Ph.D. Candidate, University of Colorado Denver
mcolsman@gmail.com
303-204-6263

APPENDIX B

Demographic Data Collection

1. First name, last name
2. Preferred email
3. Preferred email again
4. Secondary email
5. Secondary email again
6. Home phone number
7. Mobile phone number
8. Are you willing to participate in one-hour follow up phone or in-person interview?
9. Gender
    a. Female
    b. Male
10. Your highest degree:
    a. BA or BS
    b. MA or MS
    c. Multiple MA or MS
    d. PhD or EdD
    e. Other (Describe)
11. Degree major and minor
    a. BA or BS major(s)
    b. BA or BS minor(s)
    c. MA or MS major(s
    d. MA or MS minor(s)
    e. PhD or EdD emphasis
    f. Other (Describe)
12. Approximate number of credit hours of mathematics content courses included in undergraduate study
13. Approximate number of credit hours of mathematics content courses included in master's degree
14. Counting this year, how many years in total have you been teaching?
15. Counting this year, how many years in total have you taught or supported teaching mathematics?
16. Do you currently teach a pull-out mathematics for students with SLD? Please describe.
17. Do you currently support students with SLD in general education classes? Please describe.
18. Number of years (including the current year) where teaching assignment involved teaching or providing support in mathematics.
19. What level students do you teach? Check all that apply.
    a. Elementary
    b. Middle school

    c. High school

20. Ethnicity (check all that apply)
    a. African-American
    b. American Indian or Alaskan Native
    c. Asian
    d. Hispanic
    e. Pacific Islander
    f. White (not Hispanic origin)
    g. Other (Describe)
21. Which of these commonly held views about the nature of mathematics most accurately fits your perspective:
    a. Mathematics consists of rules and procedures to be memorized and practiced.
    b. Mathematics is a tool to use to solve problems and/or find solutions.
    c. Mathematics is a discipline of logic and reasoning.

APPENDIX C

Mathematics Anxiety Rating Scale: Short Version (MARS-SV)

The items in the questionnaire refer to things that may cause fear or apprehension. For each item decide which of the ratings best describes how much you are frightened by it nowadays - "Not at all" "A little" "A fair amount" "Much" or "Very much". Mark your answers on the answer sheet only. On the answer sheet, fill in "1" for Not at all; "2" for A little, "3" for A fair amount, "4" for Much or "5" for Very much.
Do not mark this question sheet. Work quickly but be sure to consider each item individually.

	Not at all	A little	A fair amount	Much	Very much
1. Taking an examination (final) in a math course.	☐	☐	☐	☐	☐
2. Thinking about an upcoming math test one week before.	☐	☐	☐	☐	☐
3. Thinking about an upcoming math test one day before.	☐	☐	☐	☐	☐
4. Thinking about an upcoming math test one hour before.	☐	☐	☐	☐	☐
5. Thinking about an upcoming math test five minutes before.	☐	☐	☐	☐	☐
6. Waiting to get a math test returned in which you expected to do well.	☐	☐	☐	☐	☐
7. Receiving your final math grade in the mail.	☐	☐	☐	☐	☐
8. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.	☐	☐	☐	☐	☐
9. Being given a "pop" quiz in a math class.	☐	☐	☐	☐	☐
10. Studying for a math test.	☐	☐	☐	☐	☐
11. Taking the math section of a college entrance exam.	☐	☐	☐	☐	☐
12. Taking an examination (quiz) in a math	☐	☐	☐	☐	☐

course.					
13. Picking up the math text book to begin working on a homework assignment.	☐	☐	☐	☐	☐
14. Being given a homework assignment of many difficult problems which is due the next class meeting.	☐	☐	☐	☐	☐
15. Getting ready to study for a math test.	☐	☐	☐	☐	☐
16. Dividing a five digit number by a two digit number in private with pencil and paper.	☐	☐	☐	☐	☐
17. Adding up 976 + 777 on paper.	☐	☐	☐	☐	☐
18. Reading a cash register receipt after your purchase.	☐	☐	☐	☐	☐
19. Figuring the sales tax on a purchase that costs more than $1.00.	☐	☐	☐	☐	☐
20. Figuring out your monthly budget.	☐	☐	☐	☐	☐
21. Being given a set of numerical problems involving addition to solve on paper.	☐	☐	☐	☐	☐
22. Having someone watch you as you total up a column of figures.	☐	☐	☐	☐	☐
23. Totaling up a dinner bill that you think overcharged you.	☐	☐	☐	☐	☐
24. Being responsible for collecting dues for an organization and keeping track of the amount.	☐	☐	☐	☐	☐
25. Studying for a driver's license test and memorizing the figure involved, such as the distance it takes to stop a car going at different speeds.	☐	☐	☐	☐	☐
26. Totaling up the dues received and the expenses of a club you belong to.	☐	☐	☐	☐	☐

27. Watching someone work with a calculator.	☐	☐	☐	☐	☐
28. Being given a set of division problems to solve.	☐	☐	☐	☐	☐
29. Being given a set of subtraction problems to solve.	☐	☐	☐	☐	☐
30. Being given a set of multiplication problems to solve. 	☐	☐	☐	☐	☐

APPENDIX D

Mathematics Beliefs Instrument

Appendix B

Mathematics Beliefs Instrument (MBI): Urban Alternative Preparation Program

Part A

"Please circle the response that most closely aligns with the NCTM Standards [illegible]"

1. Problem solving should be a SEPARATE, DISTINCT part of the mathematics curriculum.
   Agree Disagree

2. Students should share their problem-solving thinking and approaches WITH OTHER STUDENTS.
   Agree Disagree

3. Mathematics can be thought of as a language that must be MEANINGFUL if students are to communicate and apply mathematics productively.
   Agree Disagree

4. A major goal of mathematics instruction is to help children develop the belief that THEY HAVE THE POWER to control their own success in mathematics.
   Agree Disagree

5. Children should be encouraged to justify their solutions, thinking, and conclusions in a SINGLE way.
   Agree Disagree

6. The study of mathematics should include opportunities of using mathematics in OTHER CURRICULUM AREAS.
   Agree Disagree

7. The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in ISOLATION.
   Agree Disagree

8. In K-8 mathematics INCREASED emphasis should be given to reading and writing numbers SYMBOLICALLY.
   Agree Disagree

9. In K-8 mathematics, INCREASED emphasis should be given to use of CLUE WORDS (key words) to determine which operation to use in problem solving.
   Agree Disagree

10. In K-8 mathematics, skill in computation should PRECEDE word problems.
    Agree Disagree

11. Learning mathematics is a process in which students ABSORB INFORMATION, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.
    Agree Disagree

12. Mathematics SHOULD be taught as a COLLECTION of concepts, skills and algorithms.
    Agree Disagree

13. A demonstration of good reasoning should be rewarded EVEN MORE THAN students' ability to find correct answers.
    Agree Disagree

14. Appropriate calculators should be available to ALL STUDENTS at ALL TIMES.
    Agree Disagree

15. Learning mathematics must be an ACTIVE PROCESS.
    Agree Disagree

16. Children ENTER KINDERGARTEN with considerable mathematical experience, a partial understanding of many mathematical concepts and some important mathematical skills.
    Agree Disagree

Appendix B

Mathematics Beliefs Instrument (MBI): Urban Alternative Preparation Program

Part A

"Circle the response that most closely aligns with the NCTM *Standards* or intentions"

1. Problem solving should be a SEPARATE, DISTINCT part of the mathematics curriculum.
   Agree Disagree

2. Students should share their problem solving thinking and approaches WITH OTHER STUDENTS.
   Agree Disagree

3. Mathematics can be thought of as a language that must be MEANINGFUL if students are to communicate and apply mathematics productively.
   Agree Disagree

4. A major goal of mathematics instruction is to help children develop the belief that THEY HAVE THE POWER to control their own success in mathematics.
   Agree Disagree

5. Children should be encouraged to justify their solutions, thinking, and conjectures in a SINGLE way.
   Agree Disagree

6. The study of mathematics should include opportunities of using mathematics in OTHER CURRICULUM AREAS.
   Agree Disagree

7. The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in ISOLATION.
   Agree Disagree

8. In K-4 mathematics, INCREASED emphasis should be given to reading and writing numbers SYMBOLICALLY.
   Agree Disagree

9. In K-4 mathematics, INCREASED emphasis should be given to use of CLUE WORDS (key words) to determine which operation to use in problem solving.
   Agree Disagree

10. In K-4 mathematics, skill in computation should PRECEDE word problems.
    Agree Disagree

11. Learning mathematics is a process in which students ABSORB INFORMATION, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.
    Agree Disagree

12. Mathematics SHOULD be taught as a COLLECTION of concepts, skills and algorithms.
    Agree Disagree

13. A demonstration of good reasoning should be regarded EVEN MORE THAN students' ability to find correct answers.
    Agree Disagree

14. Appropriate calculators should be available to ALL STUDENTS at ALL TIMES.
    Agree Disagree

15. Learning mathematics must be an ACTIVE PROCESS.
    Agree Disagree

16. Children ENTER KINDERGARTEN with considerable mathematical experience, a partial understanding of many mathematical concepts and some important mathematical skills.
    Agree Disagree

APPENDIX E

Semi-structured Interview Protocol

1. How do you feel about mathematics?
2. What do you think when you hear the word mathematics?
3. In your survey, you indicated that the phrase(s) that best describe your view of the nature of mathematics to be: [insert here]. Can you tell me more about this?
4. How do you rate yourself in terms of anxiety about mathematics, with 1 being very comfortable and 10 being very anxious?
5. How do you rate yourself in terms of attitude about mathematics, with 1 being very unpleasant and 10 being very enjoyable?
6. What do you think contributed to your attitude toward mathematics?
7. What do you think about the way that you have been taught mathematics?
8. What do you remember best about learning mathematics in school?
9. Describe your worst experience in a mathematics class during your entire school career. Describe factors that would have made these experiences more positive.
10. Please describe an ideal mathematics classroom.
11. What is most important about teaching mathematics?
12. What is the role of the teacher in learning mathematics?
13. What is the role of students in learning mathematics?
14. How do children learn mathematics?

15. What accounts for the differences between good and poor mathematics students? What can good mathematics students do that students who struggle in mathematics do not?
16. Is there anything inherent about students with learning disabilities that influences how they learn mathematics? If so, what?
17. Two common service delivery systems are used to support students with specific learning disabilities: pullout math classes or support in a general education classrooms. In which model do you think students with specific learning disabilities learn mathematics best?
18. What factors do you consider to have had a significant influence on your beliefs about mathematics teaching?
19. To what degree has your experience teaching mathematics influenced your beliefs and attitudes about mathematics?
20. Do you think that your attitudes and beliefs about mathematics play out in your teaching? If so, how?

APPENDIX F

Compiled Phase One Data Collection

22. First name, last name
23. Preferred email
24. Preferred email again
25. Secondary email
26. Secondary email again
27. Home phone number
28. Mobile phone number
29. Are you willing to participate in one-hour follow up phone or in-person interview?
30. Gender
    a. Female
    b. Male
31. Your highest degree:
    a. BA or BS
    b. MA or MS
    c. Multiple MA or MS
    d. PhD or EdD
    e. Other (Describe)
32. Degree major and minor
    a. BA or BS major(s)
    b. BA or BS minor(s)
    c. MA or MS major(s
    d. MA or MS minor(s)
    e. PhD or EdD emphasis
    f. Other (Describe)
33. Approximate number of credit hours of mathematics content courses included in undergraduate study
34. Approximate number of credit hours of mathematics content courses included in master's degree
35. Counting this year, how many years in total have you been teaching?
36. Counting this year, how many years in total have you taught or supported teaching mathematics?
37. Do you currently teach a pull-out mathematics for students with SLD? Please describe.
38. Do you currently support students with SLD in general education classes? Please describe.
39. Number of years (including the current year) where teaching assignment involved teaching or providing support in mathematics.
40. What level students do you teach? Check all that apply.
    a. Elementary
    b. Middle school

c. High school

41. Ethnicity (check all that apply)
    a. African-American
    b. American Indian or Alaskan Native
    c. Asian
    d. Hispanic
    e. Pacific Islander
    f. White (not Hispanic origin)
    g. Other (Describe)
42. Which of these commonly held views about the nature of mathematics most accurately fits your perspective:
    a. Mathematics consists of rules and procedures to be memorized and practiced.
    b. Mathematics is a tool to use to solve problems and/or find solutions.
    c. Mathematics is a discipline of logic and reasoning.

Questions Related to Beliefs about Mathematics, Teaching Mathematics, and Learning Mathematics
(from the *Mathematics Beliefs Instrument*)

This portion of the questionnaire relates to your ideas about mathematics. Your answers to the questions that follow will help me to understand what you think mathematics is all about.

43. Problem solving should be a separate, distinct part of the mathematics curriculum.
    a. Agree
    b. Disagree
44. Students should share their problem-solving thinking and approaches with other students.
    c. Agree
    d. Disagree
45. Mathematics can be thought of as a language that must be meaningful if students are to communicate and apply mathematics productively.
    a. Agree
    b. Disagree
46. A major goal of mathematics instruction is to help children develop the belief that they have the power to control their own success in mathematics.
    a. Agree
    b. Disagree
47. Children should be encouraged to justify their solutions, thinking, and conjectures in a single way.
    a. Agree
    b. Disagree
48. The study of mathematics should include opportunities of using mathematics in other curriculum areas.
    a. Agree
    b. Disagree

49. The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation.
    a. Agree
    b. Disagree
50. In K-5 mathematics, increased emphasis should be given to reading and writing numbers symbolically.
    a. Agree
    b. Disagree
51. In K-5 mathematics, increased emphasis should be given to use of clue words (key words) to determine which operation to use in problem solving.
    a. Agree
    b. Disagree
52. In K-5 mathematics, skill in computation should precede word problems.
    a. Agree
    b. Disagree
53. Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.
    a. Agree
    b. Disagree
54. Mathematics should be taught as a collection of concepts, skills and algorithms.
    a. Agree
    b. Disagree
55. A demonstration of good reasoning should be regarded even more than students' ability to find correct answers.
    a. Agree
    b. Disagree
56. Appropriate calculators should be available to all students at all times.
    a. Agree
    b. Disagree
57. Learning mathematics must be an active process.
    a. Agree
    b. Disagree
58. Children enter kindergarten with considerable mathematical experience, a partial understanding of many mathematical concepts, and some important mathematical skills.
    a. Agree
    b. Disagree
59. Some people are good at mathematics and some aren't.
    a. True
    b. More true than false
    c. More false than true
    d. False
60. In mathematics something is either right or it is wrong.
    a. True
    b. More true than false
    c. More false than true

    d. False
61. Good mathematics teachers show students lots of different ways to look at the same question.
    a. True
    b. More true than false
    c. More false than true
    d. False
62. Good math teachers show you the exact way to answer the math question you will be tested on.
    a. True
    b. More true than false
    c. More false than true
    d. False
63. Everything important about mathematics is already known by mathematicians.
    a. True
    b. More true than false
    c. More false than true
    d. False
64. In mathematics you can be creative and discover things by yourself.
    a. True
    b. More true than false
    c. More false than true
    d. False
65. Math problems can be done correctly in only one way.
    a. True
    b. More true than false
    c. More false than true
    d. False
66. To solve most math problems you have to be taught the correct procedure.
    a. True
    b. More true than false
    c. More false than true
    d. False
67. The best way to do well in math is to memorize all the formulas.
    a. True
    b. More true than false
    c. More false than true
    d. False
68. Males are better at math than females.
    a. True
    b. More true than false
    c. More false than true
    d. False
69. Some ethnic groups are better at math than others.
    a. True
    b. More true than false

c. More false than true
d. False

70. To be good in math you must be able to solve problems quickly.
   a. True
   b. More true than false
   c. More false than true
   d. False

Questions Related to Attitudes Towards Mathematics
(from the *Mathematics Anxiety Rating Scale: Short Version*)

The items in this portion of the questionnaire refer to things that may cause fear or apprehension. For each item decide which of the ratings best describes how much you are frightened by it nowadays - "Not at all" "A little" "A fair amount" "Much" or "Very much". Work quickly but be sure to consider each item individually.

71. Taking an examination (final) in a math course.
   a. Not at all
   b. A little
   c. A fair amount
   d. Much
   e. Very much
72. Thinking about an upcoming math test one week before.
   a. Not at all
   b. A little
   c. A fair amount
   d. Much
   e. Very much
73. Thinking about an upcoming math test one day before.
   a. Not at all
   b. A little
   c. A fair amount
   d. Much
   e. Very much
74. Thinking about an upcoming math test one hour before.
   a. Not at all
   b. A little
   c. A fair amount
   d. Much
   e. Very much
75. Thinking about an upcoming math test five minutes before.
   a. Not at all
   b. A little
   c. A fair amount
   d. Much
   e. Very much
76. Waiting to get a math test returned in which you expected to do well.
   a. Not at all

b. A little
c. A fair amount
d. Much
e. Very much

77. Receiving your final math grade in the mail.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
78. Realizing that you have to take a certain number of math classes to fulfill the requirements of your major.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
79. Being given a "pop" quiz in a math class.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
80. Studying for a math test.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
81. Taking the math section of a college entrance exam.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
82. Taking an examination (quiz) in a math course.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
83. Picking up the math text book to begin working on a homework assignment.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much

e. Very much

84. Being given a homework assignment of many difficult problems which is due the next class meeting.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
85. Getting ready to study for a math test.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
86. Dividing a five digit number by a two digit number in private with a pencil and paper.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
87. Adding up 976 + 777 on paper.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
88. Reading a cash receipt after your purchase.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
89. Figuring the sales tax on a purchase that costs more than $1.00.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
90. Figuring out your monthly budget.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
91. Being given a set of numerical problems involving addition to solve on paper.

a. Not at all
b. A little
c. A fair amount
d. Much
e. Very much

92. Having someone watch you as you total up a column of figures.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
93. Totaling up a dinner bill you think overcharged you.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
94. Being responsible for collecting dues for an organization and keeping track of the amount.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
95. Studying for a driver's license test and memorizing the figures involved, such as the distances it takes to stop a car at different speeds.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
96. Totaling up the dues received and expenses of a club you belong to.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
97. Watching someone work with a calculator.
    a. Not at all
    b. A little
    c. A fair amount
    d. Much
    e. Very much
98. Being given a set of division problems to solve.
    a. Not at all
    b. A little

c. A fair amount
d. Much
e. Very much

99. Being given a set of subtraction problems to solve.
a. Not at all
b. A little
c. A fair amount
d. Much
e. Very much

100. Being given a set of multiplication problems to solve.
a. Not at all
b. A little
c. A fair amount
d. Much
e. Very much

APPENDIX G

Listing of Start and Emergent Codes for Qualitative Analysis

Table 1 Start Codes Based on Conceptual Framework and Research Question

Code	Meaning
ATTITUDE	Data related to participant's attitude toward mathematics
LEARN MATH	Data related to participant's beliefs about learning mathematics
NATURE	Data related to participant's beliefs about the nature of mathematics
ROLE OF STUDENT	Data related to participant's beliefs about the role of the student in learning mathematics
ROLE OF TEACHER	Data related to participant's beliefs about the role of the teacher in learning mathematics
TEACH MATH	Data related to participant's beliefs about teaching mathematics

Table 2 Emergent Codes Based on Common Themes Across Sub-sample

Code	Meaning
ATTRIBUTION	Data related to participant's beliefs about the attribution of mathematics learning success
CHANGE	Data related to a change in participant's attitudes or beliefs
CONTRAST	Data related to contrasts the participant drew between their learning experience and current mathematics practices
IMPACT	Data related to impact of teaching on participant's attitude or beliefs
KEYWORD	Data denoting a keyword for the participant
MLE	Data related to participant's mathematics learning experience
SCHOOL/INFORMAL MATH	Data related to participant's distinction between school mathematics and informal mathematics
SELF-REFERENT	Data related to participant's self-reference to personal learning biography

APPENDIX H

Mathematics Anxiety Rating Scale: Short Version

Raw Score and Percentile Equivalents

Percentile	***MARS-SV* Raw Score**
5 %	34
10 %	37
20 %	43
25 %	46
35 %	51
40 %	55
50 %	59
60 %	65
70 %	74
75 %	78
80 %	84
85 %	90
90 %	97
95 %	108
99 %	120

APPENDIX I

Mathematics Beliefs Instrument Codes

Code	*MBI* Statement
ProbSolv	Problem solving should be a separate, distinct part of the mathematics curriculum.
ShareThink	Students should share their problem-solving thinking and approaches with other students.
MathLang	Mathematics can be thought of as a language that must be meaningful if students are to communicate and apply mathematics productively.
GoalPower	A major goal of mathematics instruction is to help children develop the belief that they have the power to control their own success in mathematics.
JustSoln	Children should be encouraged to justify their solutions, thinking, and conjectures in a single way.
MathCurric	The study of mathematics should include opportunities of using mathematics in other curriculum areas.
MathStrands	The mathematics curriculum consists of several discrete strands such as computation, geometry, and measurement which can best be taught in isolation.
ElemSym	In K-5 mathematics, increased emphasis should be given to reading and writing numbers symbolically.
ClueWords	In K-5 mathematics, increased emphasis should be In K-5 mathematics, increased emphasis should be given to use of clue words (key words) to determine which operation to use in problem solving. given to reading and writing numbers symbolically.
CompPrec	In K-5 mathematics, skill in computation should precede word problems.
LearnAbsorb	Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.
MathCollec	Mathematics should be taught as a collection of concepts, skills, and

Code	*MBI* Statement
	algorithms.
Reasoning	A demonstration of good reasoning should be regarded even more than students' ability to find correct answers.
Calc	Appropriate calculators should be available to all students at all times.
Active	Learning mathematics must be an active process.
EnterK	Children enter kindergarten with considerable mathematical experience, a partial understanding of many mathematical concepts, and some important mathematical skills.
GoodNot	Some people are good at mathematics and some aren't.
RightWrong	In mathematics something is either right or it is wrong.
ShowMany	Good mathematics teachers show students lots of different ways to look at the same question.
ShowExact	Good math teachers show you the exact way to answer the math question you will be tested on.
MathKnown	Everything important about mathematics is already known by mathematicians.
DiscoverSelf	In mathematics you can be creative and discover things by yourself.
CorrectOne	Math problems can be done correctly in only one way.
TaughtProc	To solve most math problems you have to be taught the correct procedure.
Memorize	The best way to do well in math is to memorize all the formulas.
MalesBetter	Males are better at math than females.
EthnicBetter	Some ethnic groups are better at math than others.
SolveQuickly	To be good in math you must be able to solve problems quickly.

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