Colorado Academic Standards

Mathematics

Sixth Grade – Eighth Grade

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

Mathematics Standards Review and Revision Committee

Chairperson

Joanie Funderburk
President
Colorado Council of Teachers of Mathematics

Members

Lisa Bejarano
Teacher
Aspen Valley High School
Academy District 20

Michael Brom
Assessment and Accountability Teacher on Special Assignment
Lewis-Palmer School District 38

Ann Conaway
Teacher
Palisade High School
Mesa County Valley School District 51

Dennis DeBay
Mathematics Education Faculty
University of Colorado Denver

Greg George
K-12 Mathematics Coordinator
St. Vrain Valley School District

Cassie Harrelson
Director of Professional Practice
Colorado Education Association

Lanny Hass
Principal
Thompson Valley High School
Thompson School District

Ken Jensen
Mathematics Instructional Coach
Aurora Public Schools

Lisa Rogers
Student Achievement Coordinator
Fountain-Fort Carson School District 8

David Sawtelle
K-12 Mathematics Specialist
Colorado Springs School District 11

T. Vail Shoultz-McCole
Early Childhood Program Director
Colorado Mesa University

Ann Summers
K-12 Mathematics and Intervention Specialist
Littleton Public Schools

State Board of Education and Colorado Department of Education

Colorado State Board of Education

Angelika Schroeder (D, Chair)
2nd Congressional District
Boulder

Joyce Rankin (R, Vice Chair)
3rd Congressional District
Carbondale

Steve Durham (R)
5th Congressional District
Colorado Springs

Valentina (Val) Flores (D)
1st Congressional District
Denver

Jane Goff (D)
7th Congressional District
Arvada

Rebecca McClellan (D)
6th Congressional District
Centennial

Debora Scheffel (R)
4th Congressional District
Parker

Colorado Department of Education

Katy Anthes, Ph.D.
Commissioner of Education
Secretary to the Board of Education

Melissa Colsman, Ph.D.
Associate Commissioner of Education
Student Learning Division

Floyd Cobb, Ph.D.
Executive Director
Teaching and Learning Unit

CDE Standards and Instructional Support Office

Karol Gates
Director

Carla Aguilar, Ph.D.
Music Content Specialist

Ariana Antonio
Standards Project Manager

Joanna Bruno, Ph.D.
Science Content Specialist

Lourdes (Lulu) Buck
World Languages Content Specialist

Donna Goodwin, Ph.D.
Visual Arts Content Specialist

Stephanie Hartman, Ph.D.
Social Studies Content Specialist

Judi Hofmeister
Dance Content Specialist
Drama and Theatre Arts Content Specialist

Jamie Hurley, Ph.D.
Comprehensive Health Content Specialist
Physical Education Content Specialist

Raymond Johnson
Mathematics Content Specialist

Christine Liebe
Computer Science Content Specialist

Vince Puzick
Reading, Writing, and Communicating Content Specialist

Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards (CCSSI, 2010). Across the curriculum at every grade, students are expected to consistently have opportunities to engage in each of the eight practices. The practices aligned with each Grade Level Expectation in the Colorado Academic Standards represent the *strongest potential* alignments between content and the practices, and are not meant to exclude students from engaging in the rest of the practices.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Math Practice MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Math Practice MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Math Practice MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Practice MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Math Practice MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Math Practice MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Math Practice MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1,2)$ with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

References

Common Core State Standards Initiative. (2010). *Standards for mathematical practice*. *http://www.corestandards.org/Math/Practice*

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. (For more on modeling, see Appendix: Modeling Cycle.)

How to Read the Colorado Academic Standards

CONTENT AREA
Grade Level, Standard Category

COLORADO Department of Education

Prepared Graduates:

The *PG Statements* represent concepts and skills that all students who complete the Colorado education system must master to ensure their success in postsecondary and workforce settings.

Grade Level Expectation:

The *GLEs* are an articulation of the concepts and skills for a grade, grade band, or range that students must master to ensure their progress toward becoming a prepared graduate.

Evidence Outcomes

The *EOs* describe the evidence that demonstrates that a student is meeting the GLE at a mastery level.

Academic Context and Connections

The *ACCs* provide context for interpreting, connecting, and applying the content and skills of the GLE. This includes the *Colorado Essential Skills*, which are the critical skills needed to prepare students to successfully enter the workforce or educational opportunities beyond high school embedded within statute (C.R.S. 22-7-1005) and identified by the Colorado Workforce Development Committee.

The ACCs contain information unique to each content area. Content-specific elements of the ACCs are described below.

Colorado Academic Standards Content Area

Grade Level, Standard Category   2020 Colorado Academic Standards   GLE Code

Academic Context and Connections in Mathematics:

Colorado Essential Skills and Mathematical Practices: These statements describe how the learning of the content and skills described by the GLE and EOs connects to and supports the development of the *Colorado Essential Skills* and *Standards for Mathematical Practice* named in the parentheses.

Inquiry Questions: The sample question that are intended to promote deeper thinking, reflection, and refined understandings precisely related to the GLE.

Coherence Connections: These statements relate how the GLE relates to content within and across grade levels. The first statement indicates if a GLE is *major*, *supporting*, or *additional* work of the grade. Between 65% and 85% of the work of each grade (with P-2 at the high end of that range) should be focused on the GLEs labeled as **major work**. The remainder of the time should focus on **supporting work** and **additional work**, where it can appropriately support and compliment students' engagement in major work. **Advanced outcomes**, marked with a (+), represent content best saved for upper-level math courses in a student's final three semesters of high school. The remaining statements describe how the GLE and EOs build from content learned in prior grades, connects to content in the same grade, and supports learning in later grades.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP7. Look for and make use of structure.

Grade Level Expectation:

6.RP.A. Ratios & Proportional Relationships: Understand ratio concepts and use ratio reasoning to solve problems.

Evidence Outcomes

Students Can:

1. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was* $2:1$, *because for every* 2 *wings there was* 1 *beak." "For every vote Candidate* A *received, Candidate* C *received nearly three votes."* (CCSS: 6.RP.A.1)
2. Apply the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of* 3 *cups of flour to* 4 *cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid $75 for* 15 *hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)* (CCSS: 6.RP.A.2)
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (CCSS: 6.RP.A.3)
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (CCSS: 6.RP.A.3.a)
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took* 7 *hours to mow* 4 *lawns, then at that rate, how many lawns could be mowed in* 35 *hours? At what rate were lawns being mowed?* (CCSS: 6.RP.A.3.b)
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent. (CCSS: 6.RP.A.3.c)
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (CCSS: 6.RP.A.3.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use ratio tables to test solutions and determine equivalent ratios. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Analyze and use appropriate quantities and pay attention to units in problems that require reasoning with ratios. (MP2)
3. Construct arguments that compare quantities using ratios or rates. (MP3)
4. Use tables, tape diagrams, and double number line diagrams to provide a structure for seeing equivalency between ratios. (MP7)

Inquiry Questions:

1. How are ratios different from fractions?
2. What is the difference between a quantity and a number?
3. How is a percent also a ratio?
4. How is a rate similar to and also different from a unit rate?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In prior grades, students work with multiplication, division, and measurement. Prior knowledge with the structure of the multiplication table is an important connection for students in creating and verifying equivalent ratios written in symbolic form or in ratio tables (multiplicative comparison vs. additive comparison).
3. In Grade 6, this expectation connects with one-variable equations, inequalities, and representing and analyzing quantitative relationships between dependent and independent variables.
4. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems. In high school, students generalize rates of change to linear and nonlinear functions and use them to describe real-world scenarios.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

6.NS.A. The Number System: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Evidence Outcomes

Students Can:

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for* $\frac{2}{3} \div \frac{3}{4}$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ *because* $\frac{3}{4}$ *of* $\frac{8}{9}$ *is* $\frac{2}{3}$*. (In general,* $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$*.) How much chocolate will each person get if* 3 *people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{3}{4}$*-cup servings are in* $\frac{2}{3}$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $\frac{3}{4}$ *mi and area* $\frac{1}{2}$ *square mi?* (CCSS: 6.NS.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Create and solve word problems using division of fractions, understanding the relationship of the arithmetic to the problem being solved. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Reason about the contextualized meaning of numbers in word problems involving division of fractions, and decontextualize those numbers to perform efficient calculations. (MP2)
3. Model real-world situations involving scaling by non-whole numbers using multiplication and division by fractions. (MP4)

Inquiry Questions:

1. When dividing, is the quotient always going to be a smaller number than the dividend? Why or why not?
2. What kinds of real-world situations require the division of fractions?
3. How can the division of fractions be modeled visually?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students apply and extend previous understandings of multiplication and division to divide whole numbers by unit fractions and unit fractions by whole numbers.
3. In Grade 6, this expectation connects with solving one-step, one-variable equations and inequalities.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

COLORADO
Department of Education

Prepared Graduates:

MP6. Attend to precision.

MP7. Look for and make use of structure.

Grade Level Expectation:

6.NS.B. The Number System: Compute fluently with multi-digit numbers and find common factors and multiples.

Evidence Outcomes

Students Can:

2. Fluently divide multi-digit numbers using the standard algorithm. (CCSS: 6.NS.B.2)
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (CCSS: 6.NS.B.3)
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers $1–100$ with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* $36 + 8$ *as* $4(9 + 2)$. (CCSS: 6.NS.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Accurately add, subtract, multiply, and divide with decimals. (MP6)
2. Recognize the structures of factors and multiples when identifying the greatest common factor and least common multiple of two whole numbers. Use the greatest common factor to rewrite an expression using the distributive property. (MP7)

Inquiry Questions:

1. How do operations with decimals compare and contrast to operations with whole numbers?
2. How does rewriting the sum of two whole numbers using the distributive property yield new understanding and insights on the sum?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 5, students divide whole numbers with two-digit divisors and perform operations with decimals.
3. In Grade 6, this expectation connects with applying and extending previous understandings of arithmetic to algebraic expressions.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. Students apply the concept of greatest common factor to factor linear expressions, and extending properties of whole numbers to variable expressions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

Grade Level Expectation:

6.NS.C. The Number System: Apply and extend previous understandings of numbers to the system of rational numbers.

Evidence Outcomes

Students Can:

5. Explain why positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (CCSS: 6.NS.C.5)
6. Describe a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. (CCSS: 6.NS.C.6)
   a. Use opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; identify that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. (CCSS: 6.NS.C.6.a)
   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; explain that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (CCSS: 6.NS.C.6.b)
   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (CCSS: 6.NS.C.6.c)
7. Order and find absolute value of rational numbers. (CCSS: 6.NS.C.7)
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* (CCSS: 6.NS.C.7.a)
   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3^\circ\mathrm{C} > -7^\circ\mathrm{C}$ to express the fact $-3^\circ\mathrm{C}$ is warmer than $-7^\circ\mathrm{C}$.* (CCSS: 6.NS.C.7.b)
   c. Define the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.* (CCSS: 6.NS.C.7.c)
   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* (CCSS: 6.NS.C.7.d)
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS: 6.NS.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate integers to form hypotheses, make observations and draw conclusions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Understand the relationship among negative numbers, positive numbers, and absolute value. (MP2)
3. Explain the order of rational numbers using their location on the number line. (MP3)
4. Demonstrate how to plot points on a number line and plot ordered pairs on a coordinate plane. (MP5)

Inquiry Questions:

1. Why do we have negative numbers?
2. What relationships exist among positive and negative numbers on the number line?
3. How does the opposite of a number differ from the absolute value of that same number?
4. How does an ordered pair correspond to its given point on a coordinate plane?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students develop understanding of fractions as numbers and graph points on the coordinate plane (limited to the first quadrant) to solve real-world and mathematical problems.
3. In Grade 6, this expectation connects with reasoning about and solving one-step, one-variable equations and inequalities.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. In Grade 8, students investigate patterns of association in bivariate data.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

6.EE.A. Expressions & Equations: Apply and extend previous understandings of arithmetic to algebraic expressions.

Evidence Outcomes

Students Can:

1. Write and evaluate numerical expressions involving whole-number exponents. (CCSS: 6.EE.A.1)
2. Write, read, and evaluate expressions in which letters stand for numbers. (CCSS: 6.EE.A.2)
   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as $5 - y$.* (CCSS: 6.EE.A.2.a)
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.* (CCSS: 6.EE.A.2.b)
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.* (CCSS: 6.EE.A.2.c)
3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.* (CCSS: 6.EE.A.3)
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.* (CCSS: 6.EE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that expressions can be written in multiple forms and describe cause-and-effect relationships and patterns. (Entrepreneurial Skills: Critical Thinking/Problem Solving and Inquiry/Analysis)
2. Communicate a justification of why expressions are equivalent using arguments about properties of operations and whole numbers. (MP3)
3. See the structure of an expression like $x + 2$ as a sum but also as a single factor in the product $3(x + 2)$. (MP7)
4. Recognize equivalence in variable expressions with repeated addition (such as $y + y + y = 3y$) and repeated multiplication (such as $y \times y \times y = y^3$) and use arithmetic operations to justify the equivalence. (MP8)

Inquiry Questions:

1. How are algebraic expressions similar to and different from numerical expressions?
2. What does it mean for two variable expressions to be equivalent?
3. How might the application of the order of operations differ when using grouping symbols, such as parentheses, for numerical expressions as compared to algebraic expressions?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students understand and apply properties of operations, relationships between inverse arithmetic operations, and write and interpret numerical expressions.
3. In Grade 6, this expectation connects to fluency with multi-digit numbers, finding common factors and multiples, and one-variable equations and inequalities.
4. In future grades, students work with radicals and integer exponents and interpret the structure of more complex algebraic expressions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP6. Attend to precision.

Grade Level Expectation:

6.EE.B. Expressions & Equations: Reason about and solve one-variable equations and inequalities.

Evidence Outcomes

Students Can:

5. Describe solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS: 6.EE.B.5)
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (CCSS: 6.EE.B.6)
7. Solve real-world and mathematical problems by writing and solving equations of the form $x \pm p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (CCSS: 6.EE.B.7)
8. Write an inequality of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. Show that inequalities of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS: 6.EE.B.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate unknown values to form hypotheses, make observations, and draw conclusions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason about the values and operations of an equation both within a real-world context and abstracted from it. (MP2)
3. State precisely the meaning of variables used when setting up equations. (MP6)

Inquiry Questions:

1. What are the different ways a variable can be used in an algebraic equation or inequality? For example, how are these uses of the variable x different from each other? (a) $x + 5 = 8$; (b) $x = \frac{1}{2}$; (c) $x > 5$.
2. How is the solution to an inequality different than a solution to an equation?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students write simple expressions that record calculations with numbers, interpret numerical expressions without evaluating them, and generate ordered pairs from two numerical rules.
3. This expectation connects to several others in Grade 6: (a) understanding ratio concepts and use ratio reasoning to solve problems, (b) applying and extending previous understandings of multiplication and division to divide fractions by fractions, (c) applying and extending previous understandings of numbers to the system of rational numbers, (d) applying and extending previous understandings of arithmetic to algebraic expressions, and (e) representing and analyzing quantitative relationships between dependent and independent variables.
4. In Grade 7, students solve real-life and mathematical problems involving two-step equations and inequalities. In Grade 8, students work with radicals and integer exponents and solve linear equations and pairs of simultaneous linear equations.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

6.EE.C. Expressions & Equations: Represent and analyze quantitative relationships between dependent and independent variables.

Evidence Outcomes

Students Can:

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.* (CCSS: 6.EE.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze relationships between dependent and independent variables. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason about the operations that relate constant and variable quantities in equations with dependent and independent variables. (MP2)
3. Model with mathematics by describing real-world situations with equations and inequalities. (MP4)

Inquiry Questions:

1. How can you determine if a variable is the independent variable or the dependent variable?
2. What are the advantages of showing the relationship between an independent and dependent variable in multiple representations (table, graph, equation)?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students analyze numerical patterns and relationships, including generating and graphing ordered pairs in the first quadrant.
3. In Grade 6, this expectation connects with understanding ratio concepts and using ratio reasoning to solve problems.
4. In Grade 7, students decide if two quantities are in a proportional relationship and identify the unit rate in tables, graphs, equations, diagrams, and verbal descriptions.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

6.SP.A. Statistics & Probability: Develop understanding of statistical variability.

Evidence Outcomes

Students Can:

1. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* (CCSS: 6.SP.A.1)
2. Demonstrate that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape. (CCSS: 6.SP.A.2)
3. Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (CCSS: 6.SP.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Identify statistical questions that require the collection of data representing multiple perspectives. (Civic/Interpersonal Skills: Global/Cultural Awareness)
2. Make sense of practical problems by turning them into statistical investigations. (MP1)
3. Reason abstractly and quantitatively with data collected from statistical investigations by describing the data's center, spread, and shape. (MP2)
4. Model variability in data collected to answer statistical questions and draw conclusions based on center, spread, and shape. (MP4)

Inquiry Questions:

1. What distinguishes a statistical question from a question that is not a statistical question?
2. Why do we have numerical measures for both the center of a data set and the variability of a data set?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students represent and interpret data in dot plots/line plots, and use arithmetic to answer questions about the plots.
3. In Grade 6, students summarize and describe data distributions using numerical measures of center and spread, and terms such as cluster, peak, gap, symmetry, skew, and outlier.
4. In Grade 7, students use random sampling to draw inferences about a population and draw informal comparative inferences about two populations. In high school, students summarize, represent, and interpret data on a single count or measurement variable.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

6.SP.B. Statistics & Probability: Summarize and describe distributions.

Evidence Outcomes

Students Can:

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (CCSS: 6.SP.B.4)
5. Summarize numerical data sets in relation to their context, such as by: (CCSS: 6.SP.B.5)
   a. Reporting the number of observations. (CCSS: 6.SP.B.5.a)
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. (CCSS: 6.SP.B.5.b)
   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (CCSS: 6.SP.B.5.c)
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS: 6.SP.B.5.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Write informative texts describing statistical distributions, their measures, and how they relate to the context in which the data were gathered. (Entrepreneurial Skills: Literacy/Writing)
2. Move from context to abstraction and back to context while finding and using measures of center and variability and describing what they mean in the context of the data. (MP2)
3. Analyze data sets and draw conclusions based on the data display and measures of center and/or variability. (MP4)
4. Identify clusters, peaks, gaps, and symmetry, and describe the meaning of these and other patterns in data distributions. (MP7)

Inquiry Questions:

1. How can different data displays communicate different meanings?
2. When is it better to use the mean as a measure of center? Why?
3. When is it better to use the median as a measure of center? Why?
4. How many values of a data set do you use to calculate the range? Interquartile range? Mean absolute deviation? How does this help to compare what these measures represent?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students represent and interpret data in dot plots and line plots.
3. In Grade 6, this expectation connects with developing understanding of statistical variability.
4. In high school, students summarize, represent, and interpret data on a single count or measurement variable (including standard deviation), and make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

6.G.A. Geometry: Solve real-world and mathematical problems involving area, surface area, and volume.

Evidence Outcomes

Students Can:

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.1)
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.2)
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.3)
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that problems can be identified and possible solutions can be created with respect to using area, surface area, and volume. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of a problem by understanding the context of the problem before applying a formula. (MP1)
3. Model real-world problems involving shape and space. (MP4)
4. Strategically use coordinate planes, nets of three-dimensional figures, and area and volume formulas as tools to solve real-world problems. (MP5)

Inquiry Questions:

1. What is the difference between what area measures and what volume measures?
2. How does using decomposition aid in finding the area of composite figures?
3. How are nets of three-dimensional figures used to find surface area?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 4, students solve problems involving measurement and converting from a larger unit to a smaller unit. In Grade 5, students understand concepts of volume, relate volume to multiplication and to addition, and graph points on the coordinate plane to solve real-world and mathematical problems.
3. In Grade 6, this expectation connects with graphing points in all four quadrants of the coordinate plane and finding distances between points with the same first coordinate or the same second coordinate.
4. In Grade 7, students draw, construct, and describe geometrical figures and describe the relationships between them, and solve real-world and mathematical problems involving angle measure, area, surface area, and volume. In Grade 8, students understand congruence and similarity and understand and apply the Pythagorean Theorem.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

7.RP.A. Ratios & Proportional Relationships: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Evidence Outcomes

Students Can:

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.* (CCSS: 7.RP.A.1)
2. Identify and represent proportional relationships between quantities. (CCSS: 7.RP.A.2)
    a. Determine whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS: 7.RP.A.2.a)
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS: 7.RP.A.2.b)
    c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* (CCSS: 7.RP.A.2.c)
    d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where r is the unit rate. (CCSS: 7.RP.A.2.d)
3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (CCSS: 7.RP.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize when proportional relationships occur and apply these relationships to personal experiences. (Entrepreneurial Skills: Inquiry/Analysis)
2. Recognize, identify, and solve problems that involve proportional relationships to make predictions and describe associations among variables. (MP1)
3. Reason quantitatively with rates and their units in proportional relationships. (MP2)
4. Use repeated reasoning to test for equivalent ratios, such as reasoning that walking $\frac{1}{2}$ mile in $\frac{1}{4}$ hour is equivalent to walking 1 mile in $\frac{1}{2}$ hour and equivalent to walking 2 miles in 1 hour, the unit rate. (MP8)

Inquiry Questions:

1. How are proportional relationships related to unit rates?
2. How can proportional relationships be expressed using tables, equations, and graphs?
3. What are properties of all proportional relationships when graphed on the coordinate plane?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students understand ratio concepts and use ratio reasoning to solve problems.
3. This expectation connects with several others in Grade 7: (a) solving real-life and mathematical problems using numerical and algebraic expressions and equations, (b) investigating chance processes and developing, using, and evaluating probability models, and (c) drawing, constructing, and describing geometrical figures and describing the relationships between them.
4. In Grade 8, students (a) understand the connections between proportional relationships, lines, and linear equations, (b) define, evaluate, and compare functions, and (c) use functions to model relationships between quantities. In high school, students use proportional relationships to define trigonometric ratios, solve problems involving right triangles, and find arc lengths and areas of sectors of circles.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

7.NS.A. The Number System: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Evidence Outcomes

Students Can:

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. (CCSS: 7.NS.A.1)
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has* 0 *charge because its two constituents are oppositely charged.* (CCSS: 7.NS.A.1.a)
   b. Demonstrate $p+q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.1.b)
   c. Demonstrate subtraction of rational numbers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (CCSS: 7.NS.A.1.c)
   d. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.A.1.d)
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. (CCSS: 7.NS.A.2)
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1)=1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.2.a)
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right)=\frac{-p}{q}=\frac{p}{-q}$. Interpret quotients of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.2.b)
   c. Apply properties of operations as strategies to multiply and divide rational numbers. (CCSS: 7.NS.A.2.c)
   d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (CCSS: 7.NS.A.2.d)
3. Solve real-world and mathematical problems involving the four operations with rational numbers. *(Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)* (CCSS: 7.NS.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems with rational numbers using all four operations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Compute with rational numbers abstractly and interpret quantities in context. (MP2)
3. Justify understanding and computational accuracy of operations with rational numbers. (MP3)
4. Use additive inverses, absolute value, the distributive property, and properties of operations to reason with and operate on rational numbers. (MP7)

Inquiry Questions:

1. How do operations with integers compare to and contrast with operations with whole numbers?
2. How can operations with negative integers be modeled visually?
3. How can it be determined if the decimal form of a rational number terminates or repeats?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students use the four operations with whole numbers and fractions to solve problems.
3. In Grade 7, this expectation connects with solving real-life and mathematical problems using numerical and algebraic expressions and equations. This expectation begins the formal study of rational numbers (a number expressible in the form $\frac{a}{b}$ or $-\frac{a}{b}$ for some fraction $\frac{a}{b}$; the rational numbers include the integers) as extended from their study of fractions, which in these standards always refers to non-negative numbers.
4. In Grade 8, students extend their study of the real number system to include irrational numbers, radical expressions, and integer exponents. In high school, students work with rational exponents and complex numbers.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

7.EE.A. Expressions & Equations: Use properties of operations to generate equivalent expressions.

Evidence Outcomes

Students Can:

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (CCSS: 7.EE.A.1)
2. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by* 5%*" is the same as "multiply by* 1.05*."* (CCSS: 7.EE.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that the structures of equivalent algebraic expressions provide different ways of seeing the same problem. (MP7)

Inquiry Questions:

1. How is it determined that two algebraic expressions are equivalent?
2. What is the value of having an algebraic expression in equivalent forms?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students apply and extend previous understandings of arithmetic to algebraic expressions.
3. In Grade 8, students use equivalent expressions to analyze and solve linear equations and pairs of simultaneous linear equations. In high school, students use equivalent expressions within various families of functions to reveal key features of graphs and how those features are related to contextual situations.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

7.EE.B. Expressions & Equations: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Evidence Outcomes

Students Can:

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making* $\$25$ *an hour gets a* 10% *raise, she will make an additional* $\frac{1}{10}$ *of her salary an hour, or* $\$2.50$*, for a new salary of* $\$27.50$*. If you want to place a towel bar* $9\frac{3}{4}$ *inches long in the center of a door that is* $27\frac{1}{2}$ *inches wide, you will need to place the bar about* 9 *inches from each edge; this estimate can be used as a check on the exact computation.* (CCSS: 7.EE.B.3)
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (CCSS: 7.EE.B.4)
   a. Solve word problems leading to equations of the form $px \pm q = r$ and $p(x \pm q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is* 54 *cm. Its length is* 6 *cm. What is its width?* (CCSS: 7.EE.B.4.a)
   b. Solve word problems leading to inequalities of the form $px \pm q > r$, $px \pm q \geq r$, $px \pm q < r$, or $px \pm q \leq r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid* $\$50$ *per week plus* $\$3$ *per sale. This week you want your pay to be at least* $\$100$*. Write an inequality for the number of sales you need to make and describe the solutions.* (CCSS: 7.EE.B.4.b)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Adapt to different forms of equations and inequalities and reach solutions that make sense in context. (Personal Skills: Adaptability/Flexibility)
2. Use mental computation and estimation to check the reasonableness of their solutions. Make connections between the sequence of operations used in an algebraic approach and an arithmetic approach, understanding how simply reasoning about the numbers connects to writing and solving a corresponding algebraic equation. (MP1)
3. Represent a situation symbolically and solve, attending to the meaning of quantities and variables. (MP2)
4. Select an appropriate solution approach (calculator, mental math, drawing a diagram, etc.) based on the specific values and/or desired result of a problem. (MP5)
5. Use estimation, mental calculations, and understanding of real-world contexts to assess the reasonableness of answers to real-life and mathematical problems. (MP6)

Inquiry Questions:

1. Do the properties of operations apply to variables the same way they do to numbers? Why?
2. Why are there different ways to solve equations?
3. In what scenarios might estimation be better than an exact answer?
4. How can the reasonableness of a solution be determined?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students reason about and solve one-step, one-variable equations and inequalities.
3. In Grade 7, this expectation connects with analyzing proportional relationships, using them to solve real-world and mathematical problems, and applying and extending previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
4. In Grade 8, students work with radicals and integer exponents, analyze and solve linear equations and pairs of simultaneous linear equations, and describe functional relationships.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

Grade Level Expectation:

7.SP.A. Statistics & Probability: Use random sampling to draw inferences about a population.

Evidence Outcomes

Students Can:

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; explain that generalizations about a population from a sample are valid only if the sample is representative of that population. Explain that random sampling tends to produce representative samples and support valid inferences. (CCSS: 7.SP.A.1)
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* (CCSS: 7.SP.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Infer about a population using a random sample. (Entrepreneurial Skills: Inquiry/Analysis)
2. Make conjectures about population parameters and support arguments with sample data. (MP3)
3. Use multiple samples to informally model the variability of sample statistics like the mean. (MP4)

Inquiry Questions:

1. Why would a researcher use sampling for a study or survey?
2. Why does random sampling give more trustworthy results than nonrandom sampling in a study or survey? How might methods for obtaining a sample for a study or survey affect the results of the survey?
3. How can a winner be concluded in an election, from a sample, before counting all the ballots?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 6, students develop understanding of statistical variability.
3. In Grade 7, this expectation connects with drawing informal comparative inferences about two populations, investigating chance processes, and with developing, using, and evaluating probability models.
4. In high school, students understand and evaluate random processes underlying statistical experiments and also make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.

Grade Level Expectation:

7.SP.B. Statistics & Probability: Draw informal comparative inferences about two populations.

Evidence Outcomes

Students Can:

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is* 10 cm *greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* (CCSS: 7.SP.B.3)
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* (CCSS: 7.SP.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Interpret variability in statistical distributions and draw conclusions about the distance between their centers using units of mean absolute deviation. (Entrepreneurial Skills: Inquiry/Analysis)
2. Base arguments about the difference between two distributions on the relative variability of the distributions, not just the difference between the two distribution means. (MP3)
3. Model real-world populations with statistical distributions and compare the distributions using measures of center and variability. (MP4)

Inquiry Questions:

1. How do measures of center (such as mean) and variability (such as mean absolute deviation) work together to describe comparisons of data?
2. How can we use measures of center and variability to compare two data sets? Why is it not wise to compare two data sets using only measures of center?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 6, students study measures of center and variability to describe, compare, and contrast data sets.
3. In Grade 7, this expectation connects with using random sampling to draw inferences about a population.
4. In high school, students summarize, represent, and interpret data on a single count or measurement variable and also make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Prepared Graduates:

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

Grade Level Expectation:

7.SP.C. Statistics & Probability: Investigate chance processes and develop, use, and evaluate probability models.

Evidence Outcomes

Students Can:

5. Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS: 7.SP.C.5)
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* (CCSS: 7.SP.C.6)
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (CCSS: 7.SP.C.7)
   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* (CCSS: 7.SP.C.7.a)
   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* (CCSS: 7.SP.C.7.b)
8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (CCSS: 7.SP.C.8)
   a. Explain that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (CCSS: 7.SP.C.8.a)
   b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. (CCSS: 7.SP.C.8.b)
   c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?* (CCSS: 7.SP.C.8.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Be innovative when designing simulations to generate frequencies of compound events by using random digits, dice, coins, or other chance objects to represent the probabilities of real-world events. (Entrepreneurial Skills: Creativity/Innovation)
2. Use probability models and simulations to predict outcomes of real-world chance events both theoretically and experimentally. (MP4)
3. Use technology, manipulatives, and simulations to determine probabilities and understand chance events. (MP5)

Inquiry Questions:

1. Since the probability of getting heads on the toss of a fair coin is $\frac{1}{2}$, does that mean for every one hundred tosses of a coin exactly fifty of them will be heads? Why or why not?
2. What might a discrepancy in the predicted outcome and the actual outcome of a chance event tell us?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In prior grades, students study rational numbers and operations with rational numbers.
3. In Grade 7, probability concepts support the major work of understanding rational numbers. This expectation connects with analyzing proportional relationships, using them to solve real-world and mathematical problems, and using random sampling to draw inferences about a population.
4. In high school, students understand and evaluate random processes underlying statistical experiments, understand independence and conditional probability and use them to interpret data, and use the rules of probability to compute probabilities of compound events in a uniform probability model.

COLORADO Department of Education

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

7.G.A. Geometry: Draw, construct, and describe geometrical figures and describe the relationships between them.

Evidence Outcomes

Students Can:

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS: 7.G.A.1)
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS: 7.G.A.2)
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in cross sections of right rectangular prisms and right rectangular pyramids. (CCSS: 7.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate what side and angle measurements are necessary to determine a unique triangle. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason abstractly by deconstructing three-dimensional shapes into two-dimensional cross-sections. (MP2)
3. Describe, analyze, and generalize about the resulting cross-section of a sliced three-dimensional figure and justify their reasoning. (MP3)
4. Appropriately use paper, pencil, ruler, compass, protractor, or technology to draw geometric shapes. (MP5)

Inquiry Questions:

1. How are proportions used to solve problems involving scale drawings?
2. What are some examples of cross-sections whose shapes may be identical but are from different three-dimensional figures?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 6, students solve real-world and mathematical problems involving area, surface area, and volume.
3. In Grade 7, this expectation connects with analyzing proportional relationships and using them to solve real-world and mathematical problems.
4. In Grade 8, students understand the connections between proportional relationships, lines, and linear equations, and understand congruence and similarity using physical models, transparencies, or geometry software. In high school, students use geometric constructions as a basis for geometric proof.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP6. Attend to precision.

Grade Level Expectation:

7.G.B. Geometry: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Evidence Outcomes

Students Can:

4. State the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (CCSS: 7.G.B.4)
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure. (CCSS: 7.G.B.5)
6. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS: 7.G.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving angle measure, area, surface area, and volume. (Entrepreneurial Skills: Inquiry/Analysis)
2. Persevere with complex shapes by analyzing their component parts and applying geometric properties and measures of area and volume. (MP1)
3. Model real-world situations involving area, surface area, and volume. (MP4)
4. Reason accurately with measurement units when calculating angles, circumference, area, surface area, and volume. (MP6)

Inquiry Questions:

1. How can the formula for the area of a circle be derived from the formula for the circumference of the circle?
2. What are the angle measure relationships in supplementary, complementary, vertical, and adjacent angles?
3. What are some examples of real-world situations where one would need to find (a) area, (b) volume, and (c) surface area?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students understand concepts of angle, measure angles, and solve real-world and mathematical problems involving area, surface area, and volume.
3. In Grade 8, students understand congruence and similarity using physical models, transparencies, or geometry software, and understand and apply the Pythagorean Theorem. Students also use the formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems.

Prepared Graduates:

MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.NS.A. The Number System: Know that there are numbers that are not rational, and approximate them by rational numbers.

Evidence Outcomes

Students Can:

1. Demonstrate informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Define irrational numbers as numbers that are not rational. (CCSS: 8.NS.A.1)
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS: 8.NS.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate rational and irrational numbers and their relative approximate positions on a number line. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use technology to look for repetition in decimal expansions and use number lines to order and compare irrational numbers relative to rational numbers. (MP5)
3. Apply understanding of rational and irrational numbers to describe and work within the structure of the real number system effectively and efficiently. (MP7)
4. Recognize repetition in decimal expansions of rational numbers and recognize when a decimal expansion cannot be represented by a rational number. (MP8)

Inquiry Questions:

1. How many irrational numbers exist?
2. Why is there no real number closest to zero?
3. Can you accurately plot an irrational number on the number line? How do you know?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
3. In Grade 8, this expectation supports working with radicals and integer exponents. This concludes the introduction of all numbers that comprise the real number system.
4. In high school, students use properties of rational and irrational numbers, work with rational exponents, and extend their understanding of number systems to include complex numbers.

COLORADO
Department of Education

Prepared Graduates:

MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.EE.A. Expressions & Equations: Work with radicals and integer exponents.

Evidence Outcomes

Students Can:

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (CCSS: 8.EE.A.1)
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares (up to 100) and cube roots of small perfect cubes (up to 64). Know that $\sqrt{2}$ is irrational. (CCSS: 8.EE.A.2)
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3 *times* 10^8 *and the population of the world as* 7 *times* 10^9*, and determine that the world population is more than* 20 *times larger.* (CCSS: 8.EE.A.3)
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (CCSS: 8.EE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason about unusually large or small quantities. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use calculators or computers to compute with and approximate radicals and roots, and understand how such tools represent scientific notation. (MP5)
3. Explore the structure of numerical expressions with integer exponents to generate equivalent expressions. (MP7)
4. Look for how positive integer exponents are equivalent to repeated multiplication by the base and how negative integer exponents are equivalent to repeated division by the base. (MP8)

Inquiry Questions:

1. How is performing operations on numbers in scientific notation similar to or different from performing operations on numbers in standard notation?
2. Why does a positive number raised to a negative exponent not equal a negative number?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students use whole-number exponents to denote powers of ten. In Grades 6 and 7, they work with algebraic and numerical expressions containing whole-number exponents.
3. In Grade 8, this expectation connects with knowing that there are numbers that are not rational and approximating them by rational numbers, understanding and applying the Pythagorean Theorem, and solving real-world and mathematical problems involving volume of cylinders, cones, and spheres. In high school, students extend work with exponents to rational exponents.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.EE.B. Expressions & Equations: Understand the connections between proportional relationships, lines, and linear equations.

Evidence Outcomes

Students Can:

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (CCSS: 8.EE.B.5)
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (CCSS: 8.EE.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make connections between representations of linear growth. (Entrepreneurial Skills: Inquiry/Analysis)
2. Record information about constant rates of change in graphs and equations. (Entrepreneurial Skills: Literacy/Writing)
3. Make sense of and compare proportional relationships represented in different forms. (MP1)
4. Compare, contrast, and make claims with proportional relationships based on properties of equations, tables, and/or graphs. (MP3)
5. Explore the structure of proportional relationships expressed as equations or graphs for methods of comparison. (MP7)

Inquiry Questions:

1. How is the unit rate of a proportional relationship related to the slope of its graphical representation?
2. Why are similar triangles effective for describing slope geometrically?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students recognize and represent proportional relationships, calculate the constant of proportionality, and graph proportional relationships on the coordinate plane, recognizing that they always pass through the origin.
3. In Grade 8, this expectation connects with analyzing and solving linear equations and pairs of simultaneous linear equations, with defining, evaluating, and comparing functions, and with understanding congruence and similarity using physical models, transparencies, or geometry software.
4. In high school, students compare multiple representations of inverse proportional relationships.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.EE.C. Expressions & Equations: Analyze and solve linear equations and pairs of simultaneous linear equations.

Evidence Outcomes

Students Can:

7. Solve linear equations in one variable. (CCSS: 8.EE.C.7)
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (CCSS: 8.EE.C.7.a)
   b. Solve linear equations with rational number coefficients, including equations with variables on both sides and whose solutions require expanding expressions using the distributive property and collecting like terms. (CCSS: 8.EE.C.7.b)
8. Analyze and solve pairs of simultaneous linear equations. (CCSS: 8.EE.C.8)
   a. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (CCSS: 8.EE.C.8.a)
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be* 5 *and* 6*.* (CCSS: 8.EE.C.8.b)
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (CCSS: 8.EE.C.8.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving linear equations and systems of linear equations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Solve problems that require a system of linear equations in two variables. (MP1)
3. Model real-world problems with linear equations and systems of linear equations, with variables defined in their real-world context. (MP4)
4. Solve equations and systems of equations and express solutions with accuracy that makes sense in the real-world context modeled by the equations. (MP6)
5. Recognize the structure of equations and of systems of equations that produce one, infinitely many, or no solution. (MP7)

Inquiry Questions:

1. What is meant by a “solution” to a linear equation? What is meant by a “solution” to a system of two linear equations? How are these concepts related?
2. How is it possible for an equation to have more than one solution? How is it possible for an equation to have no solution?
3. Why can’t a system of linear equations have a solution set other than one, zero, or infinitely many solutions?
4. What connections exist between the graphical solution and the algebraic solution of a system of linear equations?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students reason about and solve one-step and two-step, one-variable equations and inequalities, use properties of operations to generate equivalent expressions, and solve real-world and mathematical problems using numerical and algebraic expressions and equations.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations and with investigating patterns of association in bivariate data.
4. In high school, students abstract and generalize about linear functions and how they compare and contrast to nonlinear functions. Students also reason about and solve systems of equations that include one or more nonlinear equations.

COLORADO Department of Education

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.F.A. Functions: Define, evaluate, and compare functions.

Evidence Outcomes

Students Can:

1. Define a function as a rule that assigns to each input exactly one output. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required for Grade 8.) (CCSS: 8.F.A.1)
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (CCSS: 8.F.A.2)
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$, $(2,4)$ and $(3,9)$, which are not on a straight line.* (CCSS: 8.F.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make connections between the information gathered through tables, equations, graphs, and verbal descriptions of functions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Define variables as quantities and interpret ordered pairs from a functional relationship with respect to those variables. (MP2)
3. With and without technology, analyze and describe functions that are not linear with the use of equations, graphs, and tables. (MP5)
4. See a function as a rule that assigns each input to exactly one output; this structure does not "turn inputs into outputs"; rather, it describes the relationship between items in two sets. (MP7)
5. Recognize patterns of linear growth in different representations of linear functions. (MP8)

Inquiry Questions:

1. Why is it important to know if a mathematical relationship is a function?
2. How can you determine if a function is linear or nonlinear?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations and with using functions to model relationships between quantities.
4. In high school, students use function notation, analyze functions using different representations, build new functions from existing functions, and extend from linear functions to quadratic, exponential, and other more advanced functions.

COLORADO Department of Education

Prepared Graduates:

MP4. Model with mathematics.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.F.B. Functions: Use functions to model relationships between quantities.

Evidence Outcomes

Students Can:

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS: 8.F.B.4)
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS: 8.F.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Describe in writing the qualitative features of linear or nonlinear functions. (Entrepreneurial Skills: Literacy/Writing)
2. Model real-world situations with linear functions. (MP4)
3. Explore properties of linear functions and how those properties appear in the structure of linear equations in slope-intercept form. (MP7)
4. Use strategies to calculate the rate of change in a linear function (slope) and use properties of linear functions to create equations. (MP8)

Inquiry Questions:

1. What is the minimum information needed to write a linear function for a relationship between two quantities?
2. What are some quantitative and qualitative features of graphs of functions?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems.
3. In Grade 8, this expectation connects with defining, evaluating, and comparing functions and with investigating patterns of association in bivariate data.
4. In high school, students use function notation, analyze functions using different representations, build new functions from existing functions, and extend from linear functions to quadratic, exponential, and other more advanced functions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.SP.A. Statistics & Probability: Investigate patterns of association in bivariate data.

Evidence Outcomes

Students Can:

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (CCSS: 8.SP.A.1)
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (CCSS: 8.SP.A.2)
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of* 1.5 *cm/hr as meaning that an additional hour of sunlight each day is associated with an additional* 1.5 *cm in mature plant height.* (CCSS: 8.SP.A.3)
4. Explain that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* (CCSS: 8.SP.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize and describe patterns in bivariate data. (Entrepreneurial Skills: Inquiry/Analysis)
2. Interpret the contextual meaning of slope and y-intercept, where applicable in a linear model fit to bivariate data. (MP2)
3. Build statistical models to explore, describe, and generalize the relationship between two variables. (MP4)
4. Use scatter plots and two-way tables to describe possible associations in bivariate data. (MP7)

Inquiry Questions:

1. In what ways is a scatter plot useful in describing and interpreting the relationship between two quantities?
2. Why would we create a linear model for a set of bivariate data?
3. How do you know when a credible prediction can be made from a linear model of bivariate data?
4. What does a pattern of association look like for categorical data?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In previous grades, students apply and extend previous understandings of numbers to the system of rational numbers.
3. In Grade 8, this expectation supports using functions to model relationships between quantities.
4. In high school, students summarize, represent, and interpret data on two categorical and quantitative variables, interpret linear models, and understand independence and conditional probability.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.G.A. Geometry: Understand congruence and similarity using physical models, transparencies, or geometry software.

Evidence Outcomes

Students Can:

1. Verify experimentally the properties of rotations, reflections, and translations: (CCSS: 8.G.A.1)
   a. Lines are taken to lines, and line segments to line segments of the same length. (CCSS: 8.G.A.1.a)
   b. Angles are taken to angles of the same measure. (CCSS: 8.G.A.1.b)
   c. Parallel lines are taken to parallel lines. (CCSS: 8.G.A.1.c)
2. Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (CCSS: 8.G.A.2)
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (CCSS: 8.G.A.3)
4. Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (CCSS: 8.G.A.4)
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* (CCSS: 8.G.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Think about how rotations, reflections, and translations of a geometric figure preserve congruence as similar to how properties of operations such as the associative, commutative, and distributive properties preserve equivalence of arithmetic and algebraic expressions. (Entrepreneurial Skills: Critical Thinking/Problem Solving and Inquiry/Analysis)
2. Explain a sequence of transformations that results in a congruent or similar triangle. (MP3)
3. Use physical models, transparencies, geometric software, or other appropriate tools to explore the relationships between transformations and congruence and similarity. (MP5)
4. Use the structure of the coordinate system to describe the locations of figures obtained with rotations, reflections, and translations. (MP7)
5. Reason that since any one rotation, reflection, or translation of a figure preserves congruence, then any sequence of those transformations must also preserve congruence. (MP8)

Inquiry Questions:

1. How are properties of rotations, reflections, translations, and dilations connected to congruence?
2. How are properties of rotations, reflections, translations, and dilations connected to similarity?
3. Why are angle measures significant regarding the similarity of two figures?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students solve problems involving angle measure, area, surface area, and volume, and draw, construct, and also describe geometrical figures and the relationships between them.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations.
4. In high school, students extend their work with transformations, apply the concepts of transformations to prove geometric theorems, and use similarity to define trigonometric functions.

COLORADO
Department of Education

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.G.B. Geometry: Understand and apply the Pythagorean Theorem.

Evidence Outcomes

Students Can:

6. Explain a proof of the Pythagorean Theorem and its converse. (CCSS: 8.G.B.6)
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS: 8.G.B.7)
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS: 8.G.B.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Think of the Pythagorean Theorem as not just a formula, but a formula that only holds true under certain conditions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Construct a viable argument about why a proof of the Pythagorean Theorem is valid. (MP3)
3. Test to see if a triangle is a right triangle by applying the Pythagorean Theorem. (MP7)
4. Use patterns to recognize and generate Pythagorean triples. (MP8)

Inquiry Questions:

1. What is the relationship between the Pythagorean Theorem and its converse? In what ways is each useful?
2. Is it always possible to use the Pythagorean Theorem to find the distance between points on the coordinate plane? How do you know?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grades 6 and 7, students solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
3. In Grade 8, this expectation connects with radicals and integer exponents, square roots, and solving simple equations in the form $x^2 = p$.
4. In high school, students (a) prove and apply trigonometric identities, (b) prove theorems involving similarity, (c) define trigonometric ratios and solve problems involving right triangles, (d) translate between the geometric description and the equation for a conic section, and (e) use coordinates to prove simple geometric theorems algebraically.

COLORADO Department of Education

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

Grade Level Expectation:

8.G.C. Geometry: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Evidence Outcomes

Students Can:

9. State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (CCSS: 8.G.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Efficiently solve problems using established volume formulas. (Professional Skills: Task/Time Management)
2. Describe how the formulas for volumes of cones, cylinders, and spheres relate to one another and to the volume formulas for solids with rectangular bases. (MP3)
3. Use appropriate precision when solving problems involving measurements and volume formulas that describe real-world shapes. (MP6)

Inquiry Questions:

1. How are the formulas of cones, cylinders, and spheres similar to each other?
2. How are the formulas of cones, cylinder, and spheres connected to the formulas for pyramids, prisms, and cubes?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 7, students solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
3. In Grade 8, this expectation connects with radicals and integer exponents.
4. In high school, students apply geometric concepts in mathematical modeling situations and to solve design problems.

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5,\ 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5,\ 5 = 5 + 0$ $5 = 1 + 4,\ 5 = 4 + 1$ $5 = 2 + 3,\ 5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5,\ 5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?,\ 3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?,\ ? + 3 = 5$

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

	Unknown Product	**Group Size Unknown** ("How many in each group? Division)	**Number of Groups Unknown** ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red had costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long as the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first.
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here, a, b, and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every a there exists $-a$ so that $a+(-a)=(-a)+a=0$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Table 4. The properties of equality. Here, a, b, and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a=b$, then $b=a$.
Transitive property of equality	If $a=b$ and $b=c$, then $a=c$.
Addition property of equality	If $a=b$, then $a+c=b+c$.
Subtraction property of equality	If $a=b$, then $a-c=b-c$.
Multiplication property of equality	If $a=b$, then $a\times c=b\times c$.
Division property of equality	If $a=b$ and $c\neq 0$, then $a\div c=b\div c$.
,Substitution property of equality	If $a=b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here, a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a<b$, $a=b$, $a>b$.

If $a>b$ and $b>c$ then $a>c$.

If $a>b$, then $b<a$.

If $a>b$, then $-a<-b$.

If $a>b$, then $a\pm c>b\pm c$.

If $a>b$ and $c>0$, then $a\times c>b\times c$.

If $a>b$ and $c<0$, then $a\times c<b\times c$.

If $a>b$ and $c>0$, then $a\div c>b\div c$.

If $a>b$ and $c<0$, then $a\div c<b\div c$.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Some examples of situations requiring modeling might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and financial investments.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.