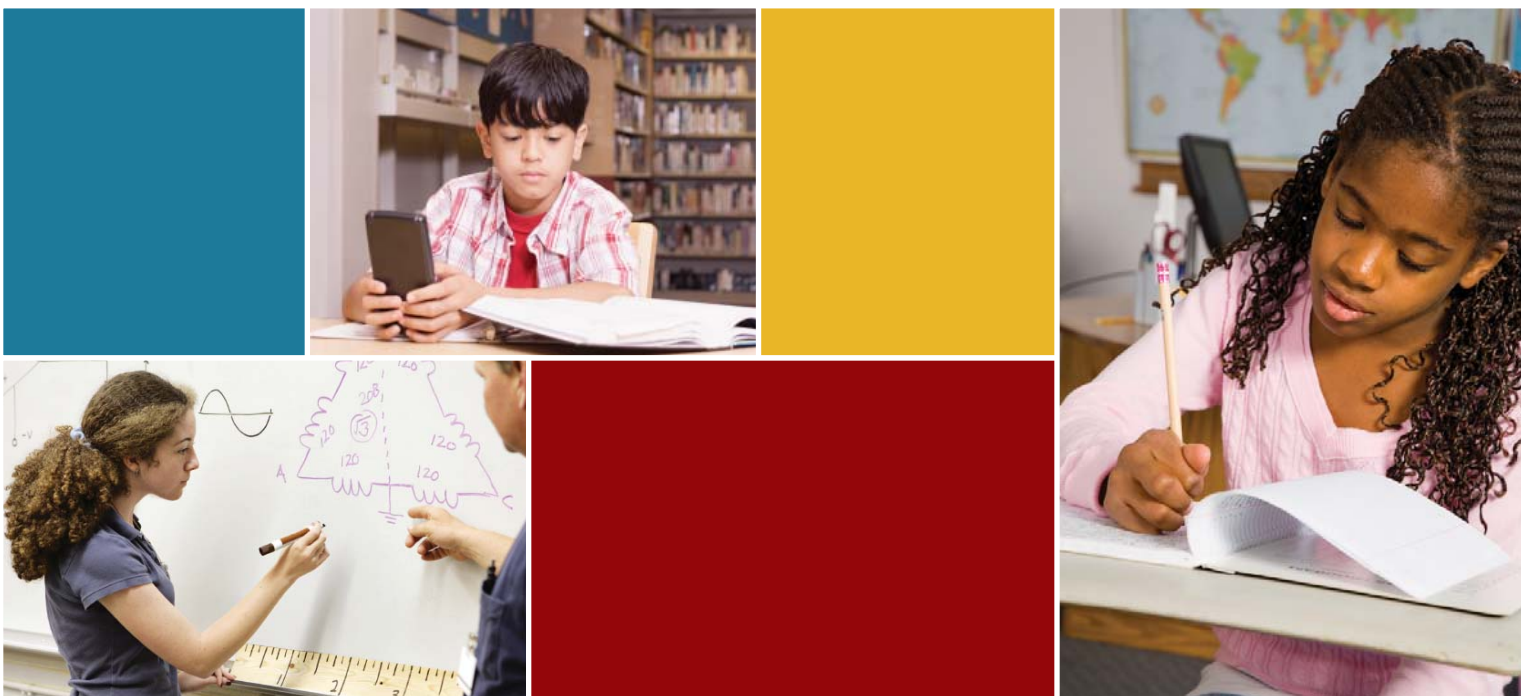


Fourth Grade

Mathematics





## Colorado Academic Standards in Mathematics and The Common Core State Standards for Mathematics

On December 10, 2009, the Colorado State Board of Education adopted the revised Mathematics Academic Standards, along with academic standards in nine other content areas, creating Colorado's first fully aligned preschool through high school academic expectations. Developed by a broad spectrum of Coloradans representing Pre-K and K-12 education, higher education, and business, utilizing the best national and international exemplars, the intention of these standards is to prepare Colorado schoolchildren for achievement at each grade level, and ultimately, for successful performance in postsecondary institutions and/or the workforce.

Concurrent to the revision of the Colorado standards was the Common Core State Standards (CCSS) initiative, whose process and purpose significantly overlapped with that of the Colorado Academic Standards. Led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), these standards present a national perspective on academic expectations for students, Kindergarten through High School in the United States.

Upon the release of the Common Core State Standards for Mathematics on June 2, 2010, the Colorado Department of Education began a gap analysis process to determine the degree to which the expectations of the Colorado Academic Standards aligned with the Common Core. The independent analysis proved a nearly 95% alignment between the two sets of standards. On August 2, 2010, the Colorado State Board of Education adopted the Common Core State Standards, and requested the integration of the Common Core State Standards and the Colorado Academic Standards.

In partnership with the dedicated members of the Colorado Standards Revision Subcommittee in Mathematics, this document represents the integration of the combined academic content of both sets of standards, maintaining the unique aspects of the Colorado Academic Standards, which include personal financial literacy, 21<sup>st</sup> century skills, school readiness competencies, postsecondary and workforce readiness competencies, and preschool expectations. The result is a world-class set of standards that are greater than the sum of their parts.

The Colorado Department of Education encourages you to review the Common Core State Standards and the extensive appendices at [www.corestandards.org](http://www.corestandards.org). While all the expectations of the Common Core State Standards are embedded and **coded with CCSS:** in this document, additional information on the development and the intentions behind the Common Core State Standards can be found on the website.

## Colorado Academic Standards Mathematics Standards

*"Pure mathematics is, in its way, the poetry of logical ideas."  
Albert Einstein*

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*"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."  
Dr. Michael Brown, Nobel Prize Laureate*

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In the 21<sup>st</sup> century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

## Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how 21<sup>st</sup> century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The “Continuum of State Standards Definitions” section that follows shows the hierarchical order of the standards components. The “Standards Template” section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

**Prepared Graduate Competencies:** The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

**Standard:** The topical organization of an academic content area.

**High School Expectations:** The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. *What do students need to know in high school?*

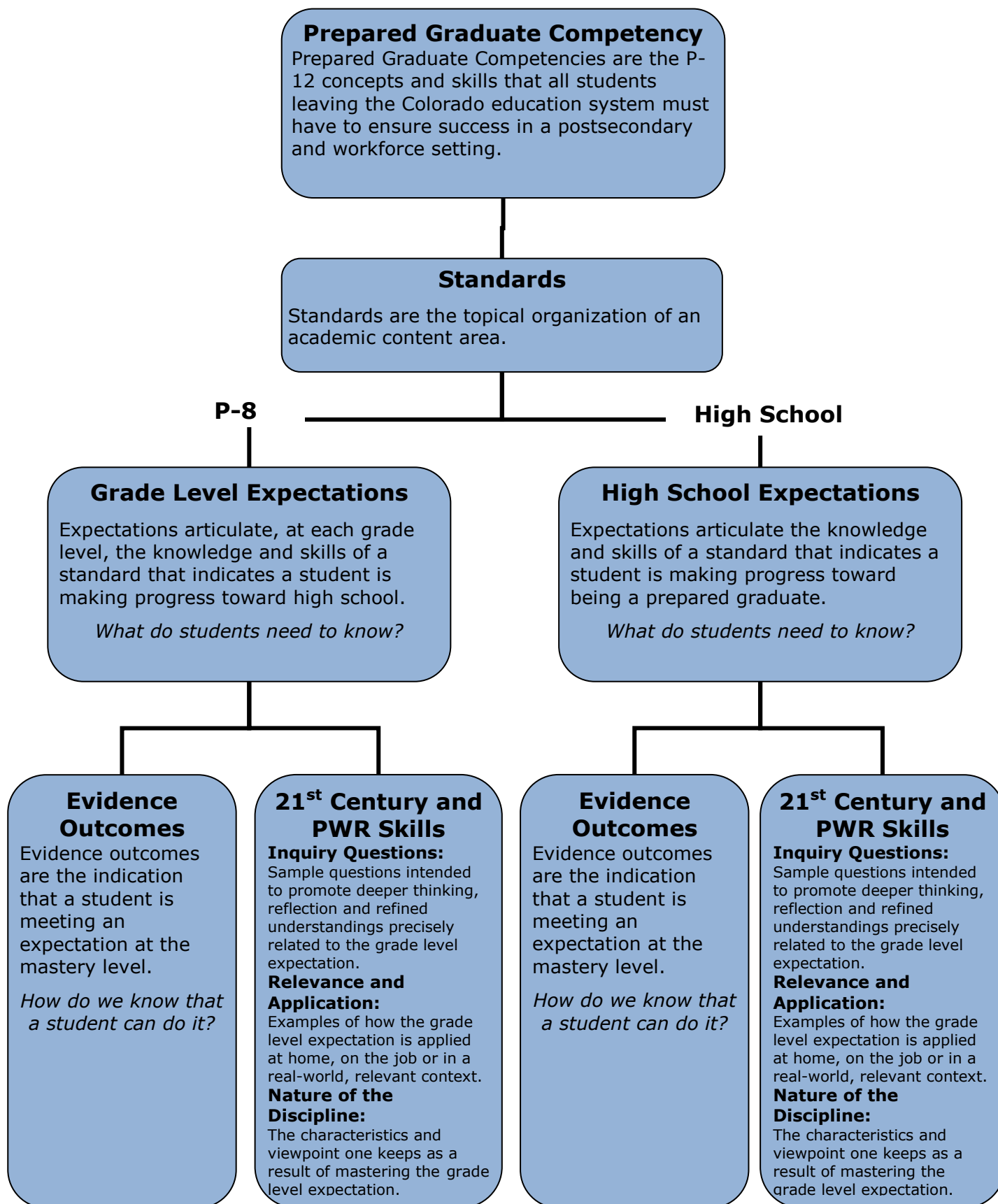
**Grade Level Expectations:** The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

**Evidence Outcomes:** The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

**21<sup>st</sup> Century Skills and Readiness Competencies:** Includes the following:

- ***Inquiry Questions:***  
Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
- ***Relevance and Application:***  
Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
- ***Nature of the Discipline:***  
The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

## Continuum of State Standards Definitions



## STANDARDS TEMPLATE

**Content Area: NAME OF CONTENT AREA**

**Standard:** The topical organization of an academic content area.

**Prepared Graduates:**

- The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

### High School and Grade Level Expectations

**Concepts and skills students master:**

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

*What do students need to know?*

**Evidence Outcomes**

**Students can:**

Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.

*How do we know that a student can do it?*

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

**Relevance and Application:**

Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

**Nature of the Discipline:**

The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

## Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

## Colorado Academic Standards Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

### 1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

### 2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

### 3.

### Data

#### Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

### 4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

### Modeling Across the Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (\*).



## **Standards for Mathematical Practice from The Common Core State Standards for Mathematics**

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

### **Mathematics | Standards for Mathematical Practice**

*The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).*

#### **1. Make sense of problems and persevere in solving them.**

*Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.*

#### **2. Reason abstractly and quantitatively.**

*Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.*

#### **3. Construct viable arguments and critique the reasoning of others.**

*Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.*

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### **4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **5. Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

#### **6. Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

#### **7. Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or

as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

#### **8. Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through  $(1, 2)$  with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

#### **Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Fourth Grade</b>	
1. Number Sense, Properties, and Operations	<ol style="list-style-type: none"> <li>The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms</li> <li>Different models and representations can be used to compare fractional parts</li> <li>Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency</li> </ol>
2. Patterns, Functions, and Algebraic Structures	<ol style="list-style-type: none"> <li>Number patterns and relationships can be represented by symbols</li> </ol>
3. Data Analysis, Statistics, and Probability	<ol style="list-style-type: none"> <li>Visual displays are used to represent data</li> </ol>
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time</li> <li>Geometric figures in the plane and in space are described and analyzed by their attributes</li> </ol>

From the Common State Standards for Mathematics, Page 27.

### **Mathematics | Grade 4**

*In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.*

*(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.*

*(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,  $15/9 = 5/3$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.*

*(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.*

## **21<sup>st</sup> Century Skills and Readiness Competencies in Mathematics**

Mathematics in Colorado's description of 21<sup>st</sup> century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21<sup>st</sup> century skills, as follows:

### Critical Thinking and Reasoning

Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

### Information Literacy

The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

### Collaboration

Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

### Self-Direction

Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

### Invention

Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

## **Colorado’s Description for School Readiness**

*(Adopted by the State Board of Education, December 2008)*

School readiness describes both the preparedness of a child to engage in and benefit from learning experiences, and the ability of a school to meet the needs of all students enrolled in publicly funded preschools or kindergartens. School readiness is enhanced when schools, families, and community service providers work collaboratively to ensure that every child is ready for higher levels of learning in academic content.

## **Colorado’s Description of Postsecondary and Workforce Readiness**

*(Adopted by the State Board of Education, June 2009)*

Postsecondary and workforce readiness describes the knowledge, skills, and behaviors essential for high school graduates to be prepared to enter college and the workforce and to compete in the global economy. The description assumes students have developed consistent intellectual growth throughout their high school career as a result of academic work that is increasingly challenging, engaging, and coherent. Postsecondary education and workforce readiness assumes that students are ready and able to demonstrate the following without the need for remediation: Critical thinking and problem-solving; finding and using information/information technology; creativity and innovation; global and cultural awareness; civic responsibility; work ethic; personal responsibility; communication; and collaboration.

## **How These Skills and Competencies are Embedded in the Revised Standards**

Three themes are used to describe these important skills and competencies and are interwoven throughout the standards: *inquiry questions; relevance and application; and the nature of each discipline*. These competencies should not be thought of stand-alone concepts, but should be integrated throughout the curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without the content to think about, it is equally impossible for students to understand the content of a discipline without grappling with complex questions and the investigation of topics.

**Inquiry Questions** – Inquiry is a multifaceted process requiring students to think and pursue understanding. Inquiry demands that students (a) engage in an active observation and questioning process; (b) investigate to gather evidence; (c) formulate explanations based on evidence; (d) communicate and justify explanations, and; (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

**Relevance and Application** – The hallmark of learning a discipline is the ability to apply the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, developing, adapting, and refining solutions for the betterment of society. The application of a discipline, including how technology assists or accelerates the work, enables students to more fully appreciate how the mastery of the grade level expectation matters after formal schooling is complete.

**Nature of Discipline** – The unique advantage of a discipline is the perspective it gives the mind to see the world and situations differently. The characteristics and viewpoint one keeps as a result of mastering the grade level expectation is the nature of the discipline retained in the mind’s eye.

# 1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

## **Prepared Graduates**

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

### **Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:**

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data



**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

**Prepared Graduates:**

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

**Grade Level Expectation: Fourth Grade**

**Concepts and skills students master:**

1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

**Evidence Outcomes**

**Students can:**

- a. Generalize place value understanding for multi-digit whole numbers (CCSS: 4.NBT)
  - i. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (CCSS: 4.NBT.1)
  - ii. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. (CCSS: 4.NBT.2)
  - iii. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons. (CCSS: 4.NBT.2)
  - iv. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.3)
- b. Use decimal notation to express fractions, and compare decimal fractions (CCSS: 4.NF)
  - i. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.<sup>1</sup> (CCSS: 4.NF.5)
  - ii. Use decimal notation for fractions with denominators 10 or 100.<sup>2</sup> (CCSS: 4.NF.6)
  - iii. Compare two decimals to hundredths by reasoning about their size.<sup>3</sup> (CCSS: 4.NF.7)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. Why isn't there a "oneths" place in decimal fractions?
2. How can a number with greater decimal digits be less than one with fewer decimal digits?
3. Is there a decimal closest to one? Why?

**Relevance and Application:**

1. Decimal place value is the basis of the monetary system and provides information about how much items cost, how much change should be returned, or the amount of savings that has accumulated.
2. Knowledge and use of place value for large numbers provides context for population, distance between cities or landmarks, and attendance at events.

**Nature of Mathematics:**

1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.
2. Mathematicians reason abstractly and quantitatively. (MP)
3. Mathematicians look for and make use of structure. (MP)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

#### Grade Level Expectation: Fourth Grade

#### Concepts and skills students master:

2. Different models and representations can be used to compare fractional parts

#### Evidence Outcomes

##### Students can:

- Use ideas of fraction equivalence and ordering to: (CCSS: 4.NF)
  - Explain equivalence of fractions using drawings and models.<sup>4</sup>
  - Use the principle of fraction equivalence to recognize and generate equivalent fractions. (CCSS: 4.NF.1)
  - Compare two fractions with different numerators and different denominators,<sup>5</sup> and justify the conclusions.<sup>6</sup> (CCSS: 4.NF.2)
- Build fractions from unit fractions by applying understandings of operations on whole numbers. (CCSS: 4.NF)
  - Apply previous understandings of addition and subtraction to add and subtract fractions.<sup>7</sup>
    - Compose and decompose fractions as sums and differences of fractions with the same denominator in more than one way and justify with visual models.
    - Add and subtract mixed numbers with like denominators.<sup>8</sup> (CCSS: 4.NF.3c)
    - Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.<sup>9</sup> (CCSS: 4.NF.3d)
  - Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS: 4.NF.4)
    - Express a fraction  $a/b$  as a multiple of  $1/b$ .<sup>10</sup> (CCSS: 4.NF.4a)
    - Use a visual fraction model to express  $a/b$  as a multiple of  $1/b$ , and apply to multiplication of whole number by a fraction.<sup>11</sup> (CCSS: 4.NF.4b)
    - Solve word problems involving multiplication of a fraction by a whole number.<sup>12</sup> (CCSS: 4.NF.4c)

#### 21<sup>st</sup> Century Skills and Readiness Competencies

##### Inquiry Questions:

- How can different fractions represent the same quantity?
- How are fractions used as models?
- Why are fractions so useful?
- What would the world be like without fractions?

##### Relevance and Application:

- Fractions and decimals are used any time there is a need to apportion such as sharing food, cooking, making savings plans, creating art projects, timing in music, or portioning supplies.
- Fractions are used to represent the chance that an event will occur such as randomly selecting a certain color of shirt or the probability of a certain player scoring a soccer goal.
- Fractions are used to measure quantities between whole units such as number of meters between houses, the height of a student, or the diameter of the moon.

##### Nature of Mathematics:

- Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)
- Mathematicians model with mathematics. (MP)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Are fluent with basic numerical, symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

#### Grade Level Expectation: Fourth Grade

##### Concepts and skills students master:

3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency

##### Evidence Outcomes

###### Students can:

- Use place value understanding and properties of operations to perform multi-digit arithmetic. (CCSS: 4.NBT)
  - Fluently add and subtract multi-digit whole numbers using standard algorithms. (CCSS: 4.NBT.4)
  - Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. (CCSS: 4.NBT.5)
  - Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 4.NBT.6)
  - Illustrate and explain multiplication and division calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.6)
- Use the four operations with whole numbers to solve problems. (CCSS: 4.OA)
  - Interpret a multiplication equation as a comparison.<sup>13</sup> (CCSS: 4.OA.1)
  - Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.1)
  - Multiply or divide to solve word problems involving multiplicative comparison.<sup>14</sup> (CCSS: 4.OA.2)
  - Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. (CCSS: 4.OA.3)
  - Represent multistep word problems with equations using a variable to represent the unknown quantity. (CCSS: 4.OA.3)
  - Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.3)
  - Using the four operations analyze the relationship between choice and opportunity cost (PFL)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- Is it possible to make multiplication and division of large numbers easy?
- What do remainders mean and how are they used?
- When is the "correct" answer not the most useful answer?

###### Relevance and Application:

- Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra.

###### Nature of Mathematics:

- Mathematicians envision and test strategies for solving problems.
- Mathematicians develop simple procedures to express complex mathematical concepts.
- Mathematicians make sense of problems and persevere in solving them. (MP)
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)
- Mathematicians look for and express regularity in repeated reasoning. (MP)

## Standard: 1. Number Sense, Properties, and Operations Fourth Grade

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<sup>1</sup> For example, express  $3/10$  as  $30/100$ , and add  $3/10 + 4/100 = 34/100$ . (CCSS: 4.NF.6)

<sup>2</sup> For example, rewrite  $0.62$  as  $62/100$ ; describe a length as  $0.62$  meters; locate  $0.62$  on a number line diagram. (CCSS: 4.NF.6)

<sup>3</sup> Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual model. (CCSS: 4.NF.7)

<sup>4</sup> Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. (CCSS: 4.NF.1)

<sup>5</sup> e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , (CCSS: 4.NF.2)

<sup>6</sup> e.g., by using a visual fraction model. (CCSS: 4.NF.2)

<sup>7</sup> Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ . (CCSS: 4.NF.3)

Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS: 4.NF.3a)

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$ . (CCSS: 4.NF.3b)

<sup>8</sup> e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS: 4.NF.3c)

<sup>9</sup> e.g., by using visual fraction models and equations to represent the problem. (CCSS: 4.NF.3d)

<sup>10</sup> For example, use a visual fraction model to represent  $5/4$  as the product  $5 \times (1/4)$ , recording the conclusion by the equation  $5/4 = 5 \times (1/4)$ . (CCSS: 4.NF.4a)

<sup>11</sup> For example,  $3 \times (2/5)$  as  $6 \times (1/5)$ , recognizing this product as  $6/5$ . (In general,  $n \times (a/b) = (n \times a)/b$ .) (CCSS: 4.NF.4b)

<sup>12</sup> e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat  $3/8$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (CCSS: 4.NF.4c)

<sup>13</sup> e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. (CCSS: 4.OA.1)

<sup>14</sup> e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (CCSS: 4.OA.2)

## 2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

### Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

#### **Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:**

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

## Content Area: Mathematics

### Standard: 2. Patterns, Functions, and Algebraic Structures

#### Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

### Grade Level Expectation: Fourth Grade

#### Concepts and skills students master:

1. Number patterns and relationships can be represented by symbols

#### Evidence Outcomes

##### Students can:

- a. Generate and analyze patterns and identify apparent features of the pattern that were not explicit in the rule itself.<sup>1</sup> (CCSS: 4.OA.5)
  - i. Use number relationships to find the missing number in a sequence
  - ii. Use a symbol to represent and find an unknown quantity in a problem situation
  - iii. Complete input/output tables
  - iv. Find the unknown in simple equations
- b. Apply concepts of squares, primes, composites, factors, and multiples to solve problems
  - i. Find all factor pairs for a whole number in the range 1–100. (CCSS: 4.OA.4)
  - ii. Recognize that a whole number is a multiple of each of its factors. (CCSS: 4.OA.4)
  - iii. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. (CCSS: 4.OA.4)
  - iv. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS: 4.OA.4)

#### 21<sup>st</sup> Century Skills and Readiness Competencies

##### Inquiry Questions:

1. What characteristics can be used to classify numbers into different groups?
2. How can we predict the next element in a pattern?
3. Why do we use symbols to represent missing numbers?
4. Why is finding an unknown quantity important?

##### Relevance and Application:

1. Use of an input/output table helps to make predictions in everyday contexts such as the number of beads needed to make multiple bracelets or number of inches of expected growth.
2. Symbols help to represent situations from everyday life with simple equations such as finding how much additional money is needed to buy a skateboard, determining the number of players missing from a soccer team, or calculating the number of students absent from school.
3. Comprehension of the relationships between primes, composites, multiples, and factors develop number sense. The relationships are used to simplify computations with large numbers, algebraic expressions, and division problems, and to find common denominators.

##### Nature of Mathematics:

1. Mathematics involves pattern seeking.
2. Mathematicians use patterns to simplify calculations.
3. Mathematicians model with mathematics. (MP)

**Standard: 2. Patterns, Functions, and Algebraic Structures**  
**Fourth Grade**

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<sup>1</sup> For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. (CCSS: 4.OA.5)

# 3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

## Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

**Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:**

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions



**Content Area: Mathematics**

**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: Fourth Grade**

**Concepts and skills students master:**

1. Visual displays are used to represent data

**Evidence Outcomes**

**Students can:**

- a. Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). (CCSS: 4.MD.4)
- b. Solve problems involving addition and subtraction of fractions by using information presented in line plots.<sup>1</sup> (CCSS: 4.MD.4)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. What can you learn by collecting data?
2. What can the shape of data in a display tell you?

**Relevance and Application:**

1. The collection and analysis of data provides understanding of how things work. For example, measuring the weather every day for a year helps to better understand weather.

**Nature of Mathematics:**

1. Mathematics helps people use data to learn about the world.
2. Mathematicians model with mathematics. (MP)
3. Mathematicians use appropriate tools strategically. (MP)
4. Mathematicians attend to precision. (MP)

**Standard: 3. Data Analysis, Statistics, and Probability**  
**Fourth Grade**

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<sup>1</sup> For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.  
(CCSS: 4.MD.4)

# 4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

## Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

### Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

#### Grade Level Expectation: Fourth Grade

##### Concepts and skills students master:

1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time

##### Evidence Outcomes

###### Students can:

- a. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (CCSS: 4.MD)
  - i. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. (CCSS: 4.MD.1)
  - ii. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.<sup>1</sup> (CCSS: 4.MD.1)
  - iii. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. (CCSS: 4.MD.2)
  - iv. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS: 4.MD.2)
  - v. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.<sup>2</sup> (CCSS: 4.MD.3)
- b. Use concepts of angle and measure angles. (CCSS: 4.MD)
  - i. Describe angles as geometric shapes that are formed wherever two rays share a common endpoint, and explain concepts of angle measurement.<sup>3</sup> (CCSS: 4.MD.5)
  - ii. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.6)
  - iii. Demonstrate that angle measure as additive.<sup>4</sup> (CCSS: 4.MD.7)
  - iv. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.<sup>5</sup> (CCSS: 4.MD.7)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. How do you decide when close is close enough?
2. How can you describe the size of geometric figures?

###### Relevance and Application:

1. Accurate use of measurement tools allows people to create and design projects around the home or in the community such as flower beds for a garden, fencing for the yard, wallpaper for a room, or a frame for a picture.

###### Nature of Mathematics:

1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade.
2. Mathematicians make sense of problems and persevere in solving them. (MP)
3. Mathematicians use appropriate tools strategically. (MP)
4. Mathematicians attend to precision. (MP)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

**Grade Level Expectation: Fourth Grade**

**Concepts and skills students master:**

2. Geometric figures in the plane and in space are described and analyzed by their attributes

**Evidence Outcomes**

**Students can:**

- a. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. (CCSS: 4.G.1)
- b. Identify points, line segments, angles, and perpendicular and parallel lines in two-dimensional figures. (CCSS: 4.G.1)
- c. Classify and identify two-dimensional figures according to attributes of line relationships or angle size.<sup>6</sup> (CCSS: 4.G.2)
- d. Identify a line of symmetry for a two-dimensional figure.<sup>7</sup> (CCSS: 4.G.3)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. How do geometric relationships help us solve problems?
- 2. Is a square still a square if it's tilted on its side?
- 3. How are three-dimensional shapes different from two-dimensional shapes?
- 4. What would life be like in a two-dimensional world?
- 5. Why is it helpful to classify things like angles or shapes?

**Relevance and Application:**

- 1. The understanding and use of spatial relationships helps to predict the result of motions such as how articles can be laid out in a newspaper, what a room will look like if the furniture is rearranged, or knowing whether a door can still be opened if a refrigerator is repositioned.
- 2. The application of spatial relationships of parallel and perpendicular lines aid in creation and building. For example, hanging a picture to be level, building windows that are square, or sewing a straight seam.

**Nature of Mathematics:**

- 1. Geometry is a system that can be used to model the world around us or to model imaginary worlds.
- 2. Mathematicians look for and make use of structure. (MP)
- 3. Mathematicians look for and express regularity in repeated reasoning. (MP)

**Standard: 4. Shape, Dimension, and Geometric Relationships**  
**Fourth Grade**

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<sup>1</sup> For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... (CCSS: 4.MD.1)

<sup>2</sup> For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. (CCSS: 4.MD.3)

<sup>3</sup> An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through  $\frac{1}{360}$  of a circle is called a "one-degree angle," and can be used to measure angles. (CCSS: 4.MD.5a)

An angle that turns through  $n$  one-degree angles is said to have an angle measure of  $n$  degrees. (CCSS: 4.MD.5b)

<sup>4</sup> When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. (CCSS: 4.MD.7)

<sup>5</sup> e.g., by using an equation with a symbol for the unknown angle measure. (CCSS: 4.MD.7)

<sup>6</sup> Based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS: 4.G.2)

<sup>7</sup> as a line across the figure such that the figure can be folded along the line into matching parts. (CCSS: 4.G.3)

Identify line-symmetric figures and draw lines of symmetry. (CCSS: 4.G.3)

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