Augenblick, Palaich
and Associates, Inc.

Cost of Enrollment and Enrollment Change in Colorado Districts

Prepared by:

Douglas Rose
Augenblick, Palaich and Associates, Inc.
Denver, Colorado

January, 2007

Executive Summary

This report was prepared by Augenblick, Palaich and Associates, Inc. (APA), a Denver-based consulting firm that has worked with state policy makers on school funding issues for more than 20 years. The report examines how Colorado's education funding system allocates funds to districts of differing size and enrollment patterns. It identifies five district size groupings (from very small to very large) and six different district enrollment patterns (which represent various stages of student enrollment decline or growth). These size and enrollment groupings are then used to analyze different aspects of district spending and cost.

The analysis indicates that Colorado's current funding system does not fully account for some of the cost nuances that districts face. This is particularly true with regard to three elements of student cost:

1. Fixed cost: Some district cost occurs before a student ever arrives. These costs, which include such items as maintaining a district headquarters and staff and the need to comply with paperwork, record-keeping, and basic legal requirements, are embedded in every district's operations.
2. Current cost: Most of this cost occurs when the student attends school in the district.
3. Post cost: Some costs reflect resources needed to serve students who attended schools in the district in prior years. Teachers, for instance, are hired and remain in their jobs despite minor fluctuations in enrollment from year to year.

To more accurately quantify these costs, we create a uniform dollar measure (called a Colorado Yearly Pupil Unit (CYPU)). CYPU units represent *average statewide current expenditures* per full time student in a given year. In 2004-05, the state average per-student spending was $6,918, which would represent one CYPU for that year. Applying this CYPU approach to the three cost elements listed above, we find:

1. Every district incurs a fixed cost of 48.4 CYPU.
2. Current costs are equal to .627 CYPU times the current enrollment.
3. Post costs are phased over a four year period. Such costs can be accounted for based on prior year enrollments so that districts would receive: .215 CYPU times the previous year's enrollment; .096 CYPU times the enrollment two years prior; .043 times the enrollment three years prior, and .019 times the enrollment four years prior.

By identifying the cost portions associated with fixed, current, and post costs, we are able to generate a comparison between the current Colorado system and a revised system which could more fully account for the cost nuances districts face. In many cases, Colorado's current system produces results similar to a revised system. The primary differences are associated with very small, declining districts and very large, growing ones. For these types of districts, the current Colorado system may be too generous in its allocation of resources. Under a revised system, such districts would likely receive fewer dollars per student, but other districts in the state would receive more.

I. Current Colorado District Spending Patterns

How much do Colorado school districts currently spend per student? How much more do large or small Colorado districts spend than the average? How much more or less do growing or shrinking districts spend? How many of the differences in spending are justified and how much is just a consequence of politics and Colorado's current education funding formula? These are the questions asked in this section. To answer them, we classify districts in two ways:

1. By their **size**, and
2. By their **pattern of enrollment change**.

The size classification is based on 1987-88 enrollment which was the earliest year of data availability. The pattern of enrollment change examines enrollment from 1987-88 to 2004-05. Using the 1987-88 enrollment figures, districts are grouped into five size categories (shown in Table 1).

Table 1: Classifying Colorado School Districts by Size

Name	FTE Enrollment	Number of Districts
Very Small:	<250	48
Small:	251-500	40
Moderate:	501-2000	45
Large:	2001-14,000	31
Very Large:	>14,000	12

The clearest district enrollment patterns are of growth, decline, and stability (or flat enrollment). From these three patterns we define six types of district enrollment:

1. "Growth" = Sustained enrollment growth over time.
2. "Decline" = Sustained enrollment decline over time.
3. "Rapid Growth" = Districts showing exponential enrollment growth (only one district qualifies for this in Colorado).
4. "Late growth" = A period of flat enrollment followed by growth.
5. "Plateau growth" = A period of growth followed by flat enrollment.
6. "Other" patterns = Districts showing combinations of the above patterns.

These enrollment patterns, and the number of Colorado districts they apply to, are shown in Table 2 below.

Table 2: Classifying Colorado School Districts by Enrollment Pattern

Name	Enrollment Pattern	Number of Districts
Rapid Growth	Exponential Growth	1
Growth	Steady Growth	28
Late Growth	Stability, then Growth	9
Plateau	Growth, then Stability	19
Decline	Steady Decline	13
Other	Complexity	104

The combination of district size and enrollment change patterns from 1987-88 to 2004-05 (shown in Table 3) shows some relationship between the two. For instance, there is a tendency for the initially larger districts to subsequently grow and for the smaller districts to show less steady growth. Table 3 also shows that just above half of the state's districts are classified as either small or very small. Colorado districts, with their yearly enrollments fitting these patterns, are shown in the graphs in the Appendix.

Table 3: Grouping Colorado Districts by Size and Enrollment Pattern

Enrollment Pattern	Very Large	Large	Moderate	Small	Very Small	Grand Total
Decline	1	1	3	4	4	13
Plateau		1	2	6	10	19
Other	3	16	30	27	28	104
Late Growth		2	1	1	5	9
Growth	8	10	7	2	1	28
Rapid Growth		1				1
District Totals	*12*	*31*	*43*	*40*	*48*	*174*

*Colorado has 178 school districts. However, due to district reorganization useful data over the time period studies was only available for 174 districts.

Spending Analysis by District Size and Enrollment Pattern

Having classified Colorado's school districts by both size and pattern of enrollment, we can now analyze district patterns in terms of per-student spending. We start with a description of spending patterns from 1992-93 through 2004-05. We classify district spending in terms of the yearly average in statewide, per pupil spending. For instance, the average spending in 1992-93 was $4,323 per student. In that year, Weld County RE-1 spent $4,110 per student. We describe the RE-1 spending as 4,110/4,323, which equals .95. We then average this spending-per-statewide-average across the 13 included years (1992-93 through 2004-05). The results are presented in the table below, expressed in terms of 2004-05 spending.

Table 4: Per-Student Spending Based on District Size and Enrollment Pattern

Enrollment Pattern	*Size* (Based on 1988 enrollment)					
	Very Large	Large	Moderate	Small	Very Small	Total
Decline	$ 6,680	$ 6,483	$ 6,933	$ 7,762	$ 11,807	$ 8,634
Plateau	$ -	$ 5,935	$ 6,087	$ 7,048	$ 9,952	$ 8,417
Other	$ 6,806	$ 6,695	$ 6,900	$ 7,471	$ 10,055	$ 7,863
Late Growth	$ -	$ 6,186	$ 6,146	$ 6,598	$ 10,328	$ 8,528
Grow	$ 7,030	$ 6,332	$ 7,037	$ 6,598	$ 7,029	$ 6,752

Rapid Grow	$ -	$ 6,750	$ -	$ -	$ -	$ 6,750
Total	$ 6,945	$ 6,515	$ 6,777	$ 7,371	$ 10,145	$ 7,681

Two observations are worth noting. First, most districts have fewer than the mean number of students. Second, smaller districts tend to spend more per student, largely because they lack the economies of scale which larger districts can benefit from. Because of these two factors, many Colorado districts spend more than the 2004-05 statewide average (which was $6,918 per student). The smallest districts spend the most, and large (but not the largest) districts spent the least per student. Districts which were declining in enrollment spent the most per student, while districts that were growing spent the least. As might be expected, very small districts with declining enrollment spent far more per student than anyone else, while large and growing districts spent comparatively little. Additionally, because districts that plateau are mostly very small, the overall average for the plateau pattern reflects the higher costs in very small districts rather than the average amounts from other size districts that plateau.

II. Is There a Different Way Colorado Might Allocate its Education Dollars?

The section above seeks to identify how district size and enrollment change currently interact with per-student spending. The analysis does not, however, analyze whether or how Colorado might consider changing or improving the way it allocates district funds. To consider any such changes, it is important that policymakers take a more precise view of the impact of student enrollment on cost. Specifically, a student's full cost impact can perhaps best be viewed in terms of three dimensions:

1) Fixed cost: Some district cost occurs before a student ever arrives. These costs, which include such items as maintaining a district headquarters and staff and the need to comply with paperwork, record-keeping, and basic legal requirements, are embedded in district operations.

2) Current cost: Most of this cost occurs when the student attends school in the district.

3) Post cost: Some costs reflect resources needed to serve students who attended schools in the district in prior years. Librarians, for instance, may remain in their jobs despite minor fluctuations in enrollment from year to year.

This section of our report provides estimates for each of these three dimensions of student cost. It does so by analyzing district enrollment and spending throughout Colorado for the years 1987-88 through 2004-2005.

The first step in the process is to identify a uniform means of representing cost over time. This is important when one considers that dollars change in value from year to year due to inflation, and that the Colorado school funding formula was also modified over time. For the purposes of our analysis, the Colorado Yearly Pupil Unit (CYPU) offers a more uniform means of representing cost. CYPU units represent *average statewide current expenditures* per full time student (FTE) in

a given year. In 2004-05, the state average per-student spending was $6,918, which would represent one CYPU for that year.

As discussed earlier, there are three dimensions of district cost. The first precedes the student and is not really a per pupil cost at all. Each district has these embedded or fixed costs simply by being a district. We estimate these costs at 48.4 CYPU. On a per student basis, the significance of this initial cost depends on the number of students. For instance, if there are 100,000 students in a district, it is a minuscule cost. If, however, there are only 100 students, the 48.4 CYPU cost is significant. The current formula accommodates these costs as part of a per pupil amount which is higher for smaller districts.

The second dimension of cost is the per pupil current cost. Colorado's state funding formula, allocates almost all its dollars to districts based on this cost. In fact, we estimate that over 99.9% of Colorado's funding matches current student attendance. The problem with this system is that not all of a student's cost is represented by current attendance. In fact, we estimate that current attendance accounts for only .627 of a CYPU. The remainder of cost is explained in the next paragraph.

The third portion of the CYPU, which we term the "post" cost, is the share needed to serve students who attended schools in the district in prior years. For example, our analysis finds that when students leave a district a different cost structure applies. That is, the cost of a student rapidly declines over time. In fact as Table 5 indicates, after a student leaves the cost is expected to steadily decline until it reaches almost zero after four years. As the data in the table shows, adding the current cost to the post costs spread out over four years equals one full CYPU.

Table 5: Current and Post Cost Estimates

	Enrollment Year				
	Current	One Year Ago	Two Years Ago	Three Years Ago	Four Years Ago
Weight (in CYPU)	.627	.215	.096	.043	.019

As presently configured, Colorado's school finance formula does not recognize student cost in the three dimensions discussed above. Instead, the state's school finance formula encourages districts to treat costs for departed students as if they continue at the same level for up to three additional years. This means, for instance, that districts with declining enrollments are funded by the state for three additional years based on the full cost of those students who left. For increasing enrollment, the current formula funds new students at full cost, rather than at the estimated first year marginal cost of .627 of the average yearly cost.

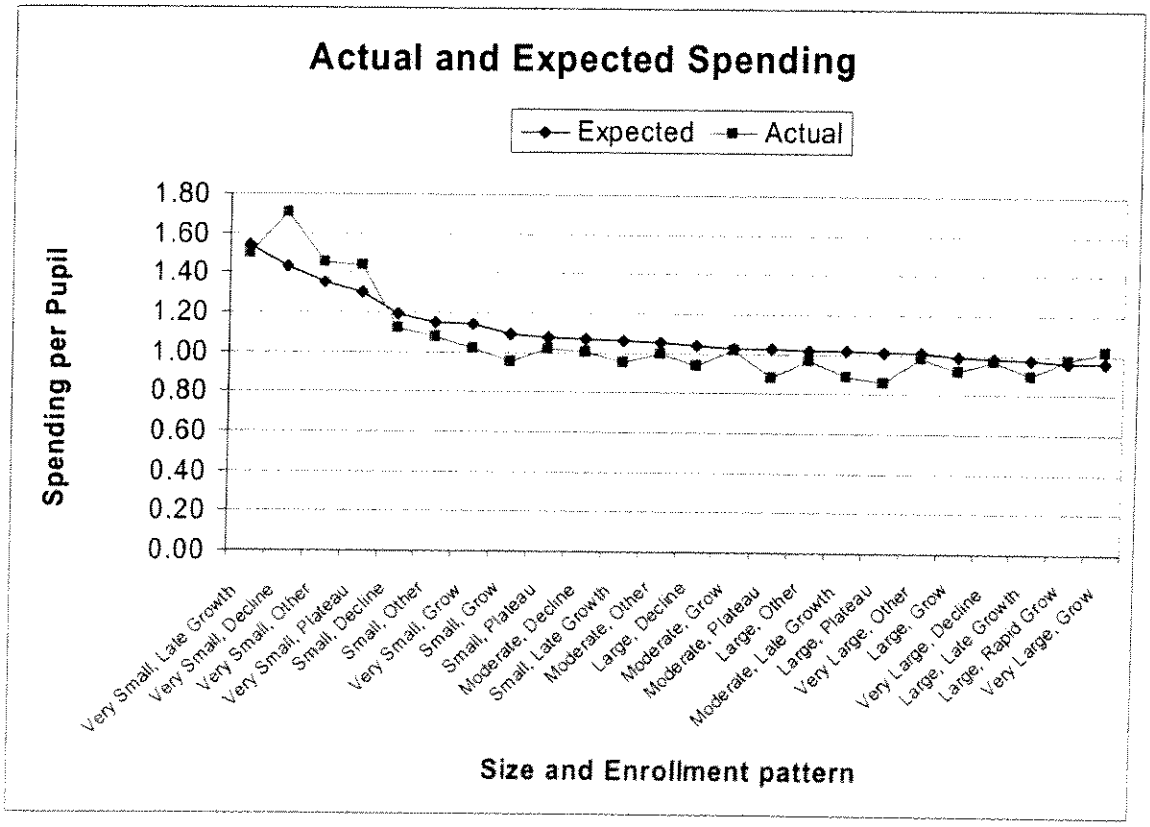
To make the funding system more precise, Colorado could account for district cost based on a five year span (current enrollment and enrollments over the prior four years). In other words, districts would receive .627 for every student currently enrolled. In addition, each district would receive: .215 CYPU times the previous year's enrollment; .096 CYPU times the enrollment 2 years prior; .043 times the enrollment 3 years prior, and .019 times the enrollment 4 years prior.

This method would allow Colorado to more accurately fund districts based on the actual current and post costs they accrue.

Were Colorado to consider changing its funding system to more precisely match the costs districts face, districts around the state could be impacted in different ways. The next section is designed to help understand how such impacts might differ.

How Districts Might be Impacted if Colorado's Funding System Were Changed

Districts of different size and growth patterns would be affected differently if Colorado's current funding structure were changed to match the enrollment-based CYPU formula discussed above. The chart below illustrates how current (or "actual") district spending might look under an "expected" CYPU-based formula. To translate the figures in the chart into dollars, simply multiply the number by the 2004-05 state average per-student spending of $6,918.



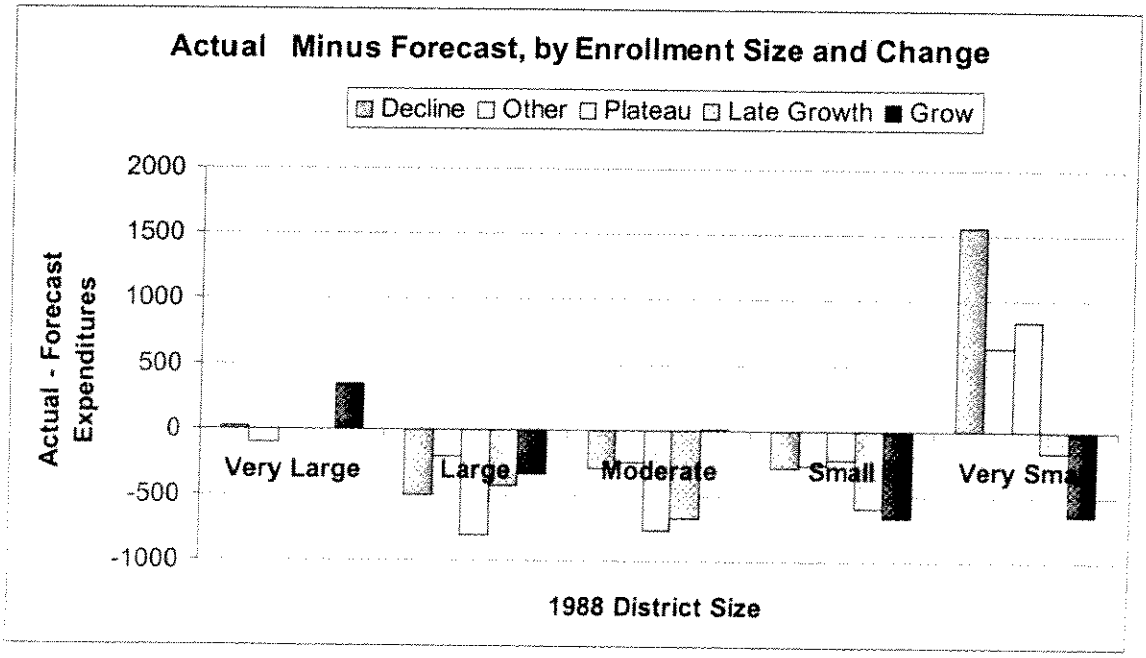
The combination of size and enrollment pattern is more complex than either one alone would suggest. Very small districts that either decline or plateau in enrollment have more dollars to spend per student under the current system than under a CYPU system. This is because, under the CYPU approach, these districts would no longer be able to count for three years the full costs of students who leave the district. Similarly large, rapidly growing districts and very large, growing districts might receive less since they would only receive .627 CYPU for each new student.

The information in the chart is displayed numerically in Table 6 below. District types that show negative numbers are those whose current spending is below what it would be under a CYPU system. There are only two size or enrollment patterns that cause districts to fare better or worse under all circumstances. Small districts do worse under the current system than they would under an enrollment-based system, no matter what their enrollment pattern. Districts with stable enrollment followed by growth (late growth), would be better funded under a CYPU system rather than the current formula no matter what their size.

Table 6: Is Current District Spending Above or Below What it Would be Under a CYPU-Based System?

	1988 Size				
1992-2005 Average	Very Large	Large	Moderate	Small	Very Small
Decline	13	-503	-299	-290	1559
Other	-106	-207	-243	-266	638
Plateau		-803	-765	-228	844
Late Growth		-433	-675	-593	-165
Grow	339	-337	9	-669	-646
Rapid Growth		170			

The same information appears in a different graphic format below. Information is not shown for individual districts because past enrollment patterns do not necessarily represent what the future will look like.



Conclusion

This report examines how Colorado's education funding system allocates funds to districts of differing size and enrollment patterns. It identifies five district size groupings (from very small to very large) and six different district enrollment patterns (which represent various stages of enrollment decline or growth). Our work uses these size and enrollment groupings to analyze different aspects of district spending and cost. Findings indicate that Colorado's current funding system does not fully account for some of the cost aspects that districts face. This is particularly true with regard to three elements of student cost: 1) a fixed cost; 2) a current cost; and 3) a "post" cost.

By creating a uniform dollar measure (which we term a CYPU) and by determining the proper cost portions associated with fixed, current, and post costs, we are able to generate a comparison between the current Colorado system and a revised system which fully recognizes the cost nuances. While this comparison indicates that, in many cases, Colorado's current system produces similar results to the revised system, the current system is more generous with very small, declining districts and very large, growing ones, while an enrollment-based system would better fund small or late-growing districts.

Technical Appendix

The data for this study are for the years 1987-88 through 2004-2005. District spending on operations is indicated by a total, plus subtotals for instruction, operations & maintenance, administration, pupil support, and other support. Additionally, data is available on other spending and non-operational spending. Enrollment data is in terms of FTE, with subcategories for free and reduced price lunch, English as a second language, and special education students.

Over time, spending has increased, as has enrollment. Spending per pupil has increased, as witness the inflation adjustments in the base amount. There have also been other adjustments in this period. For purposes of comparing over time, we fold all these yearly increases together and use the yearly changes in the average state per pupil payment. Unless otherwise mentioned, the data are adjusted by the yearly averages to reflect these changing costs. In addition, changing standards have led to an increase in the share of the budget devoted to pupil support.

Overall and instruction costs have increased at a 3% pace on average during the period. Administrative and pupil support costs have increased at slightly lower rates. Operating and maintenance costs and other support costs, however, have not on the whole increased as fast and reliably. In order to correct for these time-related trends, we adjust spending so that it is unrelated to year.

In using patterns of spending to infer what the state formula ought to be, the immediate problem is that spending is largely a result of the state formula in force. So, we run the risk of using the current arrangement to justify perpetuating the current arrangement, which is circular reasoning. In general, we wish to analyze spending in ways that are not closely linked to the current formula.

The main procedure for dealing with this problem is to focus on spending changes that are driven by enrollment changes. In particular, we are interested in the lag between enrollment changes and cost changes. Some costs, we expect, continue after the student is gone, and some costs don't immediately start on the student's arrival. For the costs of decreased enrollment, the state formula allows districts to average up to 4 years of enrollment, effectively assuming continued costs despite departed students. Because of this policy, our analyses of decreased costs are more complicated than those for increased costs, where the state has no policy of delayed payment. We expect that, if cost increases are below enrollment increases, districts will respond, within limits, as if they had discretion, and spending in the usual categories will not increase as fast as enrollment. As that enrollment continues, however, costs catch up. To model this, we model spending changes (last year divided by this year) as a function of enrollment changes (this year through five years ago divided by this year). Implicit in this is a model of spending as a function of enrollment, of which the version of spending change as a function of enrollment change is an expression.

The simplest method of doing this is to estimate a multiple regression equation of spending ratios and enrollment rations, and we do this. These regression estimates become a baseline against which to measure the adequacy of alternative approaches, for the regression provides the

minimum error. In the case of spending decreases, the minimum is not very good, because there is no clear pattern of enrollment changes causing the spending changes – the amount of spending change depends on enrollment changes outside the period examined, because some of the spending hasn't dropped earlier in response to earlier enrollment changes.

The regression coefficients, however, are too unconstrained. We wish to have weights for each year that add to one, so that we can apportion the spending by enrollment year. Additionally, we don't wish to have negative weights (more students means less money). Finally, we wish to have weights which follow some ascending or descending order. A formula which embodies these constraints is to have coefficients of the form b * x^year, where b and x are coefficients to be estimated, year is 5 for the current year and 1 for four years ago. Additionally, the weights are constrained to sum to 1. This means that the coefficient b can be expressed in terms of the coefficient x. The coefficient for the current year is $x^4(1-x)/(1-x^5)$, the coefficient for the prior year is $x^3(1-x)/(1-x^5)$, and for the earliest year is $(1-x)/(1-x^5)$. When the terms are rearranged, this provides a polynomial in X which can be solved by iteration.

Indeed, it can be solved different ways, to match different criteria. We have used the variable means (a good solution), the covariances (a poor solution), maximizing the correlation of the estimate with spending (a good solution), and minimizing the absolute deviations of an intercept from 0 and the regression coefficient from 1 when spending is regressed on the estimate (the chosen solution).

The minimum deviation solution is nearly (98.9%) as efficient as the best linear unbiased estimator. The coefficients are reported below. The apportionment of expense across years is 56.5% for the current year, 25.1% for the prior year, 11.2% for the year before that, 5% for the next earlier year, and 2.2% for earliest of the five years.

Current	Prior	3 Years	4 years	5 years
.565	.251	.112	.050	.022

Correspondingly, in determining allocations, the formula would give a weight of .565 to current students, .251 to last year's students, etc.

The next step was to test the predictions for five different size groups.

Very large >14,000 enrollment
14,000 > Large > 2,000
2,000 > Medium > 500
500 > Small > 250
250> Very Small

The overall minimum deviation solution is, with one exception, at least 93.7% and at most 100% as efficient as the most efficient estimator. The exception is for Small districts, for which the solutions is only 80.9% as efficient as the best linear unbiased estimate.

To translate from spending change to actual spending, we regress normalized spending on the enrollment plus the weighted yearly enrollment figures from the minimum deviation results. The result predicts spending as well as the best linear unbiased estimator. The combined formula (in terms of average spending per pupil, which is 1 for any year) is

District spending = 48.421 + 0.626714909 * Current Enrollment + 0.215129869 * Year-1 Enrollment +0.09567797 * Year -2 Enrollment + 0.042552315 * Year -3 Enrollment + 0.018924937 * Year -4 Enrollment

The sum of the yearly coefficients is .999 of an average per pupil spending, with the rest made up by the constant. The constant represents the cost of the district operations without any students. Hence, consolidation of two districts into one might be expected to save 48.4 times the average student spending.

To return to actual district spending, we multiply by the average state yearly spending per pupil.

Spending Categories

In addition to the analyses leading to the results reported in the text, we explored the shifts in the role of types of spending..

We examined how responsive (elastic) subcategories of expenditure are to overall levels of expenditure. Unfortunately, they track overall spending fairly closely. Instruction is such a large share of spending that its results closely track overall results, hence add little new information. The other categories are plagued with estimating problems. Essentially, we are dividing a ratio of two years in a spending category by another ratio of overall spending in two years, a procedure that can magnify small differences, especially in the case of Other Support, which is zero a number of years in many districts.

Category	Response to Overall Spending Change
Instruction	99.6%
Operations	98.5%
Administration	99.4%
Pupil Support	100.9%
Other Support	99.5%

The least responsive category is operations and maintenance. These estimates result from a regression analysis, through the origin, of the ratio of spending on a subcategory in two consecutive years to the corresponding overall spending ratio. When overall funds go up or down, spending on operations goes up or down more slowly. Instruction, administrative, and other spending change slightly less than does overall spending. Pupil support is the most flexible of expenditure categories. This implies that changes in pupil support spending are most likely to result from the availability of funding, while changes in spending on operations and maintenance depend least on the availability of funds. These conform to our initial expectations, except that we would have predicted a reversal of the results for operations and administration.

A second approach was to use the changes in the subcategories to estimate the overall spending change. The weight given to proportional changes in categories when computing overall proportional changes indicates the share of changed spending due to changes in the category. If

no special considerations applied, then these would be equal to the categories share of total spending. A difficulty with these analyses is that changes in categories of spending are highly correlated with each other, leading the more minor categories to have less reliable coefficients. One way of dealing with this is to use an intercept, which is the unallocated portion of total spending change, representing a portion that depends on circumstances. As indicated below, Other Support and Operations & Maintenance are again least flexible and show smaller shares of change than of overall spending. Much of this falls into the unallocated amount. Instruction, administration, and pupil support have change shares close to their overall shares. Whether or not operations and other support go up or down with overall apparently depends on district factors that we have not captured.

Category	Average Shares	Change Shares
Instruction	65.9%	67.3%
Operations	11.2%	0.3%
Administration	12.5%	12.4%
Pupil Support	7.2%	7.5%
Other Support	3.2%	0.4%
Unallocated		11.5%

In general, the implications so far are that the analysis of spending change by spending categories does not add much to the overall analysis.

At the risk of simply reflecting current policy, we focused on current spending, rather than spending change.

We also tried removing extreme scores. Other support is erratic, often zero, with more dramatic changes than the other categories. It is at once sticky and subject to large changes, just not small ones. We exclude dramatic changes (one-year changes quadrupling or reducing by three-quarters) in operations, pupil support, and other support.

Our results for instruction are similar to the overall results. However, the results for other support are bizarre, with almost all weight going to enrollment in the earliest year. By contrast, operations, administration, and pupil support are all closely tied to current enrollment. This latter seems to reflect current state funding policy.

	Current Year	Prior Year	2 Years Ago	3 Years Ago	4 Years Ago
Instr	0.678535	0.219766	0.071178	0.023053	0.007467
O&Main	0.802234	0.15885	0.031454	0.006228	0.001233
Admin	0.810202	0.153937	0.029248	0.005557	0.001056
PupSup	0.771192	0.176832	0.040547	0.009297	0.002132
OtherS	2.34E-06	5.93E-05	0.001501	0.037964	0.960474

Overall, the analysis of spending categories appears relatively fruitless, adding little to the overall analysis.