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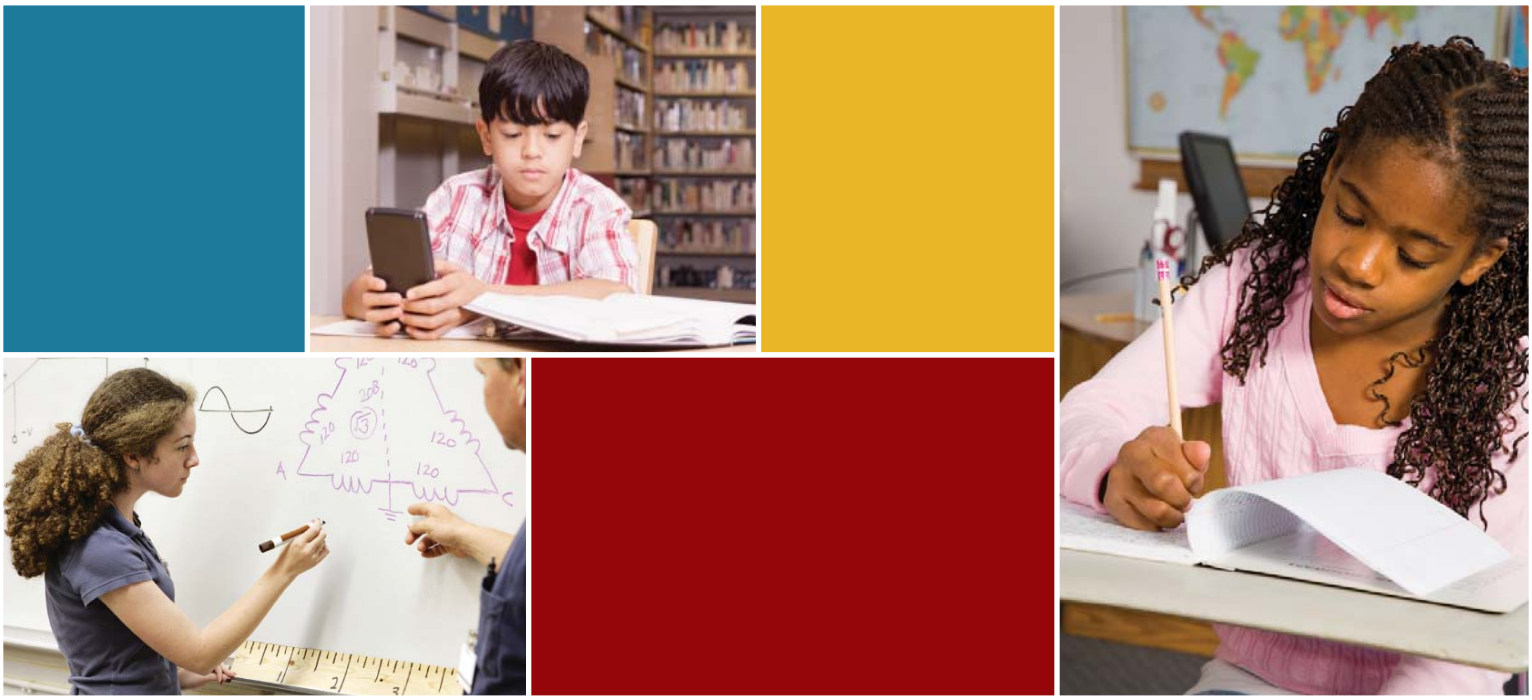
COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Colorado Academic STANDARDS

Seventh Grade

Mathematics

Mathematics



Colorado Academic Standards in Mathematics and The Common Core State Standards for Mathematics

On December 10, 2009, the Colorado State Board of Education adopted the revised Mathematics Academic Standards, along with academic standards in nine other content areas, creating Colorado's first fully aligned preschool through high school academic expectations. Developed by a broad spectrum of Coloradans representing Pre-K and K-12 education, higher education, and business, utilizing the best national and international exemplars, the intention of these standards is to prepare Colorado schoolchildren for achievement at each grade level, and ultimately, for successful performance in postsecondary institutions and/or the workforce.

Concurrent to the revision of the Colorado standards was the Common Core State Standards (CCSS) initiative, whose process and purpose significantly overlapped with that of the Colorado Academic Standards. Led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), these standards present a national perspective on academic expectations for students, Kindergarten through High School in the United States.

Upon the release of the Common Core State Standards for Mathematics on June 2, 2010, the Colorado Department of Education began a gap analysis process to determine the degree to which the expectations of the Colorado Academic Standards aligned with the Common Core. The independent analysis proved a nearly 95% alignment between the two sets of standards. On August 2, 2010, the Colorado State Board of Education adopted the Common Core State Standards, and requested the integration of the Common Core State Standards and the Colorado Academic Standards.

In partnership with the dedicated members of the Colorado Standards Revision Subcommittee in Mathematics, this document represents the integration of the combined academic content of both sets of standards, maintaining the unique aspects of the Colorado Academic Standards, which include personal financial literacy, 21st century skills, school readiness competencies, postsecondary and workforce readiness competencies, and preschool expectations. The result is a world-class set of standards that are greater than the sum of their parts.

The Colorado Department of Education encourages you to review the Common Core State Standards and the extensive appendices at www.corestandards.org. While all the expectations of the Common Core State Standards are embedded and **coded with CCSS:** in this document, additional information on the development and the intentions behind the Common Core State Standards can be found on the website.

Colorado Academic Standards
Mathematics Standards

"Pure mathematics is, in its way, the poetry of logical ideas."
Albert Einstein

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"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."
Dr. Michael Brown, Nobel Prize Laureate

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In the 21st century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how 21st century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The "Continuum of State Standards Definitions" section that follows shows the hierarchical order of the standards components. The "Standards Template" section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

Prepared Graduate Competencies: The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Standard: The topical organization of an academic content area.

High School Expectations: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. *What do students need to know in high school?*

Grade Level Expectations: The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

Evidence Outcomes: The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

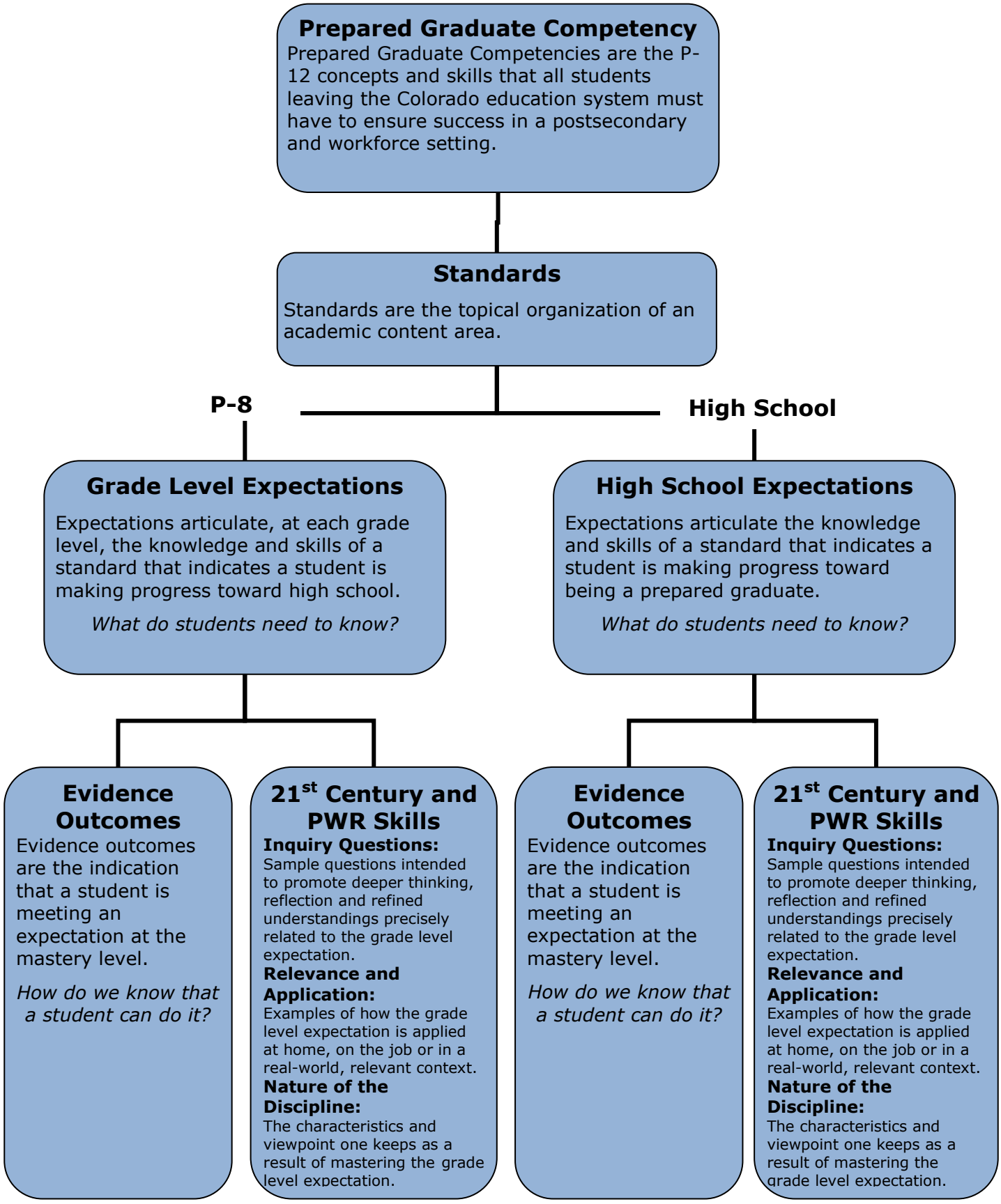
21st Century Skills and Readiness Competencies: Includes the following:

- ***Inquiry Questions:***
  Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

- ***Relevance and Application:***
  Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

- ***Nature of the Discipline:***
  The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Continuum of State Standards Definitions



STANDARDS TEMPLATE

Content Area: NAME OF CONTENT AREA

Standard: The topical organization of an academic content area.

Prepared Graduates:

- The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

High School and Grade Level Expectations

Concepts and skills students master:

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

What do students need to know?

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: Evidence outcomes are the indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*	**Inquiry Questions:** Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
	Relevance and Application: Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
	Nature of the Discipline: The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Colorado Academic Standards
Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Modeling Across the Standards
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Standards for Mathematical Practice
from
The Common Core State Standards for Mathematics

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions,
explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x^2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or

as being composed of several objects. For example, they can see 5 – 3(x – y)^2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x^2 + x + 1), and (x – 1)(x^3 + x^2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Seventh Grade	
1. Number Sense, Properties, and Operations	1. Proportional reasoning involves comparisons and multiplicative relationships among ratios 2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently
2. Patterns, Functions, and Algebraic Structures	1. Properties of arithmetic can be used to generate equivalent expressions 2. Equations and expressions model quantitative relationships and phenomena
3. Data Analysis, Statistics, and Probability	1. Statistics can be used to gain information about populations by examining samples 2. Mathematical models are used to determine probability
4. Shape, Dimension, and Geometric Relationships	1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof 2. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure

From the Common State Standards for Mathematics, Page 46.

Mathematics | Grade 7
In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve
a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures

using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

21st Century Skills and Readiness Competencies in Mathematics

Mathematics in Colorado's description of 21st century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21st century skills, as follows:

Critical Thinking and Reasoning
Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

Information Literacy
The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

Collaboration
Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

Self-Direction
Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

Invention
Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

Colorado's Description for School Readiness

(Adopted by the State Board of Education, December 2008)

School readiness describes both the preparedness of a child to engage in and benefit from learning experiences, and the ability of a school to meet the needs of all students enrolled in publicly funded preschools or kindergartens. School readiness is enhanced when schools, families, and community service providers work collaboratively to ensure that every child is ready for higher levels of learning in academic content.

Colorado's Description of Postsecondary and Workforce Readiness

(Adopted by the State Board of Education, June 2009)

Postsecondary and workforce readiness describes the knowledge, skills, and behaviors essential for high school graduates to be prepared to enter college and the workforce and to compete in the global economy. The description assumes students have developed consistent intellectual growth throughout their high school career as a result of academic work that is increasingly challenging, engaging, and coherent. Postsecondary education and workforce readiness assumes that students are ready and able to demonstrate the following without the need for remediation: Critical thinking and problem-solving; finding and using information/information technology; creativity and innovation; global and cultural awareness; civic responsibility; work ethic; personal responsibility; communication; and collaboration.

How These Skills and Competencies are Embedded in the Revised Standards

Three themes are used to describe these important skills and competencies and are interwoven throughout the standards: *inquiry questions; relevance and application; and the nature of each discipline.* These competencies should not be thought of stand-alone concepts, but should be integrated throughout the curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without the content to think about, it is equally impossible for students to understand the content of a discipline without grappling with complex questions and the investigation of topics.

Inquiry Questions – Inquiry is a multifaceted process requiring students to think and pursue understanding. Inquiry demands that students (a) engage in an active observation and questioning process; (b) investigate to gather evidence; (c) formulate explanations based on evidence; (d) communicate and justify explanations, and; (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

Relevance and Application – The hallmark of learning a discipline is the ability to apply the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, developing, adapting, and refining solutions for the betterment of society. The application of a discipline, including how technology assists or accelerates the work, enables students to more fully appreciate how the mastery of the grade level expectation matters after formal schooling is complete.

Nature of Discipline – The unique advantage of a discipline is the perspective it gives the mind to see the world and situations differently. The characteristics and viewpoint one keeps as a result of mastering the grade level expectation is the nature of the discipline retained in the mind's eye.

1.Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Proportional reasoning involves comparisons and multiplicative relationships among ratios

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Analyze proportional relationships and use them to solve real-world and mathematical problems.(CCSS: 7.RP) b. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.[1] (CCSS: 7.RP.1) c. Identify and represent proportional relationships between quantities. (CCSS: 7.RP.2) i. Determine whether two quantities are in a proportional relationship.[2] (CCSS: 7.RP.2a) ii. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS: 7.RP.2b) iii. Represent proportional relationships by equations.[3] (CCSS: 7.RP.2c) iv. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. (CCSS: 7.RP.2d) d. Use proportional relationships to solve multistep ratio and percent problems.[4] (CCSS: 7.RP.3) i. Estimate and compute unit cost of consumables (to include unit conversions if necessary) sold in quantity to make purchase decisions based on cost and practicality (PFL) ii. Solve problems involving percent of a number, discounts, taxes, simple interest, percent increase, and percent decrease (PFL)	**Inquiry Questions:** 1. What information can be determined from a relative comparison that cannot be determined from an absolute comparison? 2. What comparisons can be made using ratios? 3. How do you know when a proportional relationship exists? 4. How can proportion be used to argue fairness? 5. When is it better to use an absolute comparison? 6. When is it better to use a relative comparison? **Relevance and Application:** 1. The use of ratios, rates, and proportions allows sound decision-making in daily life such as determining best values when shopping, mixing cement or paint, adjusting recipes, calculating car mileage, using speed to determine travel time, or enlarging or shrinking copies. 2. Proportional reasoning is used extensively in the workplace. For example, determine dosages for medicine; develop scale models and drawings; adjusting salaries and benefits; or prepare mixtures in laboratories. 3. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature of Mathematics:** 1. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Apply understandings of addition and subtraction to add and subtract rational numbers including integers. (CCSS: 7.NS.1) i. Represent addition and subtraction on a horizontal or vertical number line diagram. (CCSS: 7.NS.1) ii. Describe situations in which opposite quantities combine to make 0.[5] (CCSS: 7.NS.1a) iii. Demonstrate $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. (CCSS: 7.NS.1b) iv. Show that a number and its opposite have a sum of 0 (are additive inverses). (CCSS: 7.NS.1b) v. Interpret sums of rational numbers by describing real-world contexts. (CCSS: 7.NS.1c) vi. Demonstrate subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. (CCSS: 7.NS.1c) vii. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (CCSS: 7.NS.1c) viii. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.1d) b. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers including integers. (CCSS: 7.NS.2) i. Apply properties of operations to multiplication of rational numbers.[6] (CCSS: 7.NS.2a) ii. Interpret products of rational numbers by describing real-world contexts. (CCSS: 7.NS.2a) iii. Apply properties of operations to divide integers.[7] (CCSS: 7.NS.2b) iv. Apply properties of operations as strategies to multiply and divide rational numbers. (CCSS: 7.NS.2c) v. Convert a rational number to a decimal using long division. (CCSS: 7.NS.2d) vi. Show that the decimal form of a rational number terminates in 0s or eventually repeats. (CCSS: 7.NS.2d) c. Solve real-world and mathematical problems involving the four operations with rational numbers.[8] (CCSS: 7.NS.3)	**Inquiry Questions:** 1. How do operations with rational numbers compare to operations with integers? 2. How do you know if a computational strategy is sensible? 3. Is $0.\overline{9}$ equal to one? 4. How do you know whether a fraction can be represented as a repeating or terminating decimal? **Relevance and Application:** 1. The use and understanding algorithms help individuals spend money wisely. For example, compare discounts to determine best buys and compute sales tax. 2. Estimation with rational numbers enables individuals to make decisions quickly and flexibly in daily life such as estimating a total bill at a restaurant, the amount of money left on a gift card, and price markups and markdowns. 3. People use percentages to represent quantities in real-world situations such as amount and types of taxes paid, increases or decreases in population, and changes in company profits or worker wages). **Nature of Mathematics:** 1. Mathematicians see algorithms as familiar tools in a tool chest. They combine algorithms in different ways and use them flexibly to accomplish various tasks. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians look for and make use of structure. (MP)

[1] For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour. (CCSS: 7.RP.1)
[2] e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS: 7.RP.2a)
[3] For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. (CCSS: 7.RP.2c)
[4] Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. (CCSS: 7.RP.3)
[5] For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. (CCSS: 7.NS.1a)
[6] Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. (CCSS: 7.NS.2a)
[7] Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. (CCSS: 7.NS.2b)
Interpret quotients of rational numbers by describing real-world contexts. (CCSS: 7.NS.2b)
[8] Computations with rational numbers extend the rules for manipulating fractions to complex fractions. (CCSS: 7.NS.3)

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Properties of arithmetic can be used to generate equivalent expressions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use properties of operations to generate equivalent expressions. (CCSS: 7.EE) i. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (CCSS: 7.EE.1) ii. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.[1] (CCSS: 7.EE.2)	**Inquiry Questions:** 1. How do symbolic transformations affect an equation or expression? 2. How is it determined that two algebraic expressions are equivalent? **Relevance and Application:** 1. The ability to recognize and find equivalent forms of an equation allows the transformation of equations into the most useful form such as adjusting the density formula to calculate for volume or mass. **Nature of Mathematics:** 1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians look for and express regularity in repeated reasoning. (MP)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Seventh Grade

Concepts and skills students master:
2. Equations and expressions model quantitative relationships and phenomena

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form,[2] using tools strategically. (CCSS: 7.EE.3) b. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies.[3] (CCSS: 7.EE.3) c. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (CCSS: 7.EE.4) i. Fluently solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. (CCSS: 7.EE.4a) ii. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.[4] (CCSS: 7.EE.4a) iii. Solve word problems[5] leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. (CCSS: 7.EE.4b) iv. Graph the solution set of the inequality and interpret it in the context of the problem. (CCSS: 7.EE.4b)	**Inquiry Questions:** 1. Do algebraic properties work with numbers or just symbols? Why? 2. Why are there different ways to solve equations? 3. How are properties applied in other fields of study? 4. Why might estimation be better than an exact answer? 5. When might an estimate be the only possible answer? **Relevance and Application:** 1. Procedural fluency with algebraic methods allows use of linear equations and inequalities to solve problems in fields such as banking, engineering, and insurance. For example, it helps to calculate the total value of assets or find the acceleration of an object moving at a linearly increasing speed. 2. Comprehension of the structure of equations allows one to use spreadsheets effectively to solve problems that matter such as showing how long it takes to pay off debt, or representing data collected from science experiments. 3. Estimation with rational numbers enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, how many floor tiles to buy with given dimensions, the amount of carpeting needed for a room, or fencing required for a backyard. **Nature of Mathematics:** 1. Mathematicians model with mathematics. (MP)

Standard: 2. Patterns, Functions, and Algebraic Structures
Seventh Grade

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[1] For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05." (CCSS: 7.EE.2)
[2] whole numbers, fractions, and decimals. (CCSS: 7.EE.3)
[3] For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. (CCSS: 7.EE.3)
[4] For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? (CCSS: 7.EE.4a)
[5] For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions. (CCSS: 7.EE.4b)

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Statistics can be used to gain information about populations by examining samples

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use random sampling to draw inferences about a population. (CCSS: 7.SP) i. Explain that generalizations about a population from a sample are valid only if the sample is representative of that population. (CCSS: 7.SP.1) ii. Explain that random sampling tends to produce representative samples and support valid inferences. (CCSS: 7.SP.1) iii. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. (CCSS: 7.SP.2) iv. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.[1] (CCSS: 7.SP.2) b. Draw informal comparative inferences about two populations. (CCSS: 7.SP) i. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.[2] (CCSS: 7.SP.3) ii. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.[3] (CCSS: 7.SP.4)	**Inquiry Questions:** 1. How might the sample for a survey affect the results of the survey? 2. How do you distinguish between random and bias samples? 3. How can you declare a winner in an election before counting all the ballots? **Relevance and Application:** 1. The ability to recognize how data can be biased or misrepresented allows critical evaluation of claims and avoids being misled. For example, data can be used to evaluate products that promise effectiveness or show strong opinions. 2. Mathematical inferences allow us to make reliable predictions without accounting for every piece of data. **Nature of Mathematics:** 1. Mathematicians are informed consumers of information. They evaluate the quality of data before using it to make decisions. 2. Mathematicians use appropriate tools strategically. (MP)

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Mathematical models are used to determine probability

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.[4] (CCSS: 7.SP.5) b. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.[5] (CCSS: 7.SP.6) c. Develop a probability model and use it to find probabilities of events. (CCSS: 7.SP.7) i. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (CCSS: 7.SP.7) ii. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.[6] (CCSS: 7.SP.7a) iii. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.[7] (CCSS: 7.SP.7b) d. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (CCSS: 7.SP.8) i. Explain that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (CCSS: 7.SP.8a) ii. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. (CCSS: 7.SP.8b) iii. For an event[8] described in everyday language identify the outcomes in the sample space which compose the event. (CCSS: 7.SP.8b) iv. Design and use a simulation to generate frequencies for compound events.[9] (CCSS: 7.SP.8c)	**Inquiry Questions:** 1. Why is it important to consider all of the possible outcomes of an event? 2. Is it possible to predict the future? How? 3. What are situations in which probability cannot be used? **Relevance and Application:** 1. The ability to efficiently and accurately count outcomes allows systemic analysis of such situations as trying all possible combinations when you forgot the combination to your lock or deciding to find a different approach when there are too many combinations to try; or counting how many lottery tickets you would have to buy to play every possible combination of numbers. 2. The knowledge of theoretical probability allows the development of winning strategies in games involving chance such as knowing if your hand is likely to be the best hand or is likely to improve in a game of cards. **Nature of Mathematics:** 1. Mathematicians approach problems systematically. When the number of possible outcomes is small, each outcome can be considered individually. When the number of outcomes is large, a mathematician will develop a strategy to consider the most important outcomes such as the most likely outcomes, or the most dangerous outcomes. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP)

Standard: 3. Data Analysis, Statistics, and Probability
Seventh Grade

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[1] For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. (CCSS: 7.SP.2)
[2] For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. (CCSS: 7.SP.3)
[3] For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (CCSS: 7.SP.4)
[4] Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS: 7.SP.5)
[5] For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. (CCSS: 7.SP.6)
[6] For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. (CCSS: 7.SP.7a)
[7] For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? (CCSS: 7.SP.7b)
[8] e.g., "rolling double sixes" (CCSS: 7.SP.8b)
[9] For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? (CCSS: 7.SP.8c)

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Draw construct, and describe geometrical figures and describe the relationships between them. (CCSS: 7.G) i. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS: 7.G.1) ii. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. (CCSS: 7.G.2) iii. Construct triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS: 7.G.2) iv. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. (CCSS: 7.G.3)	**Inquiry Questions:** 1. Is there a geometric figure for any given set of attributes? 2. How does scale factor affect length, perimeter, angle measure, area and volume? 3. How do you know when a proportional relationship exists? **Relevance and Application:** 1. The understanding of basic geometric relationships helps to use geometry to construct useful models of physical situations such as blueprints for construction, or maps for geography. 2. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature of Mathematics:** 1. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning. 2. The relationship between geometric figures can be modeled 3. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings. 4. Mathematicians use appropriate tools strategically. (MP) 5. Mathematicians attend to precision. (MP)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. State the formulas for the area and circumference of a circle and use them to solve problems. (CCSS: 7.G.4) b. Give an informal derivation of the relationship between the circumference and area of a circle. (CCSS: 7.G.4) c. Use properties of supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (CCSS: 7.G.5) d. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS: 7.G.6)	**Inquiry Questions:** 1. How can geometric relationships among lines and angles be generalized, described, and quantified? 2. How do line relationships affect angle relationships? 3. Can two shapes have the same volume but different surface areas? Why? 4. Can two shapes have the same surface area but different volumes? Why? 5. How are surface area and volume like and unlike each other? 6. What do surface area and volume tell about an object? 7. How are one-, two-, and three-dimensional units of measure related? 8. Why is pi an important number?
	Relevance and Application: 1. The ability to find volume and surface area helps to answer important questions such as how to minimize waste by redesigning packaging, or understanding how the shape of a room affects its energy use.
	Nature of Mathematics: 1. Geometric objects are abstracted and simplified versions of physical objects. 2. Geometers describe what is true about all cases by studying the most basic and essential aspects of objects and relationships between objects. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

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